

Chapter 20

1. **THINK** If the expansion of the gas is reversible and isothermal, then there's no change in internal energy. However, if the process is reversible and adiabatic, then there would be no change in entropy.

EXPRESS Since the gas is ideal, its pressure p is given in terms of the number of moles n , the volume V , and the temperature T by $p = nRT/V$. If the expansion is isothermal, the work done by the gas is

$$W = \int_{V_1}^{V_2} p dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln \frac{V_2}{V_1},$$

and the corresponding change in entropy is $\Delta S = \int (1/T) dQ = Q/T$, where Q is the heat absorbed (see Eq. 20-2).

ANALYZE (a) With $V_2 = 2.00V_1$ and $T = 400$ K, we obtain

$$W = nRT \ln 2.00 = (4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(400 \text{ K}) \ln 2.00 = 9.22 \times 10^3 \text{ J}.$$

(b) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22 \times 10^3 \text{ J}}{400 \text{ K}} = 23.1 \text{ J/K}.$$

(c) The change in entropy ΔS is zero for all reversible adiabatic processes.

LEARN The general expression for ΔS for reversible processes is given by Eq. 20-4:

$$\Delta S = S_f - S_i = nR \ln \left(\frac{V_f}{V_i} \right) + nC_V \ln \left(\frac{T_f}{T_i} \right).$$

Note that ΔS does not depend on how the gas changes from its initial state i to the final state f .

2. An isothermal process is one in which $T_i = T_f$, which implies $\ln (T_f/T_i) = 0$. Therefore, Eq. 20-4 leads to

$$\Delta S = nR \ln \left(\frac{V_f}{V_i} \right) \Rightarrow n = \frac{22.0}{(8.31) \ln (3.4/1.3)} = 2.75 \text{ mol}.$$

3. An isothermal process is one in which $T_i = T_f$, which implies $\ln(T_f/T_i) = 0$. Therefore, with $V_f/V_i = 2.00$, Eq. 20-4 leads to

$$\Delta S = nR \ln \left(\frac{V_f}{V_i} \right) = (2.50 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K}) \ln(2.00) = 14.4 \text{ J/K}.$$

4. From Eq. 20-2, we obtain $Q = T\Delta S = (405 \text{ K})(46.0 \text{ J/K}) = 1.86 \times 10^4 \text{ J}$.

5. We use the following relation derived in Sample Problem 20.01 — “Entropy change of two blocks coming to equilibrium.”

$$\Delta S = mc \ln(T_f / T_i).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) (2.00 \text{ kg})(75 \text{ K}) = 5.79 \times 10^4 \text{ J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With $T_f = 373.15 \text{ K}$ and $T_i = 298.15 \text{ K}$, we obtain

$$\Delta S = (2.00 \text{ kg}) \left(386 \frac{\text{J}}{\text{kg} \cdot \text{K}} \right) \ln \left(\frac{373.15}{298.15} \right) = 173 \text{ J/K}.$$

6. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333 \text{ J/g}$ from Table 19-4. Consequently,

$$\Delta S = \frac{(333 \text{ J/g})(12.0 \text{ g})}{273 \text{ K}} = 14.6 \text{ J/K}.$$

(b) The situation is similar to that described in part (a), except with $L = 2256 \text{ J/g}$, $m = 5.00 \text{ g}$, and $T = 373 \text{ K}$. We therefore find $\Delta S = 30.2 \text{ J/K}$.

7. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature T_f satisfies $m_1 c_1 (T_f - T_{i,1}) + m_2 c_2 (T_f - T_{i,2}) = 0$, which we solve for T_f :

$$\begin{aligned} T_f &= \frac{m_1 c_1 T_{i,1} + m_2 c_2 T_{i,2}}{m_1 c_1 + m_2 c_2} = \frac{(50.0 \text{ g})(386 \text{ J/kg} \cdot \text{K})(400 \text{ K}) + (100 \text{ g})(128 \text{ J/kg} \cdot \text{K})(200 \text{ K})}{(50.0 \text{ g})(386 \text{ J/kg} \cdot \text{K}) + (100 \text{ g})(128 \text{ J/kg} \cdot \text{K})} \\ &= 320 \text{ K}. \end{aligned}$$

(b) Since the two-block system is thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\begin{aligned}\Delta S &= \Delta S_1 + \Delta S_2 = m_1 c_1 \ln \left(\frac{T_f}{T_{i,1}} \right) + m_2 c_2 \ln \left(\frac{T_f}{T_{i,2}} \right) \\ &= (50.0 \text{ g})(386 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{320 \text{ K}}{400 \text{ K}} \right) + (100 \text{ g})(128 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{320 \text{ K}}{200 \text{ K}} \right) \\ &= +1.72 \text{ J/K}.\end{aligned}$$

8. We use Eq. 20-1:

$$\Delta S = \int \frac{nC_V dT}{T} = nA \int_{5.00}^{10.0} T^2 dT = \frac{nA}{3} [(10.0)^3 - (5.00)^3] = 0.0368 \text{ J/K}.$$

9. The ice warms to 0°C , then melts, and the resulting water warms to the temperature of the lake water, which is 15°C . As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If $T_i (= 263 \text{ K})$ is the initial temperature and $T_f (= 273 \text{ K})$ is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln \frac{T_f}{T_i} = (0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{273 \text{ K}}{263 \text{ K}} \right) = 0.828 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F , where L_F is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = 12.20 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln \frac{T_f}{T_i},$$

where c_w is the specific heat of water ($4190 \text{ J/kg} \cdot \text{K}$). Thus,

$$\Delta S = (0.010 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{288 \text{ K}}{273 \text{ K}} \right) = 2.24 \text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828 \text{ J/K} + 12.20 \text{ J/K} + 2.24 \text{ J/K} = 15.27 \text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0°C ,

$$Q = -mc_i(T_f - T_i) = -(0.010 \text{ kg})(2220 \text{ J/kg} \cdot \text{K})(10 \text{ K}) = -222 \text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010 \text{ kg})(333 \times 10^3 \text{ J/kg}) = -3.33 \times 10^3 \text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(15 \text{ K}) = -629 \text{ J}.$$

The total energy leaving the lake water is

$$Q = -222 \text{ J} - 3.33 \times 10^3 \text{ J} - 6.29 \times 10^2 \text{ J} = -4.18 \times 10^3 \text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18 \times 10^3 \text{ J}}{288 \text{ K}} = -14.51 \text{ J/K}.$$

The change in the entropy of the ice–lake system is $\Delta S = (15.27 - 14.51) \text{ J/K} = 0.76 \text{ J/K}$.

10. We follow the method shown in Sample Problem 20.01 — “Entropy change of two blocks coming to equilibrium.” Since

$$\Delta S = mc \int_{T_i}^{T_f} \frac{dT}{T} = mc \ln(T_f/T_i),$$

then with $\Delta S = 50 \text{ J/K}$, $T_f = 380 \text{ K}$, $T_i = 280 \text{ K}$, and $m = 0.364 \text{ kg}$, we obtain $c = 4.5 \times 10^2 \text{ J/kg} \cdot \text{K}$.

