Chapter 24

1. **THINK** Ampere is the SI unit for current. An ampere is one coulomb per second.

EXPRESS To calculate the total charge through the circuit, we note that $1\ \text{A} = 1\,\text{C/s}$ and $1\ \text{h} = 3600\,\text{s}$.

ANALYZE (a) Thus,

$$84\ \text{A}\cdot\text{h} = \left(84\,\frac{\text{C}\cdot\text{h}}{\text{s}}\right)\left(3600\,\frac{\text{s}}{\text{h}}\right) = 3.0\times10^{5}\ \text{C}.$$

(b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0 \times 10^5\ \text{C})(12\ \text{V}) = 3.6 \times 10^6\ \text{J}$.

LEARN Potential difference is the change of potential energy per unit charge. Unlike electric field, potential difference is a scalar quantity.

2. The magnitude is $\Delta U = e\Delta V = 1.2 \times 10^9\ \text{eV} = 1.2\ \text{GeV}$.

3. (a) The change in energy of the transferred charge is

$$\Delta U = q\,\Delta V = (30\ \text{C})(1.0\times 10^9\ \text{V}) = 3.0 \times 10^{10}\ \text{J}.$$

(b) If all this energy is used to accelerate a 1000-kg car from rest, then $\Delta U = K = \frac{1}{2}mv^2$, and we find the car's final speed to be

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2(3.0\times10^{10}\ \text{J})}{1000\ \text{kg}}} = 7.7\times10^{3}\ \text{m/s}.$$

4. (a) $E = F/e = \left(3.9\times10^{-15}\ \text{N}\right)/\left(1.60\times10^{-19}\ \text{C}\right) = 2.4\times10^{4}\ \text{N/C} = 2.4\times10^{4}\,\text{V/m}.$

(b) $\Delta V = E\Delta s = \left(2.4\times10^{4}\ \text{N/C}\right)\left(0.12\,\text{m}\right) = 2.9\times10^{3}\ \text{V}.$

5. **THINK** The electric field produced by an infinite sheet of charge is normal to the sheet and is uniform.

EXPRESS The magnitude of the electric field produced by the infinite sheet of charge is $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by

$$\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x.$$

ANALYZE Thus, for $\sigma = 0.10\times10^{-6}\ \text{C/m}^2$ and $\Delta V = 50\ \text{V}$, we have

$$\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(50\ \text{V})}{0.10\times10^{-6}\ \text{C/m}^2} = 8.8\times10^{-3}\ \text{m}.$$

LEARN Equipotential surfaces are always perpendicular to the electric field lines. Figure 24-5(a) depicts the electric field lines and equipotential surfaces for a uniform electric field.

6. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -(3.94\times10^{-19}\ \text{J})/(-1.60\times10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (since C and B are on the same equipotential line).

7. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E}\cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E}\cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0\ \text{V}$.

8. (a) By Eq. 24-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E}\cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30\ \text{V}$.

(b) For any region within $0 < x < 3\text{m}$, $-\int \vec{E}\cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E} \cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields V_{max} = 40 V.

(c) In view of our result in part (b), we see that now (to find V = 0) we are looking for some X > 3 m such that the "area" from x = 3 m to x = X is 40 V. Using the formula for a triangle (3 < x < 4) and a rectangle (4 < x < X), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40.$$

Therefore, X = 5.5 m.

9. (a) The work done by the electric field is

$$W = \int_i^f q_0 \vec{E} \cdot d\vec{s} = \frac{q_0\sigma}{2\varepsilon_0} \int_0^d dz = \frac{q_0\sigma d}{2\varepsilon_0} = \frac{(1.60\times10^{-19}\,\text{C})(5.80\times10^{-12}\ \text{C/m}^2)(0.0356\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}$$
$$= 1.87\times10^{-21}\,\text{J}.$$

(b) Since

$$V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0,$$

with V_0 set to be zero on the sheet, the electric potential at P is

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80\times10^{-12}\ \text{C/m}^2)(0.0356\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = -1.17\times10^{-2}\ \text{V}.$$

10. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\text{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{\text{i}} = -(4.2\times10^3\ \text{N/C})\hat{\text{i}}\,.$$

In the "outside" region where x > 0.5 m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} = -(1.4\times10^3\ \text{N/C})\hat{\text{i}}.$$

Therefore, by Eq. 24-18, we have

$$\Delta V = -\int_0^{0.8} \vec{E} \cdot d\vec{s} = -\int_0^{0.5} \left|\vec{E}_{\text{in}}\right| dx - \int_{0.5}^{0.8} \left|\vec{E}_{\text{out}}\right| dx = -\left(4.2\times10^3\right)(0.5) - \left(1.4\times10^3\right)(0.3)$$
$$= 2.5\times10^3\ \text{V}.$$

11. (a) The potential as a function of *r* is

$$V(r)=V(0)-\int_0^r E(r)dr=0-\int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr=-\frac{qr^2}{8\pi\varepsilon_0 R^3}$$
$$=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\text{C})(0.0145\ \text{m})^2}{2(0.0231\ \text{m})^3}=-2.68\times10^{-4}\ \text{V}.$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have

$$V(R)=-\frac{q}{8\pi\varepsilon_0 R}=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\text{C})}{2(0.0231\ \text{m})}=-6.81\times10^{-4}\ \text{V}.$$

12. The charge is

$$q=4\pi\varepsilon_0 RV=\frac{(10\text{m})(-1.0\text{V})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}=-1.1\times10^{-9}\text{C}.$$

13. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR=\frac{(200\ \text{V})(0.15\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}=3.3\times10^{-9}\ \text{C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma=\frac{q}{4\pi R^2}=\frac{3.3\times10^{-9}\ \text{C}}{4\pi(0.15\ \text{m})^2}=1.2\times10^{-8}\ \text{C/m}^2.$$

14. (a) The potential difference is

$$V_A-V_B=\frac{q}{4\pi\varepsilon_0 r_A}-\frac{q}{4\pi\varepsilon_0 r_B}=\left(1.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{2.0\ \text{m}}-\frac{1}{1.0\ \text{m}}\right)$$
$$=-4.5\times10^3\ \text{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

15. **THINK** The electric potential for a spherically symmetric charge distribution falls off as $1/r$, where *r* is the radial distance from the center of the charge distribution.

EXPRESS The electric potential *V* at the surface of a drop of charge *q* and radius *R* is given by $V = q/4\pi\varepsilon_0 R$.

ANALYZE (a) With V = 500 V and $q = 30\times10^{-12}$ C, we find the radius to be

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(30\times10^{-12}\ \mathrm{C}\right)}{500\ \mathrm{V}} = 5.4\times10^{-4}\ \mathrm{m}.$$

(b) After the two drops combine to form one big drop, the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of the original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\ \mathrm{V}) \approx 790\ \mathrm{V}.$$

LEARN A positively charged configuration produces a positive electric potential, and a negatively charged configuration produces a negative electric potential. Adding more charge increases the electric potential.

16. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(6.00\times10^{-12}\mathrm{C})}{0.39\ \mathrm{m}}$$
$$= 2.21\ \mathrm{V}.$$

17. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d}\right] = \frac{q}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(5.00\times10^{-15}\mathrm{C})}{2(4.00\times10^{-2}\ \mathrm{m})}$$
$$= 5.62\times10^{-4}\ \mathrm{V}.$$

The zero of the electric potential was taken to be at infinity.

18. When the charge q_2 is infinitely far away, the potential at the origin is due only to the charge q_1 :

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 d} = 5.76\times10^{-7}\ \mathrm{V}.$$

Thus, $q_1/d = 6.41 \times 10^{-17}$ C/m. Next, we note that when q_2 is located at $x = 0.080$ m, the net potential vanishes ($V_1 + V_2 = 0$). Therefore,

$$0 = \frac{kq_2}{0.08\text{ m}} + \frac{kq_1}{d}$$

Thus, we find $q_2 = -(q_1/d)(0.08\text{ m}) = -5.13 \times 10^{-18}\text{ C} = -32\,e$.

19. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

20. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

21. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.47\times3.34\times10^{-30}\text{ C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\text{ m}\right)^2} = 1.63\times10^{-5}\text{ V}.$$

22. From Eq. 24-30 and Eq. 24-14, we have (for $\theta_i = 0^\circ$)

$$W_a = q\Delta V = e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{p\cos\theta_i}{4\pi\varepsilon_0 r^2}\right) = \frac{ep\cos\theta}{4\pi\varepsilon_0 r^2}(\cos\theta - 1)$$

with r = 20 × 10^{-9} m. For θ = 180º the graph indicates W_a = –4.0 × 10^{-30} J, from which we can determine p. The magnitude of the dipole moment is therefore 5.6×10^{-37} C·m.

23. (a) From Eq. 24-35, we find the potential to be

$$V = 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2 + \sqrt{(L^2/4) + d^2}}{d}\right]$$
$$= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.68\times10^{-12}\,\mathrm{C/m})\ln\left[\frac{(0.06\ \mathrm{m}/2) + \sqrt{(0.06\ \mathrm{m})^2/4 + (0.08\ \mathrm{m})^2}}{0.08\ \mathrm{m}}\right]$$
$$= 2.43\times10^{-2}\ \mathrm{V}.$$

(b) The potential at P is V = 0 due to superposition.

24. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(25.6\times10^{-12}\,\mathrm{C})}{3.71\times10^{-2}\ \mathrm{m}}$$
$$= -6.20\ \mathrm{V}.$$

We note that the result is exactly what one would expect for a point-charge –Q at a distance R. This “coincidence” is due, in part, to the fact that V is a scalar quantity.

25. (a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right) = -\frac{5Q_1}{4\pi\varepsilon_0 R} = -\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{8.20\times10^{-2}\ \mathrm{m}} = -2.30\ \mathrm{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2 + D^2}$, so the electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2 + D^2}} - \frac{6Q_1}{\sqrt{R^2 + D^2}}\right] = -\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2 + D^2}}$$
$$= -\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{\sqrt{(8.20\times10^{-2}\ \mathrm{m})^2 + (6.71\times10^{-2}\ \mathrm{m})^2}}$$
$$= -1.78\ \mathrm{V}.$$

26. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_o}\ln\left(\frac{L + \sqrt{L^2 + d^2}}{D + \sqrt{D^2 + d^2}}\right) = \frac{2.0\times10^{-6}}{4\pi\varepsilon_0}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 2.18 \times 10^4 \text{ V}.$$

27. Letting d denote 0.010 m, we have

$$V = \frac{Q_1}{4\pi\varepsilon_0 d} + \frac{3Q_1}{8\pi\varepsilon_0 d} - \frac{3Q_1}{16\pi\varepsilon_0 d} = \frac{Q_1}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(30\times10^{-9}\text{C})}{2(0.01 \text{ m})}$$
$$= 1.3\times10^4 \text{ V}.$$

28. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$
$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(56.1\times10^{-15}\text{C})}{0.12 \text{ m}}\ln\left(1+\frac{0.12 \text{ m}}{0.025 \text{ m}}\right)$$
$$= 7.39\times10^{-3} \text{ V}.$$

29. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 24-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_o}\frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R}$$
$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(7.21\times10^{-12}\text{C})}{2.00 \text{ m}}$$
$$= 3.24\times10^{-2} \text{ V}.$$

30. The dipole potential is given by Eq. 24-30 (with $\theta = 90°$ in this case)

$$V=\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}=\frac{p\cos 90°}{4\pi\varepsilon_0 r^2}=0$$

since cos(90°) = 0 . The potential due to the short arc is $q_1/4\pi\varepsilon_0 r_1$ and that caused by the long arc is $q_2/4\pi\varepsilon_0 r_2$. Since q_1 = +2 μC, r_1 = 4.0 cm, q_2 = –3 μC, and r_2 = 6.0 cm, the potentials of the arcs cancel. The result is zero.

31. **THINK** Since the disk is uniformly charged, when the full disk is present each quadrant contributes equally to the electric potential at *P*.

EXPRESS Electrical potential is a scalar quantity. The potential at *P* due to a single quadrant is one-fourth the potential due to the entire disk. We first find an expression for the potential at *P* due to the entire disk. To do so, consider a ring of charge with radius *r* and (infinitesimal) width *dr*. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is at a distance $\sqrt{r^2+D^2}$ from *P*, so the potential it produces at *P* is

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+D^2}}=\frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

ANALYZE Integrating over *r*, the total potential at *P* is

$$V=\frac{\sigma}{2\varepsilon_0}\int_0^R\frac{r dr}{\sqrt{r^2+D^2}}=\frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\Big|_0^R=\frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right].$$

Therefore, the potential V_{sq} at *P* due to a single quadrant is

$$V_{sq}=\frac{V}{4}=\frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right]=\frac{(7.73\times10^{-15}\,\text{C/m}^2)}{8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\text{ m})^2+(0.259\text{ m})^2}-0.259\text{ m}\right]$$
$$=4.71\times10^{-5}\ \text{V}.$$

LEARN Consider the limit $D\gg R.$ The potential becomes

$$V_{sq}=\frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right]\approx\frac{\sigma}{8\varepsilon_0}\left[D\left(1+\frac{1}{2}\frac{R^2}{D^2}+\cdots\right)-D\right]$$
$$=\frac{\sigma}{8\varepsilon_0}\frac{R^2}{2D}=\frac{\pi R^2\sigma/4}{4\pi\varepsilon_0 D}=\frac{q_{sq}}{4\pi\varepsilon_0 D}$$

where $q_{sq}=\pi R^2\sigma/4$ is the charge on the quadrant. In this limit, we see that the potential resembles that due to a point charge q_{sq}.

32. Equation 24-32 applies with $dq = \lambda\,dx = bx\,dx$ (along $0 \le x \le 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20}\frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36 \text{ V}.$$

(b) Now $r = \sqrt{x^2+d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20}\frac{bxdx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Big|_0^{0.20} = 18 \text{ V}.$$

33. Consider an infinitesimal segment of the rod, located between x and $x + dx$. It has length dx and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from P_1 is $d + x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at P_1, we integrate over the length of the rod and obtain

$$\begin{aligned}V &= \frac{c}{4\pi\varepsilon_0}\int_0^L\frac{xdx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right]\\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(28.9\times10^{-12}\text{C/m}^2)\left[0.120\text{ m} - (0.030\text{ m})\ln\left(1+\frac{0.120\text{ m}}{0.030\text{ m}}\right)\right]\\ &= 1.86\times10^{-2}\ \text{V}.\end{aligned}$$

34. The magnitude of the electric field is given by

$$|E| = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{2(5.0\text{V})}{0.015\text{m}} = 6.7\times10^2\ \text{V/m}.$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly toward the negatively charged one.

35. We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = -2(2.0\text{V}/\text{m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V}/\text{m}^2)x^2 - 3.0\text{V}/\text{m}^2)y^2\right) = 2(3.0\text{V}/\text{m}^2)y.$$

We evaluate at $x = 3.0$ m and $y = 2.0$ m to obtain

$$\vec{E} = (-12\ \text{V/m})\hat{\text{i}} + (12\ \text{V/m})\hat{\text{j}}.$$

36. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{\text{i}} = -\frac{d}{dx}(1500x^2)\hat{\text{i}} = (-3000x)\hat{\text{i}} = (-3000\,\text{V/m}^2)\,(0.0130\,\text{m})\hat{\text{i}}$$
$$= (-39\,\text{V/m})\hat{\text{i}}.$$

(a) Thus, the magnitude of the electric field is E = 39 V/m.

(b) The direction of $\vec{E}$ is $-\hat{\text{i}}$, or toward plate 1.

37. **THINK** The component of the electric field $\vec{E}$ in a given direction is the negative of the rate at which potential changes with distance in that direction.