11. **THINK** The aluminum sample gives off energy as heat to water. Thermal equilibrium is reached when both the aluminum and the water come to a common final temperature T_f .

EXPRESS The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum–water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The change in entropy is $\Delta S = \int dQ/T$.

ANALYZE (a) The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K})(100^\circ\text{C}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(20^\circ\text{C})}{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})} \\ &= 57.0^\circ\text{C} = 330 \text{ K}. \end{aligned}$$

(b) Now temperatures must be given in Kelvins: $T_{ai} = 393 \text{ K}$, $T_{wi} = 293 \text{ K}$, and $T_f = 330 \text{ K}$. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \left(\frac{T_f}{T_{ai}} \right) = (0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{330 \text{ K}}{373 \text{ K}} \right) \\ &= -22.1 \text{ J/K}. \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \left(\frac{T_f}{T_{wi}} \right) = (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{330 \text{ K}}{293 \text{ K}} \right) \\ &= +24.9 \text{ J/K}. \end{aligned}$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1 \text{ J/K} + 24.9 \text{ J/K} = +2.8 \text{ J/K}.$$

LEARN The system is closed and the process is irreversible. For aluminum the entropy change is negative ($\Delta S_a < 0$) since $T_f < T_{ai}$. However, for water, entropy increases because $T_f > T_{wi}$. The overall entropy change for the aluminum-water system is positive, in accordance with the second law of thermodynamics.

12. We concentrate on the first term of Eq. 20-4 (the second term is zero because the final and initial temperatures are the same, and because $\ln(1) = 0$). Thus, the entropy change is

$$\Delta S = nR \ln(V_f/V_i).$$

Noting that $\Delta S = 0$ at $V_f = 0.40 \text{ m}^3$, we are able to deduce that $V_i = 0.40 \text{ m}^3$. We now examine the point in the graph where $\Delta S = 32 \text{ J/K}$ and $V_f = 1.2 \text{ m}^3$; the above expression can now be used to solve for the number of moles. We obtain $n = 3.5 \text{ mol}$.

13. This problem is similar to Sample Problem 20.01 — “Entropy change of two blocks coming to equilibrium.” The only difference is that we need to find the mass m of each of the blocks. Since the two blocks are identical, the final temperature T_f is the average of the initial temperatures:

$$T_f = \frac{1}{2}(T_i + T_f) = \frac{1}{2}(305.5 \text{ K} + 294.5 \text{ K}) = 300.0 \text{ K}.$$

Thus from $Q = mc\Delta T$ we find the mass m :

$$m = \frac{Q}{c\Delta T} = \frac{215 \text{ J}}{(386 \text{ J/kg} \cdot \text{K})(300.0 \text{ K} - 294.5 \text{ K})} = 0.101 \text{ kg}.$$

(a) The change in entropy for block L is

$$\Delta S_L = mc \ln \left(\frac{T_f}{T_{iL}} \right) = (0.101 \text{ kg})(386 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{300.0 \text{ K}}{305.5 \text{ K}} \right) = -0.710 \text{ J/K}.$$

(b) Since the temperature of the reservoir is virtually the same as that of the block, which gives up the same amount of heat as the reservoir absorbs, the change in entropy $\Delta S'_L$ of the reservoir connected to the left block is the opposite of that of the left block: $\Delta S'_L = -\Delta S_L = +0.710 \text{ J/K}$.

(c) The entropy change for block R is

$$\Delta S_R = mc \ln \left(\frac{T_f}{T_{iR}} \right) = (0.101 \text{ kg})(386 \text{ J/kg} \cdot \text{K}) \ln \left(\frac{300.0 \text{ K}}{294.5 \text{ K}} \right) = +0.723 \text{ J/K}.$$

(d) Similar to the case in part (b) above, the change in entropy $\Delta S'_R$ of the reservoir connected to the right block is given by $\Delta S'_R = -\Delta S_R = -0.723 \text{ J/K}$.

(e) The change in entropy for the two-block system is

$$\Delta S_L + \Delta S_R = -0.710 \text{ J/K} + 0.723 \text{ J/K} = +0.013 \text{ J/K}.$$

(f) The entropy change for the entire system is given by

$$\Delta S = \Delta S_L + \Delta S'_L + \Delta S_R + \Delta S'_R = \Delta S_L - \Delta S_L + \Delta S_R - \Delta S_R = 0,$$

which is expected of a reversible process.

14. (a) Work is done only for the *ab* portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W = \int_{V_0}^{4V_0} p_0 dV = p_0(4.00V_0 - 1.00V_0) = 3.00p_0V_0 \Rightarrow \frac{W}{p_0V_0} = 3.00.$$

(b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy Q absorbed by the gas as heat is $Q = nC_V \Delta T$, where C_V is the molar specific heat at constant volume and ΔT is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = 3R/2$. Use the ideal gas law to find that the initial temperature is

$$T_b = \frac{p_b V_b}{nR} = \frac{4p_0 V_0}{nR}$$

and that the final temperature is

$$T_c = \frac{p_c V_c}{nR} = \frac{(2p_0)(4V_0)}{nR} = \frac{8p_0 V_0}{nR}.$$

Thus,

$$Q = \frac{3}{2} nR \left(\frac{8p_0 V_0}{nR} - \frac{4p_0 V_0}{nR} \right) = 6.00 p_0 V_0.$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0 V_0$ or $\Delta E_{\text{int}}/p_0 V_0 = 6.00$. Since $n = 1$ mol, this can also be written $Q = 6.00RT_0$.

(c) For a complete cycle, $\Delta E_{\text{int}} = 0$.

(d) Since the process is at constant volume, use $dQ = nC_V dT$ to obtain

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln \frac{T_c}{T_b}.$$

Substituting $C_V = \frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b} = \frac{p_c V_c}{p_b V_b} = \frac{(2p_0)(4V_0)}{p_0(4V_0)} = 2.$$

Thus, $\Delta S = \frac{3}{2} nR \ln 2$. Since $n = 1$, this is $\Delta S = \frac{3}{2} R \ln 2 = 8.64 \text{ J/K}$.

(e) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

15. (a) The final mass of ice is $(1773 \text{ g} + 227 \text{ g})/2 = 1000 \text{ g}$. This means 773 g of water froze. Energy in the form of heat left the system in the amount mL_F , where m is the mass of the water that froze and L_F is the heat of fusion of water. The process is isothermal, so the change in entropy is

$$\Delta S = Q/T = -mL_F/T = -(0.773 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = -943 \text{ J/K}.$$

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943 \text{ J/K}.$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water–ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir–water–ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner–water–ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

16. In coming to equilibrium, the heat lost by the 100 cm^3 of liquid water (of mass $m_w = 100 \text{ g}$ and specific heat capacity $c_w = 4190 \text{ J/kg}\cdot\text{K}$) is absorbed by the ice (of mass m_i , which melts and reaches $T_f > 0^\circ\text{C}$). We begin by finding the equilibrium temperature:

$$\begin{aligned} \sum Q &= 0 \\ Q_{\text{warm water cools}} + Q_{\text{ice warms to } 0^\circ} + Q_{\text{ice melts}} + Q_{\text{melted ice warms}} &= 0 \\ c_w m_w (T_f - 20^\circ) + c_i m_i (0^\circ - (-10^\circ)) + L_F m_i + c_w m_i (T_f - 0^\circ) &= 0 \end{aligned}$$

which yields, after using $L_F = 333000 \text{ J/kg}$ and values cited in the problem, $T_f = 12.24^\circ$ which is equivalent to $T_f = 285.39 \text{ K}$. Sample Problem 20.01 — “Entropy change of two blocks coming to equilibrium” shows that

$$\Delta S_{\text{temp change}} = mc \ln \left(\frac{T_2}{T_1} \right)$$

for processes where $\Delta T = T_2 - T_1$, and Eq. 20-2 gives $\Delta S_{\text{melt}} = L_F m/T_o$ for the phase change experienced by the ice (with $T_o = 273.15 \text{ K}$). The total entropy change is (with T in Kelvins)

$$\begin{aligned} \Delta S_{\text{system}} &= m_w c_w \ln \left(\frac{285.39}{293.15} \right) + m_i c_i \ln \left(\frac{273.15}{263.15} \right) + m_i c_w \ln \left(\frac{285.39}{273.15} \right) + \frac{L_F m_i}{273.15} \\ &= (-11.24 + 0.66 + 1.47 + 9.75) \text{ J/K} = 0.64 \text{ J/K}. \end{aligned}$$

17. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting $f = 5$ in Eq. 19-51. Thus, $C_V = 5R/2$, $C_p = 7R/2$, and

$\gamma = 7/5$. In addition to various equations from Chapter 19, we also make use of Eq. 20-4 of this chapter. We note that we are asked to use the ideal gas constant as R and not plug in its numerical value. We also recall that isothermal means constant temperature, so $T_2 = T_1$ for the $1 \rightarrow 2$ process. The statement (at the end of the problem) regarding “per mole” may be taken to mean that n may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form is used to obtain

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right) = \frac{p_1}{3} \Rightarrow \frac{p_2}{p_1} = \frac{1}{3} = 0.333.$$

(b) The adiabatic relations Eq. 19-54 and Eq. 19-56 lead to

$$p_3 = p_1 \left(\frac{V_1}{V_3} \right)^\gamma = \frac{p_1}{3^{1.4}} \Rightarrow \frac{p_3}{p_1} = \frac{1}{3^{1.4}} = 0.215.$$

(c) Similarly, we find

$$T_3 = T_1 \left(\frac{V_1}{V_3} \right)^{\gamma-1} = \frac{T_1}{3^{0.4}} \Rightarrow \frac{T_3}{T_1} = \frac{1}{3^{0.4}} = 0.644.$$

• process $1 \rightarrow 2$

(d) The work is given by Eq. 19-14:

$$W = nRT_1 \ln (V_2/V_1) = RT_1 \ln 3 = 1.10RT_1.$$

Thus, $W/nRT_1 = \ln 3 = 1.10$.

(e) The internal energy change is $\Delta E_{\text{int}} = 0$, since this is an ideal gas process without a temperature change (see Eq. 19-45). Thus, the energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$, or $Q/nRT_1 = \ln 3 = 1.10$.

(f) $\Delta E_{\text{int}} = 0$ or $\Delta E_{\text{int}} / nRT_1 = 0$

(g) The entropy change is $\Delta S = Q/T_1 = 1.10R$, or $\Delta S/R = 1.10$.

• process $2 \rightarrow 3$

(h) The work is zero, since there is no volume change. Therefore, $W/nRT_1 = 0$.

(i) The internal energy change is

$$\Delta E_{\text{int}} = nC_V (T_3 - T_2) = (1) \left(\frac{5}{2} R \right) \left(\frac{T_1}{3^{0.4}} - T_1 \right) \approx -0.889 RT_1 \Rightarrow \frac{\Delta E_{\text{int}}}{nRT_1} \approx -0.889.$$

This ratio (-0.889) is also the value for Q/nRT_1 (by either the first law of thermodynamics or by the definition of C_V).

(j) $\Delta E_{\text{int}}/nRT_1 = -0.889$.

(k) For the entropy change, we obtain

$$\frac{\Delta S}{R} = n \ln \left(\frac{V_3}{V_1} \right) + n \frac{C_V}{R} \ln \left(\frac{T_3}{T_1} \right) = (1) \ln(1) + (1) \left(\frac{5}{2} \right) \ln \left(\frac{T_1/3^{0.4}}{T_1} \right) = 0 + \frac{5}{2} \ln(3^{-0.4}) \approx -1.10 .$$

• process $3 \rightarrow 1$

(l) By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process $1 \rightarrow 2$), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then,

$$W = Q - \Delta E_{\text{int}} = -0.889RT_1,$$

or $W/nRT_1 = -0.889$.

(m) $Q = 0$ in an adiabatic process.

(n) $\Delta E_{\text{int}}/nRT_1 = +0.889$.

(o) $\Delta S/nR = 0$.

18. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or “area under the curve”) of a curve in a TS diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a “straight-line” path in the TS diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2} \right) \Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process that forms a straight line in a graph that plots T versus S). This leads to

$$Q = (300 \text{ K}) (15 \text{ J/K}) = 4.5 \times 10^3 \text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n \left(\frac{3}{2} R \right) \Delta T = (2.0 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (200 \text{ K} - 400 \text{ K}) = -5.0 \times 10^3 \text{ J}.$$

(c) By the first law of thermodynamics, $W = Q - \Delta E_{\text{int}} = 4.5 \text{ kJ} - (-5.0 \text{ kJ}) = 9.5 \text{ kJ}$.

19. We note that the connection between molar heat capacity and the degrees of freedom of a monatomic gas is given by setting $f = 3$ in Eq. 19-51. Thus, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$.

(a) Since this is an ideal gas, Eq. 19-45 holds, which implies $\Delta E_{\text{int}} = 0$ for this process. Equation 19-14 also applies, so that by the first law of thermodynamics,

$$Q = 0 + W = nRT_1 \ln V_2/V_1 = p_1 V_1 \ln 2 \rightarrow Q/p_1 V_1 = \ln 2 = 0.693.$$

(b) The gas law in ratio form implies that the pressure decreased by a factor of 2 during the isothermal expansion process to $V_2 = 2.00V_1$, so that it needs to increase by a factor of 4 in this step in order to reach a final pressure of $p_2 = 2.00p_1$. That same ratio form now applied to this constant-volume process, yielding $4.00 = T_2/T_1$, which is used in the following:

$$Q = nC_V \Delta T = n \left(\frac{3}{2} R \right) (T_2 - T_1) = \frac{3}{2} nRT_1 \left(\frac{T_2}{T_1} - 1 \right) = \frac{3}{2} p_1 V_1 (4 - 1) = \frac{9}{2} p_1 V_1$$

or $Q/p_1 V_1 = 9/2 = 4.50$.