EXPRESS With $V = 2.00xyz^2$, we apply Eq. 24-41 to calculate the *x*, *y*, and *z* components of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

which, at (x, y, z) = (3.00 m, –2.00 m, 4.00 m), gives

$$(E_x, E_y, E_z) = (64.0\ \text{V/m}, -96.0\ \text{V/m}, 96.0\ \text{V/m}).$$

ANALYZE The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0\ \text{V/m})^2 + (-96.0\ \text{V/m})^2 + (96.0\ \text{V/m})^2}$$
$$= 150\,\text{V/m} = 150\,\text{N/C}.$$

LEARN If the electric potential increases along some direction, say x, with $\partial V/\partial x > 0$, then there is a corresponding nonvanishing component of $\vec{E}$ in the opposite direction ($-E_x \neq 0$).

38. (a) From the result of Problem 24-28, the electric potential at a point with coordinate x is given by

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right).$$

At $x = d$ we obtain

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L}{d}\right) = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(43.6\times10^{-15}\mathrm{C})}{0.135\ \mathrm{m}}\ln\left(1+\frac{0.135\ \mathrm{m}}{d}\right)$$
$$= (2.90\times10^{-3}\ \mathrm{V})\ln\left(1+\frac{0.135\ \mathrm{m}}{d}\right).$$

(b) We differentiate the potential with respect to x to find the x component of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right) = -\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right) = -\frac{Q}{4\pi\varepsilon_0 x(x-L)}$$
$$= -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(43.6\times10^{-15}\mathrm{C})}{x(x+0.135\ \mathrm{m})} = -\frac{(3.92\times10^{-4}\ \mathrm{N\cdot m^2/C})}{x(x+0.135\ \mathrm{m})},$$

or

$$|E_x| = \frac{(3.92\times10^{-4}\ \mathrm{N\cdot m^2/C})}{x(x+0.135\ \mathrm{m})}.$$

(c) Since $E_x < 0$, its direction relative to the positive x axis is $180°$.

(d) At $x = d = 6.20$ cm, we obtain

$$|E_x| = \frac{(3.92\times10^{-4}\ \mathrm{N\cdot m^2/C})}{(0.0620\ \mathrm{m})(0.0620\ \mathrm{m}+0.135\ \mathrm{m})} = 0.0321\ \mathrm{N/C}.$$

(e) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus, the transverse component of the electric field E_y is zero.

39. The electric field (along some axis) is the (negative of the) derivative of the potential V with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{\partial V}{\partial x} = -\left(\frac{-500\ \mathrm{V}}{0.20\ \mathrm{m}}\right) = 2500\ \mathrm{V/m} = 2500\ \mathrm{N/C}$$
$$E_y = -\frac{\partial V}{\partial y} = -\left(\frac{300\ \mathrm{V}}{0.30\ \mathrm{m}}\right) = -1000\ \mathrm{V/m} = -1000\ \mathrm{N/C}.$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of –21.8° (with respect to the positive *x* axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of q has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180° to that of $\vec{E}$). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\,\text{C})[(2500\text{ N/C})\hat{\text{i}} - (1000\text{ N/C})\hat{\text{j}}] = (-4.0\times10^{-16}\,\text{N})\hat{\text{i}} + (1.60\times10^{-16}\text{ N})\hat{\text{j}}.$$

40. (a) Consider an infinitesimal segment of the rod from x to $x + dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus,

$$\begin{aligned} V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2}-y\right) \\ &= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\text{C/m}^2)\left(\sqrt{(0.100\text{ m})^2+(0.0356\text{ m})^2}-0.0356\text{ m}\right) \\ &= 3.16\times10^{-2}\text{ V}. \end{aligned}$$

(b) The y component of the field there is

$$\begin{aligned} E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2}-y\right) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) \\ &= (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\text{C/m}^2)\left(1-\frac{0.0356\text{ m}}{\sqrt{(0.100\text{ m})^2+(0.0356\text{ m})^2}}\right) \\ &= 0.298\text{ N/C}. \end{aligned}$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at (x, y). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

41. We apply conservation of energy for the particle with $q = 7.5 \times 10^{-6}$ C (which has zero initial kinetic energy):

$$U_0 = K_f + U_f,$$

where $U = \dfrac{q\,Q}{4\pi\varepsilon_0 r}$.

(a) The initial value of r is 0.60 m and the final value is (0.6 + 0.4) m = 1.0 m (since the particles repel each other). Conservation of energy, then, leads to K_f = 0.90 J.

(b) Now the particles attract each other so that the final value of r is 0.60 – 0.40 = 0.20 m. Use of energy conservation yields K_f = 4.5 J in this case.

42. (a) We use Eq. 24-43 with $q_1 = q_2 = -e$ and r = 2.00 nm:

$$U = k\frac{q_1 q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(1.60\times10^{-19}\,\mathrm{C})^2}{2.00\times10^{-9}\,\mathrm{m}} = 1.15\times10^{-19}\,\mathrm{J}.$$

(b) Since $U > 0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

43. **THINK** The work required to set up the arrangement is equal to the potential energy of the system.

EXPRESS We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = \frac{2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.30\times10^{-12}\,\mathrm{C})^2}{0.640\ \mathrm{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$
$$= -1.92\times10^{-13}\,\mathrm{J}.$$

LEARN The work done in assembling the system is negative. This means that an external agent would have to supply $W_{\text{ext}} = +1.92\times10^{-13}\,\mathrm{J}$ in order to take apart the arrangement completely.

44. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with r = 0.020 m)

$$W = \frac{(3e-2e+2e)(6e)}{4\pi\varepsilon_0 r} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(18)(1.60\times10^{-19}\,\mathrm{C})^2}{0.020\,\mathrm{m}} = 2.1\times10^{-25}\ \mathrm{J}.$$

45. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$,

and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for v is

$$v = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(3.1\times10^{-6}\,\mathrm{C})^2}{20\times10^{-6}\mathrm{kg}}\left(\frac{1}{0.90\times10^{-3}\mathrm{m}} - \frac{1}{2.5\times10^{-3}\mathrm{m}}\right)}$$
$$= 2.5\times10^3\ \mathrm{m/s}.$$

46. Let r = 1.5 m, x = 3.0 m, q_1 = –9.0 nC, and q_2 = –6.0 pC. The work done by an external agent is given by

$$W = \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + x^2}}\right)$$
$$= \left(-9.0\times10^{-9}\ \mathrm{C}\right)\left(-6.0\times10^{-12}\mathrm{C}\right)\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\cdot\left[\frac{1}{1.5\ \mathrm{m}} - \frac{1}{\sqrt{(1.5\ \mathrm{m})^2 + (3.0\ \mathrm{m})^2}}\right]$$
$$= 1.8\times10^{-10}\,\mathrm{J}.$$

47. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in Chapter 14). Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where $m = 9.11\times10^{-31}$ kg, $e = 1.60\times10^{-19}$ C, $q = 10000e$, and r = 0.010 m. This yields v = 22490 m/s $\approx 2.2\times10^4$ m/s .

48. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19}\mathrm{C})(125\ \mathrm{V})}{9.11\times10^{-31}\ \mathrm{kg}}} = 6.63\times10^6\ \mathrm{m/s}.$$

49. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial

kinetic energy is $K_i = \frac{1}{2}mv^2$, where *m* is the mass of an electron and *v* is the initial speed of the moving electron. The final kinetic energy is zero. Thus,

$$K_i = U_f \quad \Rightarrow \quad \frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d.$$

Hence,

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0 dm}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\ \text{C})^2}{(0.010\ \text{m})(9.11\times10^{-31}\ \text{kg})}} = 3.2\times10^2\ \text{m/s}.$$

50. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{q_2Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$

51. (a) Let $\ell = 0.15\,\text{m}$ be the length of the rectangle and $w = 0.050$ m be its width. Charge q_1 is a distance ℓ from point *A* and charge q_2 is a distance *w*, so the electric potential at *A* is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\ell} + \frac{q_2}{w}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.15\,\text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.050\ \text{m}}\right)$$
$$= 6.0\times10^4\,\text{V}.$$

(b) Charge q_1 is a distance *w* from point *b* and charge q_2 is a distance ℓ, so the electric potential at *B* is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{w} + \frac{q_2}{\ell}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.050\ \text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}\right)$$
$$= -7.8\times10^5\,\text{V}.$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at *A* and U_B is the potential energy when q_3 is at *B*, then the work done in moving the charge from *B* to *A* is

$$W = U_A - U_B = q_3(V_A - V_B) = (3.0 \times 10^{-6}\ \text{C})(6.0 \times 10^4\ \text{V} + 7.8 \times 10^5\ \text{V}) = 2.5\ \text{J}.$$

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

52. From Eq. 24-30 and Eq. 24-7, we have (for θ = 180º)

$$U = qV = -e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{ep}{4\pi\varepsilon_0 r^2}$$

where r = 0.020 m. Using energy conservation, we set this expression equal to 100 eV and solve for p. The magnitude of the dipole moment is therefore $p = 4.5 \times 10^{-12}\ \mathrm{C\cdot m}$.

53. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.0\times10^{-6}\ \mathrm{C}\right)^2}{1.00\ \mathrm{m}} = 0.225\ \mathrm{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.0\times10^{-6}\ \mathrm{C}\right)^2}{\left(1.00\ \mathrm{m}\right)^2} = 0.225\ \mathrm{N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\ \mathrm{N}}{5.0\times10^{-3}\ \mathrm{kg}} = 45.0\ \mathrm{m/s^2}$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\ \mathrm{N}}{10\times10^{-3}\ \mathrm{kg}} = 22.5\ \mathrm{m/s^2}.$$

(c) Energy is conserved. The initial potential energy is U = 0.225 J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B.$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A / m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain

$$U = \tfrac{1}{2}(m_A / m_B)(m_A + m_B)v_A^2.$$

Thus,

$$v_A = \sqrt{\frac{2Um_B}{m_A(m_A + m_B)}} = \sqrt{\frac{2(0.225 \text{ J})(10\times10^{-3} \text{ kg})}{(5.0\times10^{-3} \text{ kg})(5.0\times10^{-3} \text{ kg} + 10\times10^{-3} \text{ kg})}} = 7.75 \text{ m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0\times10^{-3} \text{ kg}}{10\times10^{-3} \text{ kg}}\right)(7.75 \text{ m/s}) = -3.87 \text{ m/s},$$

or $|v_B| = 3.87$ m/s.

54. (a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative x direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is 1.0×10^7 m/s.

55. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2}m_e v_i^2$, where $v_i = 3.2 \times 10^5$ m/s. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2}m_e v_i^2,$$

or

$$r = \frac{2e^2}{3(4\pi\varepsilon_0)m_e v_i^2} = \frac{2\left(1.6\times10^{-19} \text{ C}\right)^2\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)}{3\left(9.11\times10^{-19} \text{ kg}\right)\left(3.2\times10^5 \text{ m/s}\right)^2} = 1.6\times10^{-9}\text{m}.$$

56. When particle 3 is at $x = 0.10$ m, the total potential energy vanishes. Using Eq. 24-43, we have (with meters understood at the length unit)

$$0 = \frac{q_1 q_2}{4\pi\varepsilon_0 d} + \frac{q_1 q_3}{4\pi\varepsilon_0 (d + 0.10 \text{ m})} + \frac{q_3 q_2}{4\pi\varepsilon_0 (0.10 \text{ m})}$$

This leads to

$$q_3\left(\frac{q_1}{d+0.10\text{ m}}+\frac{q_2}{0.10\text{ m}}\right)=-\frac{q_1q_2}{d}$$

which yields $q_3 = -5.7\ \mu$C.

57. **THINK** Mechanical energy is conserved in the process.

EXPRESS The electric potential at (0, y) due to the two charges Q held fixed at $(\pm x, 0)$ is

$$V=\frac{2Q}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Thus, the potential energy of the particle of charge q at (0, y) is

$$U=qV=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Conservation of mechanical energy ($K_i + U_i = K_f + U_f$) gives

$$K_f=K_i+U_i-U_f=K_i+\frac{2Qq}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_i^2}}-\frac{1}{\sqrt{x^2+y_f^2}}\right),$$

where y_i and y_f are the initial and final coordinates of the moving charge along the y axis.

ANALYZE (a) With $q=-15\times10^{-6}$ C, $Q = 50 \times 10^{-6}$ C, $x = \pm3$ m, $y_i = 4$ m, and $y_f = 0$, we obtain

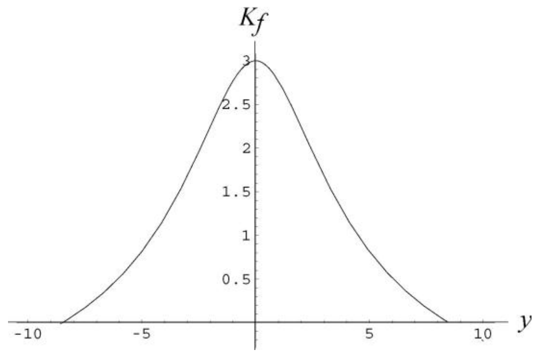
$$\begin{aligned}K_f &=1.2\text{ J}+\frac{2(50\times10^{-6}\text{ C})(-15\times10^{-6}\text{ C})}{4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{1}{\sqrt{(3.0\text{ m})^2+(4.0\text{ m})^2}}-\frac{1}{\sqrt{(3.0\text{ m})^2}}\right)\\ &=3.0\text{ J}.\end{aligned}$$

(b) We set $K_f = 0$ and solve for y_f (choosing the negative root, as indicated in the problem statement):

$$K_i+U_i=U_f\quad\Rightarrow\quad 1.2\text{ J}+\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_i^2}}=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_f^2}}\ .$$

Substituting the values given, we have $U_i=-2.7$ J, and $y_f = -8.5$ m.

LEARN The dependence of the final kinetic energy of the particle on y is plotted below. From the plot, we see that $K_f = 3.0$ J at $y = 0$, and $K_f = 0$ at $y = -8.5$ m. The particle oscillates between the two end-points $y_f = \pm 8.5$ m.



58. (a) When the proton is released, its energy is $K + U = 4.0$ eV + 3.0 eV (the latter value is inferred from the graph). This implies that if we draw a horizontal line at the 7.0 volt "height" in the graph and find where it intersects the voltage plot, then we can determine the turning point. Interpolating in the region between 1.0 cm and 3.0 cm, we find the turning point is at roughly $x = 1.7$ cm.

(b) There is no turning point toward the right, so the speed there is nonzero, and is given by energy conservation:

$$v = \sqrt{\frac{2(7.0\text{ eV})}{m}} = \sqrt{\frac{2(7.0\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})}{1.67 \times 10^{-27}\text{ kg}}} = 20\text{ km/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the proton follows immediately from $\vec{F} = q\vec{E}$, where $q = +e$ for the proton. In the region just to the left of $x = 3.0$ cm, the field is $\vec{E} = (+300\text{ V/m})\hat{\text{i}}$ and the force is $F = +4.8 \times 10^{-17}$ N.

(d) The force $\vec{F}$ points in the $+x$ direction, as the electric field $\vec{E}$.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = (-200\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 3.2 \times 10^{-17}$ N.

(f) The force $\vec{F}$ points in the $-x$ direction, as the electric field $\vec{E}$.

59. (a) The electric field between the plates is leftward in Fig, 24-59 since it points toward lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2} m_p v_0^2 + qV_1 = \frac{1}{2} m_p v^2 + qV_2 \; .$$

Using q = +1.6 × 10^{-19} C, m_p = 1.67 × 10^{-27} kg, v_0 = 90 × 10^3 m/s, V_1 = –70 V, and $V_2 = -50$ V, we obtain the final speed v = 6.53 × 10^4 m/s. We note that the value of d is not used in the solution.

60. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +2.16 \times 10^{-13}\ \text{J} .$$

With R = 0.0800 m, we find Q = –1.20 × 10^{-5} C.

(b) The work is the same, so the increase in the potential energy is ΔU = + 2.16 × 10^{-13} J.

61. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where N = odd and N = even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$U_{1,N=\text{even}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right), \quad U_{1,N=\text{odd}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right)$$

where $\theta = \dfrac{2\pi}{N}$. For configuration 2, we find

$$U_{2,N=\text{even}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)} + 2\right), \; U_{2,N=\text{odd}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right)$$

where $\theta' = \dfrac{2\pi}{N-1}$. The results are all of the form

$$U_{1\text{or}2}\,\frac{ke^2}{2R} \times \text{a pure number.}$$

In our table below we have the results for those “pure numbers” as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) $N = 12$ is the smallest value such that $U_2 < U_1$.

(b) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two — one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

62. (a) Since the two conductors are connected V_1 and V_2 must be equal to each other.

Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$, or

(b) $q_1/q = 1/3 = 0.333$.

(c) Similarly, $q_2/q = 2/3 = 0.667$.

(d) The ratio of surface charge densities is $\dfrac{\sigma_1}{\sigma_2} = \dfrac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\dfrac{q_1}{q_2}\right)\left(\dfrac{R_2}{R_1}\right)^2 = 2.00.$

63. **THINK** The electric potential is the sum of the contributions of the individual spheres.

EXPRESS Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere.

For parts (b) and (c), we note that the distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere as well as the charge on the other sphere.

ANALYZE (a) The potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0\, d/2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.0\times10^{-8}\,\mathrm{C} - 3.0\times10^{-8}\,\mathrm{C}\right)}{1.0\ \mathrm{m}} = -1.8\times10^2\,\mathrm{V}.$$

(b) The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{1.0\times10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}} - \frac{3.0\times10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}}\right] = 2.9\times10^3\,\mathrm{V}.$$

(c) Similarly, the potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{1.0\times10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}} - \frac{3.0\times10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}}\right] = -8.9\times10^3\,\mathrm{V}.$$

LEARN In the limit where $d \to \infty$, the spheres are isolated from each other and the electric potentials at the surface of each individual sphere become

$$V_{10} = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.0\times10^{-8}\,\mathrm{C})}{0.030\,\mathrm{m}} = 3.0\times10^3\,\mathrm{V},$$

and

$$V_{20} = \frac{q_2}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-3.0\times10^{-8}\,\mathrm{C})}{0.030\,\mathrm{m}} = -8.99\times10^3\,\mathrm{V}.$$

64. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also +400 V.

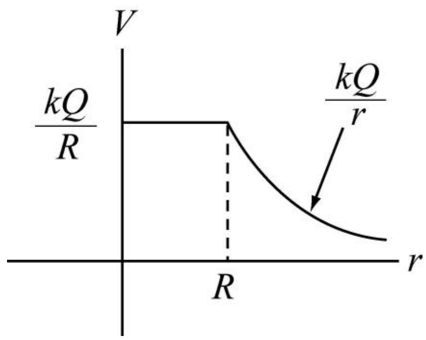
65. **THINK** If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = Q/4\pi\varepsilon_0 R$, where Q is the charge on the sphere and R is its radius.

EXPRESS From $V = Q/4\pi\varepsilon_0 R$, we find the charge to be $Q = 4\pi\varepsilon_0 RV$.

ANALYZE With $R = 0.15$ m and $V = 1500$ V, we have

$$Q = 4\pi\varepsilon_0 RV = \frac{(0.15\ \mathrm{m})(1500\ \mathrm{V})}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = 2.5\times10^{-8}\,\mathrm{C}.$$

LEARN A plot of the electric potential as a function of r is shown to the right with $k = 1/4\pi\varepsilon_0$. Note that the potential is constant inside the conducting sphere.



66. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\rm enc}}{r},$$

where $q_{\rm enc}$ is the charge enclosed in a sphere of radius r centered at the origin.

(a) For r = 4.00 m, R_2 = 1.00 m, and R_1 = 0.500 m, with $r > R_2 > R_1$ we have

$$E(r) = \frac{q_1+q_2}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\,\mathrm{C}+1.00\times10^{-6}\,\mathrm{C})}{(4.00\ \mathrm{m})^2} = 1.69\times10^3\ \mathrm{V/m}.$$

(b) For $R_2 > r$ = 0.700 m $> R_2$,

$$E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\,\mathrm{C})}{(0.700\ \mathrm{m})^2} = 3.67\times10^4\ \mathrm{V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, E = 0.

The electric potential may be obtained using Eq. 24-18:

$$V(r) - V(r') = \int_r^{r'} E(r)\,dr.$$

(d) For r = 4.00 m $> R_2 > R_1$, we have

$$V(r) = \frac{q_1+q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\,\mathrm{C}+1.00\times10^{-6}\,\mathrm{C})}{(4.00\ \mathrm{m})} = 6.74\times10^3\ \mathrm{V}.$$

(e) For r = 1.00 m = $R_2 > R_1$, we have

$$V(r) = \frac{q_1+q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\,\mathrm{C}+1.00\times10^{-6}\,\mathrm{C})}{(1.00\ \mathrm{m})} = 2.70\times10^4\ \mathrm{V}.$$

(f) For $R_2 > r$ = 0.700 m $> R_2$,

$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.700\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=3.47\times10^4\ \text{V}.$$

(g) For $R_2 > r = 0.500$ m $= R_2$,

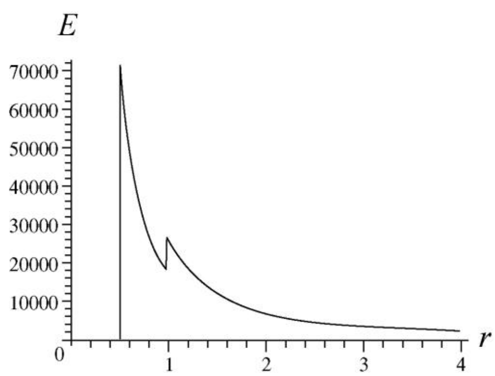
$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.500\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=4.50\times10^4\ \text{V}.$$

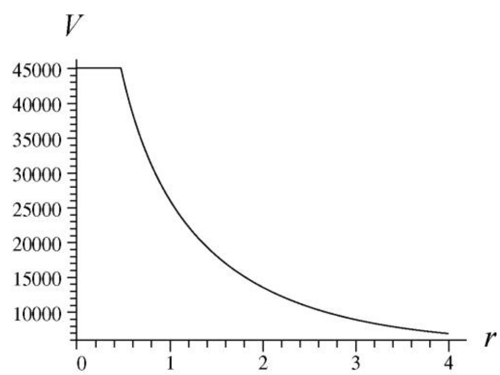
(h) For $R_2 > R_1 > r$,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.500\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=4.50\times10^4\ \text{V}.$$

(i) At $r = 0$, the potential remains constant, $V=4.50\times10^4$ V.

(j) The electric field and the potential as a function of r are depicted below:





67. (a) The magnitude of the electric field is

$$E=\frac{\sigma}{\varepsilon_0}=\frac{q}{4\pi\varepsilon_0R^2}=\frac{(3.0\times10^{-8}\text{C})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{(0.15\,\text{m})^2}=1.2\times10^4\ \text{N/C}.$$

(b) $V = RE$ = (0.15 m)(1.2×10^4 N/C) = 1.8×10^3 V.

(c) Let the distance be x. Then

$$\Delta V=V(x)-V=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x}-\frac{1}{R}\right)=-500\,\text{V},$$

which gives

$$x=\frac{R\Delta V}{-V-\Delta V}=\frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V}+500\,\text{V}}=5.8\times10^{-2}\,\text{m}.$$

68. The potential energy of the two-charge system is

$$U=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-6}\,\text{C}\right)\left(-4.00\times10^{-6}\,\text{C}\right)}{\sqrt{(3.50+2.00)^2+(0.500-1.50)^2}\,\text{cm}}$$
$$=-1.93\text{ J}.$$

Thus, –1.93 J of work is needed.

69. **THINK** To calculate the potential, we first apply Gauss' law to calculate the electric field of the charged cylinder of radius *R*. The Gaussian surface is a cylindrical surface that is concentric with the cylinder.