(c) The work done during the isothermal expansion process may be obtained by using Eq. 19-14:

$$W = nRT_1 \ln V_2/V_1 = p_1 V_1 \ln 2.00 \rightarrow W/p_1 V_1 = \ln 2 = 0.693.$$

(d) In step 2 where the volume is kept constant, $W = 0$.

(e) The change in internal energy can be calculated by combining the above results and applying the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(p_1 V_1 \ln 2 + \frac{9}{2} p_1 V_1 \right) - (p_1 V_1 \ln 2 + 0) = \frac{9}{2} p_1 V_1$$

or $\Delta E_{\text{int}}/p_1 V_1 = 9/2 = 4.50$.

(f) The change in entropy may be computed by using Eq. 20-4:

$$\begin{aligned} \Delta S &= R \ln \left(\frac{2.00V_1}{V_1} \right) + C_V \ln \left(\frac{4.00T_1}{T_1} \right) = R \ln 2.00 + \left(\frac{3}{2} R \right) \ln (2.00)^2 \\ &= R \ln 2.00 + 3R \ln 2.00 = 4R \ln 2.00 = 23.0 \text{ J/K}. \end{aligned}$$

The second approach consists of an isothermal (constant T) process in which the volume halves, followed by an isobaric (constant p) process.

(g) Here the gas law applied to the first (isothermal) step leads to a volume half as big as the original. Since $\ln(1/2.00) = -\ln 2.00$, the reasoning used above leads to

$$Q = -p_1 V_1 \ln 2.00 \Rightarrow Q/p_1 V_1 = -\ln 2.00 = -0.693.$$

(h) To obtain a final volume twice as big as the original, in this step we need to increase the volume by a factor of 4.00. Now, the gas law applied to this isobaric portion leads to a temperature ratio $T_2/T_1 = 4.00$. Thus,

$$Q = C_p \Delta T = \frac{5}{2} R(T_2 - T_1) = \frac{5}{2} R T_1 \left(\frac{T_2}{T_1} - 1 \right) = \frac{5}{2} p_1 V_1 (4 - 1) = \frac{15}{2} p_1 V_1$$

or $Q/p_1 V_1 = 15/2 = 7.50$.

(i) During the isothermal compression process, Eq. 19-14 gives

$$W = nRT_1 \ln V_2/V_1 = p_1 V_1 \ln (-1/2.00) = -p_1 V_1 \ln 2.00 \Rightarrow W/p_1 V_1 = -\ln 2 = -0.693.$$

(j) The initial value of the volume, for this part of the process, is $V_i = V_1/2$, and the final volume is $V_f = 2V_1$. The pressure maintained during this process is $p' = 2.00p_1$. The work is given by Eq. 19-16:

$$W = p' \Delta V = p' (V_f - V_i) = (2.00p_1) \left(2.00V_1 - \frac{1}{2}V_1 \right) = 3.00p_1 V_1 \Rightarrow W/p_1 V_1 = 3.00.$$

(k) Using the first law of thermodynamics, the change in internal energy is

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(\frac{15}{2} p_1 V_1 - p_1 V_1 \ln 2.00 \right) - (3p_1 V_1 - p_1 V_1 \ln 2.00) = \frac{9}{2} p_1 V_1$$

or $\Delta E_{\text{int}}/p_1 V_1 = 9/2 = 4.50$. The result is the same as that obtained in part (e).

(l) Similarly, $\Delta S = 4R \ln 2.00 = 23.0 \text{ J/K}$. the same as that obtained in part (f).

20. (a) The final pressure is

$$p_f = (5.00 \text{ kPa}) e^{(V_i - V_f)/a} = (5.00 \text{ kPa}) e^{(1.00 \text{ m}^3 - 2.00 \text{ m}^3)/1.00 \text{ m}^3} = 1.84 \text{ kPa}.$$

(b) We use the ratio form of the gas law to find the final temperature of the gas:

$$T_f = T_i \left(\frac{p_f V_f}{p_i V_i} \right) = (600 \text{ K}) \frac{(1.84 \text{ kPa})(2.00 \text{ m}^3)}{(5.00 \text{ kPa})(1.00 \text{ m}^3)} = 441 \text{ K}.$$

For later purposes, we note that this result can be written “exactly” as $T_f = T_i (2e^{-1})$. In our solution, we are avoiding using the “one mole” datum since it is not clear how precise it is.

(c) The work done by the gas is

$$\begin{aligned} W &= \int_i^f p dV = \int_{V_i}^{V_f} (5.00 \text{ kPa}) e^{(V_i - V)/a} dV = (5.00 \text{ kPa}) e^{V_i/a} \cdot \left[-a e^{-V/a} \right]_{V_i}^{V_f} \\ &= (5.00 \text{ kPa}) e^{1.00} (1.00 \text{ m}^3) (e^{-1.00} - e^{-2.00}) \\ &= 3.16 \text{ kJ}. \end{aligned}$$

(d) Consideration of a two-stage process, as suggested in the hint, brings us simply to Eq. 20-4. Consequently, with $C_V = \frac{3}{2} R$ (see Eq. 19-43), we find

$$\begin{aligned} \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) + n \left(\frac{3}{2} R \right) \ln \left(\frac{T_f}{T_i} \right) = nR \left(\ln 2 + \frac{3}{2} \ln (2e^{-1}) \right) = \frac{p_i V_i}{T_i} \left(\ln 2 + \frac{3}{2} \ln 2 + \frac{3}{2} \ln e^{-1} \right) \\ &= \frac{(5000 \text{ Pa})(1.00 \text{ m}^3)}{600 \text{ K}} \left(\frac{5}{2} \ln 2 - \frac{3}{2} \right) \\ &= 1.94 \text{ J/K}. \end{aligned}$$

21. We consider a three-step reversible process as follows: the supercooled water drop (of mass m) starts at state 1 ($T_1 = 268 \text{ K}$), moves on to state 2 (still in liquid form but at $T_2 = 273 \text{ K}$), freezes to state 3 ($T_3 = T_2$), and then cools down to state 4 (in solid form, with $T_4 = T_1$). The change in entropy for each of the stages is given as follows:

$$\Delta S_{12} = mc_w \ln (T_2/T_1),$$

$$\Delta S_{23} = -mL_F/T_2,$$

$$\Delta S_{34} = mc_I \ln (T_4/T_3) = mc_I \ln (T_1/T_2) = -mc_I \ln (T_2/T_1).$$