EXPRESS We imagine a cylindrical Gaussian surface *A* of radius *r* and length *h* concentric with the cylinder. Then, by Gauss' law,

$$\oint_A \vec{E}\cdot d\vec{A}=2\pi rhE=\frac{q_{\text{enc}}}{\varepsilon_0},$$

where q_{enc} is the amount of charge enclosed by the Gaussian cylinder. Inside the charged cylinder ($r<R$), $q_{\text{enc}}=0$, so the electric field is zero. On the other hand, outside the cylinder ($r>R$), $q_{\text{enc}}=\lambda h$ so the magnitude of the electric field is

$$E=\frac{q/h}{2\pi\varepsilon_0 r}=\frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ is the linear charge density and *r* is the distance from the line to the point where the field is measured. The potential difference between two points 1 and 2 is

$$V(r_2)-V(r_1)=-\int_{r_1}^{r_2}E(r)\,dr.$$

ANALYZE (a) The radius of the cylinder (0.020 m, the same as R_B) is denoted *R*, and the field magnitude there (160 N/C) is denoted E_B. From the equation above, we see that the electric field beyond the surface of the cylinder is inversely proportional with *r*:

$$E=E_B\frac{R_B}{r},\quad r\ge R_B.$$

Thus, if $r = R_C = 0.050$ m, we obtain

$$E_C = E_B \frac{R_B}{R_C} = (160\text{ N/C})\left(\frac{0.020\text{ m}}{0.050\text{ m}}\right) = 64\,\text{N/C}.$$

(b) The potential difference between V_B and V_C is

$$V_B - V_C = -\int_{R_C}^{R_B} \frac{E_B R_B}{r}\,dr = E_B R_B \ln\left(\frac{R_C}{R_B}\right) = (160\text{ N/C})(0.020\text{ m})\ln\left(\frac{0.050\text{ m}}{0.020\text{ m}}\right)$$
$$= 2.9\,\text{V}.$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

LEARN The electric potential at a distance $r > R_B$ can be written as

$$V(r) = V_B - E_B R_B \ln\left(\frac{r}{R_B}\right).$$

We see that $V(r)$ decreases logarithmically with r.

70. (a) We use Eq. 24-18 to find the potential: $V_{\text{wall}} - V = -\int_r^R E\,dr$, or

$$0 - V = -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \quad \Rightarrow \quad -V = -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right).$$

Consequently, $V = \rho(R^2 - r^2)/4\varepsilon_0$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1\times10^{-3}\,\text{C/m}^3}{4\left(8.85\times10^{-12}\,\text{C/V}\cdot\text{m}\right)}\left((0.05\,\text{m})^2 - 0\right) = -7.8\times10^4\,\text{V}.$$

Thus, the difference is $|V_{\text{center}}| = 7.8\times10^4\,\text{V}$.

71. **THINK** The component of the electric field $\vec{E}$ in any direction is the negative of the rate at which potential changes with distance in that direction.

EXPRESS From Eq. 24-30, the electric potential of a dipole at a point a distance r away is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

where p is the magnitude of the dipole moment $\vec{p}$ and θ is the angle between $\vec{p}$ and the position vector of the point. The potential at infinity is taken to be zero.

ANALYZE On the dipole axis $\theta = 0$ or π, so $|\cos\,\theta| = 1$. Therefore, magnitude of the electric field is

$$\left|E(r)\right| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

LEARN Take the z axis to be the dipole axis. For $r = z > 0\,(\theta = 0)$, $E = p/2\pi\varepsilon_0 z^3$. On the other hand, for $r = -z < 0\ \ (\theta = \pi)$, $E = -p/2\pi\varepsilon_0 z^3$.

72. Using Eq. 24-18, we have

$$\Delta V\ =\ -\int_2^3 \frac{A}{r^4}\ dr\ =\ \frac{A}{3}\left(\frac{1}{2^3} - \frac{1}{3^3}\right) = A(0.029/\text{m}^3).$$

73. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.10\,\text{m}} = 3.6\times10^5\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6\times10^5\ \text{V}}{0.10\,\text{m}} = 3.6\times10^6\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

74. The work done is equal to the change in the (total) electric potential energy U of the system, where

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_3 q_2}{4\pi\varepsilon_0 r_{23}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}}$$

and the notation r_{13} indicates the distance between q_1 and q_3 (similar definitions apply to r_{12} and r_{23}).

(a) We consider the difference in U where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ (r_{13} doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in U where initially $r_{23} = a$ and $r_{13} = a$, and finally r_{23} is again equal to a and r_{13} is also again equal to a (and of course, r_{12} doesn't change in this case). Thus, we obtain $\Delta U = 0$.

75. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37 \times 10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so

$$V = ER = (100 \text{ V/m})\,(6.37 \times 10^6 \text{ m}) = 6.4 \times 10^8 \text{ V}.$$

76. Using Gauss' law, $q = \varepsilon_0\Phi = +495.8$ nC. Consequently,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.958\times10^{-7}\text{ C})}{0.120 \text{ m}} = 3.71\times10^4 \text{ V}.$$

77. The potential difference is

$$\Delta V = E\Delta s = (1.92 \times 10^5 \text{ N/C})(0.0150 \text{ m}) = 2.90 \times 10^3 \text{ V}.$$

78. The charges are equidistant from the point where we are evaluating the potential — which is computed using Eq. 24-27 (or its integral equivalent). Equation 24-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{R}$$
$$= \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.52\times10^{-12}\text{ C})}{0.0850 \text{ m}} = 0.956 \text{ V}.$$

79. The electric potential energy in the presence of the dipole is

$$U = qV_{\text{dipole}} = \frac{qp\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{(-e)(ed)\cos\theta}{4\pi\varepsilon_0 r^2} .$$

Noting that $\theta_i = \theta_f = 0°$, conservation of energy leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 m d}\left(\frac{1}{25} - \frac{1}{49}\right)} = 7.0 \times 10^5 \text{ m/s} .$$

80. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 24-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)+\frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2+r^2}-z\right).$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = 0.200R$ and $z = 2.00R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = \frac{(6.20\times10^{-12}\,\text{C/m}^2)(0.130\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right)$$
$$=1.03\times10^{-2}\text{ V}.$$

81. (a) When the electron is released, its energy is

$$K + U = 3.0\text{ eV} - 6.0\text{ eV}$$

(the latter value is inferred from the graph along with the fact that $U = qV$ and $q = -e$). Because of the minus sign (of the charge) it is convenient to imagine the graph multiplied by a minus sign so that it represents potential energy in eV. Thus, the 2 V value shown at $x = 0$ would become –2 eV, and the 6 V value at $x = 4.5$ cm becomes –6 eV, and so on. The total energy (– 3.0 eV) is constant and can then be represented on our (imagined) graph as a horizontal line at – 3.0 V. This intersects the potential energy plot at a point we recognize as the turning point. Interpolating in the region between 1.0 cm and 4.0 cm, we find the turning point is at $x = 1.75\text{ cm} \approx 1.8\text{ cm}$.

(b) There is no turning point toward the right, so the speed there is nonzero. Noting that the kinetic energy at $x = 7.0$ cm is

$$K = -3.0\text{ eV} - (-5.0\text{ eV}) = 2.0\text{ eV},$$

we find the speed using energy conservation:

$$v=\sqrt{\frac{2K}{m_e}}=\sqrt{\frac{2\left(2.0\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{9.11\times10^{-31}\text{ kg}}}=8.4\times10^{5}\text{ m/s}.$$

(c) The electric field at any point P is the (negative of the) slope of the voltage graph evaluated at P. Once we know the electric field, the force on the electron follows immediately from $\vec{F}=q\vec{E}$, where $q = -e$ for the electron. In the region just to the left of $x=4.0\text{ cm}$, the electric field is $\vec{E}=(-133\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F=2.1\times10^{-17}\text{ N}$.

(d) The force points in the $+x$ direction.

(e) In the region just to the right of x = 5.0 cm, the field is $\vec{E}$ = +100 V/m $\hat{\text{i}}$ and the force is $\vec{F}$ = (–1.6 x 10^{-17} N) $\hat{\text{i}}$. Thus, the magnitude of the force is $F = 1.6\times10^{-17}$ N.

(f) The minus sign indicates that $\vec{F}$ points in the $-x$ direction.