Thus the net entropy change for the water drop is

$$\begin{aligned} \Delta S &= \Delta S_{12} + \Delta S_{23} + \Delta S_{34} = m(c_w - c_I) \ln \left(\frac{T_2}{T_1} \right) - \frac{mL_F}{T_2} \\ &= (1.00 \text{ g})(4.19 \text{ J/g} \cdot \text{K} - 2.22 \text{ J/g} \cdot \text{K}) \ln \left(\frac{273 \text{ K}}{268 \text{ K}} \right) - \frac{(1.00 \text{ g})(333 \text{ J/g})}{273 \text{ K}} \\ &= -1.18 \text{ J/K}. \end{aligned}$$

22. (a) We denote the mass of the ice (which turns to water and warms to T_f) as m and the mass of original water (which cools from 80° down to T_f) as m' . From $\Sigma Q = 0$ we have

$$L_F m + cm (T_f - 0^\circ) + cm' (T_f - 80^\circ) = 0.$$

Since $L_F = 333 \times 10^3 \text{ J/kg}$, $c = 4190 \text{ J/(kg}\cdot^\circ\text{C)}$, $m' = 0.13 \text{ kg}$, and $m = 0.012 \text{ kg}$, we find $T_f = 66.5^\circ\text{C}$, which is equivalent to 339.67 K .

(b) Using Eq. 20-2, the process of ice at 0° C turning to water at 0° C involves an entropy change of

$$\frac{Q}{T} = \frac{L_F m}{273.15 \text{ K}} = 14.6 \text{ J/K}.$$

(c) Using Eq. 20-1, the process of $m = 0.012 \text{ kg}$ of water warming from 0° C to 66.5° C involves an entropy change of

$$\int_{273.15}^{339.67} \frac{cm dT}{T} = cm \ln \left(\frac{339.67}{273.15} \right) = 11.0 \text{ J/K}.$$

(d) Similarly, the cooling of the original water involves an entropy change of

$$\int_{353.15}^{339.67} \frac{cm' dT}{T} = cm' \ln \left(\frac{339.67}{353.15} \right) = -21.2 \text{ J/K}.$$

(e) The net entropy change in this calorimetry experiment is found by summing the previous results; we find (by using more precise values than those shown above) $\Delta S_{\text{net}} = 4.39 \text{ J/K}$.

23. With $T_L = 290 \text{ K}$, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow T_H = \frac{T_L}{1 - \varepsilon} = \frac{290 \text{ K}}{1 - 0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: $T_H = 483 \text{ K}$. If we replace $\varepsilon = 0.40$ in the above calculation with $\varepsilon = 0.50$, we obtain a (final) high temperature equal to $T'_H = 580 \text{ K}$. The difference is

$$T'_H - T_H = 580 \text{ K} - 483 \text{ K} = 97 \text{ K}.$$

24. The answers to this exercise do not depend on the engine being of the Carnot design. Any heat engine that intakes energy as heat (from, say, consuming fuel) equal to $|Q_H| = 52 \text{ kJ}$ and exhausts (or discards) energy as heat equal to $|Q_L| = 36 \text{ kJ}$ will have these values of efficiency ε and net work W .

(a) Equation 20-12 gives $\varepsilon = 1 - \left| \frac{Q_L}{Q_H} \right| = 0.31 = 31\%$.

(b) Equation 20-8 gives $W = |Q_H| - |Q_L| = 16 \text{ kJ}$.

25. We solve (b) first.

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20-13. Therefore,

$$T_H = \frac{T_H - T_L}{\varepsilon} = \frac{75 \text{ K}}{0.22} = 341 \text{ K}$$

which is equivalent to 68°C .

(a) The temperature of the cold reservoir is $T_L = T_H - 75 = 341 \text{ K} - 75 \text{ K} = 266 \text{ K}$.

26. Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{373 \text{ K}}{7 \times 10^8 \text{ K}} = 0.9999995$$

quoting more figures than are significant. As a percentage, this is $\varepsilon = 99.99995\%$.

27. **THINK** The thermal efficiency of the Carnot engine depends on the temperatures of the reservoirs.

EXPRESS The efficiency of the Carnot engine is given by

$$\varepsilon_C = \frac{T_H - T_L}{T_H},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in kelvin scale. The work done by the engine is $|W| = \varepsilon |Q_H|$.

ANALYZE (a) The efficiency of the engine is

$$\varepsilon_c = \frac{T_H - T_L}{T_H} = \frac{(235 - 115) \text{ K}}{(235 + 273) \text{ K}} = 0.236 = 23.6\%$$
 .

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by

$$|W| = \varepsilon |Q_H| = 0.236(6.30 \times 10^4 \text{ J}) = 1.49 \times 10^4 \text{ J}.$$

LEARN Expressing the efficiency as $\varepsilon_c = 1 - T_L/T_H$, we see that ε_c approaches unity (100% efficiency) in the limit $T_L/T_H \rightarrow 0$. This is an impossible dream. An alternative version of the second law of thermodynamics is: *there are no perfect engines.*

28. All terms are assumed to be positive. The total work done by the two-stage system is $W_1 + W_2$. The heat-intake (from, say, consuming fuel) of the system is Q_1 , so we have (by Eq. 20-11 and Eq. 20-8)

$$\varepsilon = \frac{W_1 + W_2}{Q_1} = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = 1 - \frac{Q_3}{Q_1}.$$

Now, Eq. 20-10 leads to

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

where we assume Q_2 is absorbed by the second stage at temperature T_2 . This implies the efficiency can be written

$$\varepsilon = 1 - \frac{T_3}{T_1} = \frac{T_1 - T_3}{T_1}.$$

29. (a) The net work done is the rectangular “area” enclosed in the pV diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0 p_0.$$

Inserting the values stated in the problem, we obtain $W = 2.27 \text{ kJ}$.

(b) We compute the energy added as heat during the “heat-intake” portions of the cycle using Eq. 19-39, Eq. 19-43, and Eq. 19-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) = n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) = p_0V_0\left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \\ &= \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used. Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8 \text{ kJ}$.

(c) The efficiency is given by Eq. 20-11:

$$\varepsilon = \frac{W}{|Q_H|} = \frac{2}{13} = 0.154 = 15.4\%.$$

(d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used.

(e) This is greater than our result in part (c), as expected from the second law of thermodynamics.

30. (a) Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{333 \text{ K}}{373 \text{ K}} = 0.107.$$

We recall that a watt is joule-per-second. Thus, the (net) work done by the cycle per unit time is the given value 500 J/s. Therefore, by Eq. 20-11, we obtain the heat input per unit time:

$$\varepsilon = \frac{W}{|Q_H|} \Rightarrow \frac{0.500 \text{ kJ/s}}{0.107} = 4.67 \text{ kJ/s}.$$

(b) Considering Eq. 20-8 on a per unit time basis, we find $(4.67 - 0.500) \text{ kJ/s} = 4.17 \text{ kJ/s}$ for the rate of heat exhaust.

31. (a) We use $\varepsilon = |W/Q_H|$. The heat absorbed is $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2 \text{ kJ}}{0.25} = 33 \text{ kJ}$.