82. (a) The potential would be

$$\begin{aligned}V_e &= \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e\sigma_e k\\ &= 4\pi\left(6.37\times10^6\text{ m}\right)\left(1.0\,\text{electron/m}^2\right)\left(-1.6\times10^{-9}\text{ C/electron}\right)\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\\ &= -0.12\,\text{V}.\end{aligned}$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\text{V}}{6.37\times10^6\,\text{m}} = -1.8\times10^{-8}\text{ N/C},$$

or $|E| = 1.8\times10^{-8}$ N/C.

(c) The minus sign in E indicates that $\vec{E}$ is radially inward.

83. (a) Using d = 2 m, we find the potential at P:

$$\begin{aligned}V_P &= \frac{2e}{4\pi\varepsilon_0 d} + \frac{-2e}{4\pi\varepsilon_0(2d)} = \frac{e}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})}{2.00\text{ m}}\\ &= 7.192\times10^{-10}\text{ V}.\end{aligned}$$

Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since $U = qV$, then the movable particle's contribution of the potential energy when it is at $r = \infty$ is zero, and its contribution to U_{system} when it is at P is

$$U = qV_P = 2(1.6\times10^{-19}\text{C})(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

Thus, the work done is approximately equal to $W_{\text{app}} = 2.30\times10^{-28}$ J.

(c) Now, combining the contribution to U_{system} from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{(4.00\text{ m})^2+(2.00\text{ m})^2}} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\text{C})^2}{\sqrt{20.0}\text{ m}}$$
$$= -2.058\times10^{-28}\text{ J}$$

we obtain

$$U_{\text{system}} = W_{\text{app}} + U_{\text{fixed}} = 2.43\times10^{-29}\text{ J}.$$

84. The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged sphere:

$$V_A = V_S = \frac{q}{4\pi\varepsilon_0 R}$$

where $q = 30\times10^{-9}$ C and $R = 0.030$ m. For points beyond the surface of the sphere, the potential follows Eq. 24-26:

$$V_B = \frac{q}{4\pi\varepsilon_0 r}$$

where $r = 0.050$ m.

(a) We see that

$$V_S - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

(b) Similarly,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

85. We note that the net potential (due to the "fixed" charges) is zero at the first location ("at ∞") being considered for the movable charge q (where $q = +2e$). Thus, with $D = 4.00$ m and $e = 1.60\times10^{-19}$ C, we obtain

$$V = \frac{+2e}{4\pi\varepsilon_0(2D)} + \frac{+e}{4\pi\varepsilon_0 D} = \frac{2e}{4\pi\varepsilon_0 D} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{C})}{4.00\text{ m}}$$
$$= 7.192\times10^{-10}\text{ V} \; .$$

The work required is equal to the potential energy in the final configuration:

$$W_{\text{app}} = qV = (2e)(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

86. Since the electric potential is a scalar quantity, this calculation is far simpler than it would be for the electric field. We are able to simply take half the contribution that

would be obtained from a complete (whole) sphere. If it were a whole sphere (of the same density) then its charge would be q_{whole} = 8.00 μC. Then

$$V = \frac{1}{2} V_{\text{whole}} = \frac{1}{2}\frac{q_{\text{whole}}}{4\pi\varepsilon_o r} = \frac{1}{2}\frac{8.00 \times 10^{-6}\,\text{C}}{4\pi\varepsilon_o(0.15\text{ m})} = 2.40 \times 10^5 \text{ V} .$$

87. **THINK** The work done is equal to the change in potential energy.

EXPRESS The initial potential energy of the system is

$$U_i = \frac{2q^2}{4\pi\varepsilon_0 L} + U_0$$

where q is the charge on each particle, L is the length of the triangle side, and U_0 is the potential energy associated with the interaction of the two fixed charges. After moving to the midpoint of the line joining the two fixed charges, the final energy of the configuration is

$$U_f = \frac{2q^2}{4\pi\varepsilon_0 (L/2)} + U_0 .$$

Thus, the work done by the external agent is

$$W = \Delta U = U_f - U_i = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{2}{L} - \frac{1}{L}\right) = \frac{2q^2}{4\pi\varepsilon_0 L}.$$

ANALYZE Substituting the values given, we have

$$W = \frac{2q^2}{4\pi\varepsilon_0 L} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.12\text{ C})^2}{1.7\text{ m}} = 1.5\times10^8\ \text{J}.$$

At a rate of P = 0.83 × 10^3 joules per second, it would take W/P = 1.8 × 10^5 seconds or about 2.1 days to do this amount of work.

LEARN Since all three particles are positively charged, positive work is required by the external agent in order to bring them closer.

88. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance

$$r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2} .$$

The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V = \frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\mathrm{C}\right)}{0.020\ \mathrm{m}}$$
$$= 2.5\times10^6\,\mathrm{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0\times10^{-6}\,\mathrm{C}\right)\left(2.54\times10^6\,\mathrm{V}\right) = 5.1\ \mathrm{J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U = W + \frac{q^2}{4\pi\varepsilon_0 d} = 5.1\ \mathrm{J} + \frac{\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(2.0\times10^{-6}\,\mathrm{C}\right)^2}{0.020\,\mathrm{m}} = 6.9\ \mathrm{J}.$$

89. The net potential at point P (the place where we are to place the third electron) due to the fixed charges is computed using Eq. 24-27 (which assumes $V \to 0$ as $r \to \infty$):

$$V_P = \frac{-e}{4\pi\varepsilon_0 d} + \frac{-e}{4\pi\varepsilon_0 d} = -\frac{2e}{4\pi\varepsilon_0 d}.$$

Thus, with $d = 2.00 \times 10^{-6}$ m and $e = 1.60 \times 10^{-19}$ C, we find

$$V_P = -\frac{2e}{4\pi\varepsilon_0 d} = -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(1.60\times10^{-19}\mathrm{C})}{2.00\times10^{-6}\ \mathrm{m}} = -1.438\times10^{-3}\ \mathrm{V}\ .$$

Then the required "applied" work is, by Eq. 24-14,

$$W_{\mathrm{app}} = (-e)\,V_P = 2.30 \times 10^{-22}\ \mathrm{J}.$$

90. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. The charge Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Q_q}{4\pi\varepsilon_0 r^2}=\frac{mv^2}{r} \quad\Rightarrow\quad mv^2=\frac{Qq}{4\pi\varepsilon_0 r},$$

and the kinetic energy is

$$K=\frac{1}{2}mv^2=\frac{Qq}{8\pi\varepsilon_0 r}.$$

The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E=K+U=\frac{Qq}{8\pi\varepsilon_0 r}-\frac{Qq}{4\pi\varepsilon_0 r}=-\frac{Qq}{8\pi\varepsilon_0 r}.$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W=E_2-E_1=-\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

91. The initial speed v_i of the electron satisfies

$$K_i=\tfrac{1}{2}m_e v_i^2=e\Delta V,$$

which gives

$$v_i=\sqrt{\frac{2e\Delta V}{m_e}}=\sqrt{\frac{2\left(1.60\times10^{-19}\text{ J}\right)\left(625\text{ V}\right)}{9.11\times10^{-31}\text{ kg}}}=1.48\times10^{7}\text{ m/s}.$$

92. The net electric potential at point P is the sum of those due to the six charges:

$$V_P=\sum_{i=1}^{6}V_{Pi}=\sum_{i=1}^{6}\frac{q_i}{4\pi\varepsilon_0 r_i}=\frac{10^{-15}}{4\pi\varepsilon_0}\left[\frac{5.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{-3.00}{\sqrt{d^2+(d/2)^2}}\right.$$
$$\left.+\frac{3.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{+5.00}{\sqrt{d^2+(d/2)^2}}\right]=\frac{9.4\times10^{-16}}{4\pi\varepsilon_0(2.54\times10^{-2})}$$
$$=3.34\times10^{-4}\text{ V}.$$

93. **THINK** To calculate the potential at point B due to the charged ring, we note that all points on the ring are at the same distance from B.

EXPRESS Let point B be at $(0, 0, z)$. The electric potential at B is given by

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

where q is the charge on the ring. The potential at infinity is taken to be zero.