(b) The heat exhausted is then $|Q_L| = |Q_H| - |W| = 33 \text{ kJ} - 8.2 \text{ kJ} = 25 \text{ kJ}$.

(c) Now we have $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2 \text{ kJ}}{0.31} = 26 \text{ kJ}$.

(d) Similarly, $|Q_C| = |Q_H| - |W| = 26 \text{ kJ} - 8.2 \text{ kJ} = 18 \text{ kJ}$.

32. From Fig. 20-28, we see $Q_H = 4000 \text{ J}$ at $T_H = 325 \text{ K}$. Combining Eq. 20-11 with Eq. 20-13, we have

$$\frac{W}{Q_H} = 1 - \frac{T_C}{T_H} \Rightarrow W = 923 \text{ J}.$$

Now, for $T'_H = 550$ K, we have

$$\frac{W}{Q'_H} = 1 - \frac{T_C}{T'_H} \Rightarrow Q'_H = 1692 \text{ J} \approx 1.7 \text{ kJ}.$$

33. THINK Our engine cycle consists of three steps: isochoric heating (a to b), adiabatic expansion (b to c), and isobaric compression (c to a).

EXPRESS Energy is added as heat during the portion of the process from a to b . This portion occurs at constant volume (V_b), so $Q_H = nC_V \Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a)V_b.$$

Thus, $Q_H = \frac{3}{2}(p_b - p_a)V_b$. On the other hand, energy leaves the gas as heat during the portion of the process from c to a . This is a constant pressure process, so

$$Q_L = nC_p \Delta T = nC_p (T_a - T_c) = nC_p \left(\frac{p_a V_a}{nR} - \frac{p_c V_c}{nR} \right) = \frac{C_p}{R} p_a (V_a - V_c).$$

where C_p is the molar specific heat for constant-pressure process.

ANALYZE (a) V_b and p_b are given. We need to find p_a . Now p_a is the same as p_c and points c and b are connected by an adiabatic process. With $p_c V_c^\gamma = p_b V_b^\gamma$ for the adiabat, we have ($\gamma = 5/3$ for monatomic gas)

$$p_a = p_c = \left(\frac{V_b}{V_c} \right)^\gamma p_b = \left(\frac{1}{8.00} \right)^{5/3} (1.013 \times 10^6 \text{ Pa}) = 3.167 \times 10^4 \text{ Pa}.$$

Thus, the energy added as heat is

$$Q_H = \frac{3}{2}(p_b - p_a)V_b = \frac{3}{2}(1.013 \times 10^6 \text{ Pa} - 3.167 \times 10^4 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3) = 1.47 \times 10^3 \text{ J}.$$

(b) The energy leaving the gas as heat going from c to a is

$$Q_L = \frac{5}{2} p_a (V_a - V_c) = \frac{5}{2} (3.167 \times 10^4 \text{ Pa})(-7.00)(1.00 \times 10^{-3} \text{ m}^3) = -5.54 \times 10^2 \text{ J},$$

or $|Q_L| = 5.54 \times 10^2 \text{ J}$. The substitutions $V_a - V_c = V_a - 8.00 V_a = -7.00 V_a$ and $C_p = \frac{5}{2} R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = Q_H - Q_L = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_H = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

LEARN To summarize, the heat engine in this problem intakes energy as heat (from, say, consuming fuel) equal to $|Q_H| = 1.47 \text{ kJ}$ and exhausts energy as heat equal to $|Q_L| = 554 \text{ J}$; its efficiency and net work are $\varepsilon = 1 - |Q_L|/|Q_H|$ and $W = |Q_H| - |Q_L|$. The less the exhaust heat $|Q_L|$, the more efficient is the engine.

34. (a) Using Eq. 19-54 for process $D \rightarrow A$ gives

$$p_D V_D^\gamma = p_A V_A^\gamma \quad \Rightarrow \quad \frac{p_0}{32} (8V_0)^\gamma = p_0 V_0^\gamma$$

which leads to $8^\gamma = 32 \Rightarrow \gamma = 5/3$. The result (see Sections 19-9 and 19-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \rightarrow B$:

$$Q_H = nC_p \Delta T = n \left(\frac{5}{2} R \right) T_A \left(\frac{T_B}{T_A} - 1 \right) = nRT_A \left(\frac{5}{2} \right) (2 - 1) = p_0 V_0 \left(\frac{5}{2} \right)$$

and the exhaust heat is that liberated during process $C \rightarrow D$:

$$Q_L = nC_p \Delta T = n \left(\frac{5}{2} R \right) T_D \left(1 - \frac{T_L}{T_D} \right) = nRT_D \left(\frac{5}{2} \right) (1 - 2) = -\frac{1}{4} p_0 V_0 \left(\frac{5}{2} \right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4} T_A$ (from the gas law in ratio form). Therefore, Eq. 20-12 leads to

$$\varepsilon = 1 - \left| \frac{Q_L}{Q_H} \right| = 1 - \frac{1}{4} = 0.75 = 75\%.$$

35. (a) The pressure at 2 is $p_2 = 3.00p_1$, as given in the problem statement. The volume is $V_2 = V_1 = nRT_1/p_1$. The temperature is

$$T_2 = \frac{p_2 V_2}{nR} = \frac{3.00 p_1 V_1}{nR} = 3.00 T_1 \Rightarrow \frac{T_2}{T_1} = 3.00.$$

(b) The process $2 \rightarrow 3$ is adiabatic, so $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$. Using the result from part (a), $V_3 = 4.00V_1$, $V_2 = V_1$, and $\gamma = 1.30$, we obtain

$$\frac{T_3}{T_1} = \frac{T_3}{T_2 / 3.00} = 3.00 \left(\frac{V_2}{V_3} \right)^{\gamma-1} = 3.00 \left(\frac{1}{4.00} \right)^{0.30} = 1.98.$$

(c) The process $4 \rightarrow 1$ is adiabatic, so $T_4 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$. Since $V_4 = 4.00V_1$, we have

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4} \right)^{\gamma-1} = \left(\frac{1}{4.00} \right)^{0.30} = 0.660.$$

(d) The process $2 \rightarrow 3$ is adiabatic, so $p_2 V_2^\gamma = p_3 V_3^\gamma$ or $p_3 = (V_2/V_3)^\gamma p_2$. Substituting $V_3 = 4.00V_1$, $V_2 = V_1$, $p_2 = 3.00p_1$, and $\gamma = 1.30$, we obtain

$$\frac{p_3}{p_1} = \frac{3.00}{(4.00)^{1.30}} = 0.495.$$

(e) The process $4 \rightarrow 1$ is adiabatic, so $p_4 V_4^\gamma = p_1 V_1^\gamma$ and

$$\frac{p_4}{p_1} = \left(\frac{V_1}{V_4} \right)^\gamma = \frac{1}{(4.00)^{1.30}} = 0.165,$$

where we have used $V_4 = 4.00V_1$.