ANALYZE With $q = 16 \times 10^{-6}$ C, $z = 0.040$ m, and $R = 0.0300$ m, we find the potential difference between points A (located at the origin) and B to be

$$\begin{aligned} V_B - V_A &= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{R}\right) \\ &= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(16.0\times10^{-6}\ \mathrm{C})\left(\frac{1}{\sqrt{(0.030\ \mathrm{m})^2+(0.040\ \mathrm{m})^2}} - \frac{1}{0.030\ \mathrm{m}}\right) \\ &= -1.92\times10^6\ \mathrm{V}. \end{aligned}$$

LEARN In the limit $z \gg R$, the potential approaches its "point-charge" limit:

$$V \approx \frac{q}{4\pi\varepsilon_0 z}.$$

94. (a) Using Eq. 24-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.50\times10^{-8}\mathrm{C})}{30\ \mathrm{V}} = 4.5\,\mathrm{m}.$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

95. **THINK** To calculate the electric potential, we first apply Gauss' law to calculate the electric field of the spherical shell. The Gaussian surface is a sphere that is concentric with the shell.

EXPRESS At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so the flux through the surface is given by $\Phi = \oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = q_{\text{enc}}/\varepsilon_0,$ where r is the radius of the Gaussian surface and q_{enc} is the charge enclosed. (i) In the region $r < r_1,$ the enclosed charge is $q_{\text{enc}} = 0$ and therefore,

$E = 0$. (ii) In the region $r_1 < r < r_2$, the volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)},$$

where Q is the total charge on the spherical shell. Thus, the charge enclosed by the Gaussian surface is

$$q_{\text{enc}} = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right).$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

(iii) In the region $r > r_2$, the charge enclosed is $q_{\text{enc}} = Q$, and the electric field is like that of a point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

ANALYZE (a) For $r > r_2$ the field is like that of a point charge, and so is the potential:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where the potential was taken to be zero at infinity.

(b) In the region $r_1 < r < r_2$, we have

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$V = V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr$$
$$= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right).$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V=\frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3-r_1^3}\left(\frac{3r_2^2}{2}-\frac{r^2}{2}-\frac{r_1^3}{r}\right).$$

Since $\rho=3Q/4\pi\left(r_2^3-r_1^3\right)$ this can also be written as

$$V(r)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r^2}{2}-\frac{r_1^3}{r}\right).$$

(c) For $r<r_1$, the electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V=\frac{Q}{4\pi\varepsilon_0}\frac{3\left(r_2^2-r_1^2\right)}{2\left(r_2^3-r_1^3\right)},$$

or in terms of the charge density $V=\dfrac{\rho}{2\varepsilon_0}\left(r_2^2-r_1^2\right).$

(d) Using the expression for $V(r)$ found in (b), we have

$$V(r_1)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r_1^2}{2}-\frac{r_1^3}{r_1}\right)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{3r_1^2}{2}\right)=\frac{\rho}{2\varepsilon_0}\left(r_2^2-r_1^2\right)$$

and

$$V(r_2)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r_2^2}{2}-\frac{r_1^3}{r_2}\right)=\frac{\rho}{3\varepsilon_0}\left(r_2^2-\frac{r_1^3}{r_2}\right)=\frac{\rho}{3\varepsilon_0 r_2}\left(r_2^3-r_1^3\right)=\frac{3Q/4\pi}{3\varepsilon_0 r_2}=\frac{Q}{4\pi\varepsilon_0 r_2}\ .$$

So the solutions agree at $r = r_1$ and at $r = r_2$.

LEARN Electric potential must be continuous at the boundaries at $r = r_1$ and $r = r_2$. In the region where the electric field is zero, no work is required to move the charge around. Thus, there’s no change in potential energy and the electric potential is constant.

96. (a) We use Gauss’ law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside.

Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3} \quad \Rightarrow \quad E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R}.$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}\left(3R^2 - r^2\right).$$

(b) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R},$$

or $|\Delta V| = q/8\pi\varepsilon_0 R$.

97. **THINK** The increase in electric potential at the surface of the copper sphere is proportional to the increase in electric charge.

EXPRESS The electric potential at the surface of a sphere of radius R is given by $V = q/4\pi\varepsilon_0 R,$ where q is the charge on the sphere. Thus, $q = 4\pi\varepsilon_0 RV.$ The number of electrons entering the copper sphere is $N = q/e,$ but this must be equal to $(\lambda/2)t,$ where λ is the decay rate of the nickel.

ANALYZE (a) With $R = 0.010$ m, when $V = 1000$ V, the net charge on the sphere is

$$q = 4\pi\varepsilon_0 RV = \frac{(0.010\text{ m})(1000\text{ V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-9}\ \text{C}.$$

Dividing q by e yields

$$N = (1.11\times10^{-9}\ \text{C})/(1.6\times10^{-19}\ \text{C}) = 6.95\times10^{9}$$

electrons that entered the copper sphere. So the time required is

$$t = \frac{N}{\lambda/2} = \frac{6.95\times10^{9}}{(3.7\times10^{8}\,/\text{s})/2} = 38\ \text{s}\,.$$

(b) The energy deposited by each electron that enters the sphere is $E_0 = 100$ keV = 1.6 × 10^{-14} J. Using the given heat capacity, we note that a temperature increase of ΔT = 5.0 K = 5.0 ºC required

$$E = C\Delta T = (14\ \text{J/K})(5.0\ \text{K}) = 70\ \text{J}$$

of energy. Dividing this by E_0 gives the number of electrons needed to enter the sphere (in order to achieve that temperature change):

$$N' = \frac{E}{E_0} = \frac{70\ \text{J}}{1.6\times10^{-14}\ \text{J}} = 4.375\times10^{15}$$

Thus, the time needed is

$$t' = \frac{N'}{\lambda/2} = \frac{4.375\times10^{15}}{(3.7\times10^{8}\,/\text{s})/2} = 2.36\times10^{7}\ \text{s}$$

or roughly 270 days.

LEARN As more electrons get into copper, more energy is deposited, and the copper sample gets hotter.

98. (a) The potential difference is

$$\begin{aligned}\Delta V &= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{15\times10^{-6}\ \text{C}}{0.060\ \text{m}} - \frac{5.0\times10^{-6}\ \text{C}}{0.030\ \text{m}}\right)\\ &= 7.49\times10^{5}\ \text{V}.\end{aligned}$$

(b) By connecting the two metal spheres with a wire, we now have one conductor, and any excess charge must reside on the surface of the conductor. Therefore, the charge on the small sphere is zero.

(c) Since all the charges reside on the surface of the large sphere, we have

$$Q' = Q + q = 15.0\ \mu\text{C} + 5.00\ \mu\text{C} = 20.0\ \mu\text{C}.$$

99. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

100. The distance r being looked for is that where the alpha particle has (momentarily) zero kinetic energy. Thus, energy conservation leads to

$$K_0 + U_0 = K + U \;\Rightarrow\; (0.48\times10^{-12}\text{ J}) + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r_0} = 0 + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r}\;.$$

If we set $r_0 = \infty$ (so $U_0 = 0$) then we obtain $r = 8.8\times10^{-14}$ m.

101. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$U_{\text{up-up}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e/3)(2e/3)}{r} = \frac{4ke}{9r}e = \frac{4(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{9(1.32\times10^{-15}\text{m})}e$$
$$= 4.84\times10^5\text{eV} = 0.484\,\text{MeV}.$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(2e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r}\right] = 0.$$

102. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere.

The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$.

Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$V(R) = V\big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50\times10^8\ \text{C}\right)}{0.160\,\text{m}}$$
$$= 8.43\times10^2\ \text{V}.$$

103. Since the electric potential energy is not changed by the introduction of the third particle, we conclude that the net electric potential evaluated at P caused by the original two particles must be zero:

$$\frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} = 0\,.$$

Setting $r_1 = 5d/2$ and $r_2 = 3d/2$ we obtain $q_1 = -5q_2/3$, or $q_1/q_2 = -5/3 \approx -1.7$.