(f) The efficiency of the cycle is $\varepsilon = W/Q_{12}$, where W is the total work done by the gas during the cycle and Q_{12} is the energy added as heat during the $1 \rightarrow 2$ portion of the cycle, the only portion in which energy is added as heat. The work done during the portion of the cycle from 2 to 3 is $W_{23} = \int p dV$. Substitute $p = p_2 V_2^\gamma / V^\gamma$ to obtain

$$W_{23} = p_2 V_2^\gamma \int_{V_2}^{V_3} V^{-\gamma} dV = \left(\frac{p_2 V_2^\gamma}{\gamma-1} \right) (V_2^{1-\gamma} - V_3^{1-\gamma}).$$

Substitute $V_2 = V_1$, $V_3 = 4.00V_1$, and $p_3 = 3.00p_1$ to obtain

$$W_{23} = \left(\frac{3p_1 V_1}{1-\gamma} \right) \left(1 - \frac{1}{4^{\gamma-1}} \right) = \left(\frac{3nRT_1}{\gamma-1} \right) \left(1 - \frac{1}{4^{\gamma-1}} \right).$$

Similarly, the work done during the portion of the cycle from 4 to 1 is

$$W_{41} = \left(\frac{p_1 V_1^\gamma}{\gamma - 1} \right) (V_4^{1-\gamma} - V_1^{1-\gamma}) = - \left(\frac{p_1 V_1}{\gamma - 1} \right) \left(1 - \frac{1}{4^{\gamma-1}} \right) = - \left(\frac{nRT_1}{\gamma - 1} \right) \left(1 - \frac{1}{4^{\gamma-1}} \right).$$

No work is done during the $1 \rightarrow 2$ and $3 \rightarrow 4$ portions, so the total work done by the gas during the cycle is

$$W = W_{23} + W_{41} = \left(\frac{2nRT_1}{\gamma - 1} \right) \left(1 - \frac{1}{4^{\gamma-1}} \right).$$

The energy added as heat is

$$Q_{12} = nC_V (T_2 - T_1) = nC_V (3T_1 - T_1) = 2nC_V T_1,$$

where C_V is the molar specific heat at constant volume. Now

$$\gamma = C_p/C_V = (C_V + R)/C_V = 1 + (R/C_V),$$

so $C_V = R/(\gamma - 1)$. Here C_p is the molar specific heat at constant pressure, which for an ideal gas is $C_p = C_V + R$. Thus, $Q_{12} = 2nRT_1/(\gamma - 1)$. The efficiency is

$$\varepsilon = \frac{2nRT_1}{\gamma - 1} \left(1 - \frac{1}{4^{\gamma-1}} \right) \frac{\gamma - 1}{2nRT_1} = 1 - \frac{1}{4^{\gamma-1}}.$$

With $\gamma = 1.30$, the efficiency is $\varepsilon = 0.340$ or 34.0%.

36. (a) Using Eq. 20-14 and Eq. 20-16, we obtain

$$|W| = \frac{|Q_L|}{K_C} = (1.0 \text{ J}) \left(\frac{300 \text{ K} - 280 \text{ K}}{280 \text{ K}} \right) = 0.071 \text{ J}.$$

(b) A similar calculation (being sure to use absolute temperature) leads to 0.50 J in this case.

(c) With $T_L = 100 \text{ K}$, we obtain $|W| = 2.0 \text{ J}$.

(d) Finally, with the low temperature reservoir at 50 K, an amount of work equal to $|W| = 5.0 \text{ J}$ is required.

37. **THINK** The performance of the refrigerator is related to its rate of doing work.

EXPRESS The coefficient of performance for a refrigerator is given by

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields

$Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W , so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$.

ANALYZE In one hour, $|Q_H| = 7.54 \text{ MJ}$. With $K = 3.8$, the work done is

$$|W| = \frac{7.54 \text{ MJ}}{3.8 + 1} = 1.57 \text{ MJ}.$$

The rate at which work is done is $P = |W|/\Delta t = (1.57 \times 10^6 \text{ J})/(3600 \text{ s}) = 440 \text{ W}$.

LEARN The greater the value of K , the less the amount of work $|W|$ required to transfer the heat.

38. Equation 20-10 still holds (particularly due to its use of absolute values), and energy conservation implies $|W| + Q_L = Q_H$. Therefore, with $T_L = 268.15 \text{ K}$ and $T_H = 290.15 \text{ K}$, we find

$$|Q_H| = |Q_L| \left(\frac{T_H}{T_L} \right) = (|Q_H| - |W|) \left(\frac{290.15}{268.15} \right)$$

which (with $|W| = 1.0 \text{ J}$) leads to $|Q_H| = |W| \left(\frac{1}{1 - 268.15/290.15} \right) = 13 \text{ J}$.

39. **THINK** A large (small) value of coefficient of performance K means that less (more) work would be required to transfer the heat

EXPRESS A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by

$$K = \frac{T_L}{T_H - T_L},$$

where T_H is the temperature of the higher-temperature reservoir, and T_L the temperature of the lower-temperature reservoir, in Kelvin scale. Equivalently, the coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$.

ANALYZE For the refrigerator of this problem, $T_H = 96^\circ \text{ F} = 309 \text{ K}$ and $T_L = 70^\circ \text{ F} = 294 \text{ K}$, so

$$K = (294 \text{ K})/(309 \text{ K} - 294 \text{ K}) = 19.6.$$

Thus, with $|W| = 1.0 \text{ J}$, the amount of heat removed from the room is

$$|Q_L| = K|W| = (19.6)(1.0 \text{ J}) = 20 \text{ J}.$$

LEARN The Carnot air conditioner in this problem (with $K = 19.6$) are much more efficient than that of the typical room air conditioners ($K \approx 2.5$).

40. (a) Equation 20-15 provides

$$K_C = \frac{|Q_L|}{|Q_H| - |Q_L|} \Rightarrow |Q_H| = |Q_L| \left(\frac{1 + K_C}{K_C} \right)$$

which yields $|Q_H| = 49 \text{ kJ}$ when $K_C = 5.7$ and $|Q_L| = 42 \text{ kJ}$.

(b) From Section 20-5 we obtain

$$|W| = |Q_H| - |Q_L| = 49.4 \text{ kJ} - 42.0 \text{ kJ} = 7.4 \text{ kJ}$$

if we take the initial 42 kJ datum to be accurate to three figures. The given temperatures are not used in the calculation; in fact, it is possible that the given room temperature value is not meant to be the high temperature for the (reversed) Carnot cycle — since it does not lead to the given K_C using Eq. 20-16.

41. We are told $K = 0.27K_C$, where

$$K_C = \frac{T_L}{T_H - T_L} = \frac{294 \text{ K}}{307 \text{ K} - 294 \text{ K}} = 23$$

where the Fahrenheit temperatures have been converted to Kelvins. Expressed on a per unit time basis, Eq. 20-14 leads to

$$\frac{|W|}{t} = \frac{|Q_L|/t}{K} = \frac{4000 \text{ Btu/h}}{(0.27)(23)} = 643 \text{ Btu/h}.$$

Appendix D indicates $1 \text{ Btu/h} = 0.0003929 \text{ hp}$, so our result may be expressed as $|W|/t = 0.25 \text{ hp}$.

42. The work done by the motor in $t = 10.0 \text{ min}$ is $|W| = Pt = (200 \text{ W})(10.0 \text{ min})(60 \text{ s/min}) = 1.20 \times 10^5 \text{ J}$. The heat extracted is then

$$|Q_L| = K|W| = \frac{T_L|W|}{T_H - T_L} = \frac{(270 \text{ K})(1.20 \times 10^5 \text{ J})}{300 \text{ K} - 270 \text{ K}} = 1.08 \times 10^6 \text{ J}.$$

43. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be

$$\varepsilon = \frac{T_1 - T_2}{T_1} \Rightarrow \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be

$$K = \frac{T_4}{T_3 - T_4} \Rightarrow \frac{Q_4}{W} = \frac{T_4}{T_3 - T_4}.$$

Now $Q_4 = Q_3 - W$, so

$$(Q_3 - W)/W = T_4/(T_3 - T_4).$$

The work done by the engine is used to drive the refrigerator, so W is the same for the two. Solve the engine equation for W and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1 (T_1 - T_2)} - 1.$$

Solving for Q_3/Q_1 , we obtain

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1 \right) \left(\frac{T_1 - T_2}{T_1} \right) = \left(\frac{T_3}{T_3 - T_4} \right) \left(\frac{T_1 - T_2}{T_1} \right) = \frac{1 - (T_2/T_1)}{1 - (T_4/T_3)}.$$

With $T_1 = 400$ K, $T_2 = 150$ K, $T_3 = 325$ K, and $T_4 = 225$ K, the ratio becomes $Q_3/Q_1 = 2.03$.

44. (a) Equation 20-13 gives the Carnot efficiency as $1 - T_L/T_H$. This gives 0.222 in this case. Using this value with Eq. 20-11 leads to $W = (0.222)(750 \text{ J}) = 167 \text{ J}$.

(b) Now, Eq. 20-16 gives $K_C = 3.5$. Then, Eq. 20-14 yields $|W| = 1200/3.5 = 343 \text{ J}$.

45. We need nine labels:

Label	Number of molecules on side 1	Number of molecules on side 2
I	8	0
II	7	1
III	6	2
IV	5	3
V	4	4
VI	3	5
VII	2	6
VIII	1	7
IX	0	8

The multiplicity W is computed using Eq. 20-20. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 20-21)

$$S = k \ln W = (1.38 \times 10^{-23} \text{ J/K}) \ln(56) = 5.6 \times 10^{-23} \text{ J/K}.$$

In this way, we generate the following table:

Label	W	S
I	1	0
II	8	$2.9 \times 10^{-23} \text{ J/K}$
III	28	$4.6 \times 10^{-23} \text{ J/K}$
IV	56	$5.6 \times 10^{-23} \text{ J/K}$
V	70	$5.9 \times 10^{-23} \text{ J/K}$
VI	56	$5.6 \times 10^{-23} \text{ J/K}$
VII	28	$4.6 \times 10^{-23} \text{ J/K}$
VIII	8	$2.9 \times 10^{-23} \text{ J/K}$
IX	1	0

46. (a) We denote the configuration with n heads out of N trials as $(n; N)$. We use Eq. 20-20:

$$W(25; 50) = \frac{50!}{(25!)(50-25)!} = 1.26 \times 10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of N independent molecules, the total number of available states of the N -particle system is

$$N_{\text{total}} = 2 \times 2 \times 2 \times \cdots \times 2 = 2^N.$$

With $N = 50$, we obtain $N_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25; 50) = \frac{W(25; 50)}{2^{50}} = \frac{1.26 \times 10^{14}}{1.13 \times 10^{15}} = 11.1\%.$$

With $N = 100$, we obtain

$$(d) W(N/2, N) = N! / [(N/2)!]^2 = 1.01 \times 10^{29},$$

(e) $N_{\text{total}} = 2^N = 1.27 \times 10^{30}$,

(f) and $p(N/2; N) = W(N/2, N) / N_{\text{total}} = 8.0\%$.

Similarly, for $N = 200$, we obtain

(g) $W(N/2, N) = 9.25 \times 10^{58}$,

(h) $N_{\text{total}} = 1.61 \times 10^{60}$,

(i) and $p(N/2; N) = 5.7\%$.

(j) As N increases, the number of available microscopic states increases as 2^N , so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent there decreases with an increase in N .

47. **THINK** The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

EXPRESS Given N molecules, if the box is divided into m equal parts, with n_1 molecules in the first, n_2 in the second, ..., such that $n_1 + n_2 + \dots + n_m = N$. There are $N!$ arrangements of the N molecules, but $n_1!$ are simply rearrangements of the n_1 molecules in the first part, $n_2!$ are rearrangements of the n_2 molecules in the second, ... These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_1! n_2! n_3! \dots n_m!}.$$

ANALYZE (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. Using the argument above, we find the multiplicity to be

$$W = \frac{N!}{n_L! n_C! n_R!}.$$

Note that $n_L + n_C + n_R = N$.

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!(N/3)!(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!(N/2)!}{(N/3)!(N/3)!(N/3)!}.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!50!}{33!33!34!} = 4.2 \times 10^{16}.$$

LEARN The more parts the box is divided into, the greater the number of configurations. This exercise illustrates the statistical view of entropy, which is related to W as $S = k \ln W$.

48. (a) A good way to (mathematically) think of this is to consider the terms when you expand:

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$

The coefficients correspond to the multiplicities. Thus, the smallest coefficient is 1.

(b) The largest coefficient is 6.

(c) Since the logarithm of 1 is zero, then Eq. 20-21 gives $S = 0$ for the least case.

(d) $S = k \ln(6) = 2.47 \times 10^{-23} \text{ J/K}$.

49. From the formula for heat conduction, Eq. 19-32, using Table 19-6, we have

$$H = kA \frac{T_H - T_C}{L} = (401) (\pi(0.02)^2) 270/1.50$$

which yields $H = 90.7 \text{ J/s}$. Using Eq. 20-2, this is associated with an entropy rate-of-decrease of the high temperature reservoir (at 573 K) equal to

$$\Delta S/t = -90.7/573 = -0.158 \text{ (J/K)/s}.$$

And it is associated with an entropy rate-of-increase of the low temperature reservoir (at 303 K) equal to

$$\Delta S/t = +90.7/303 = 0.299 \text{ (J/K)/s}.$$

The net result is $(0.299 - 0.158) \text{ (J/K)/s} = 0.141 \text{ (J/K)/s}$.

50. For an isothermal ideal gas process, we have $Q = W = nRT \ln(V_f/V_i)$. Thus,

$$\Delta S = Q/T = W/T = nR \ln(V_f/V_i)$$

(a) $V_f/V_i = (0.800)/(0.200) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34 \text{ J/K}$.

(b) $V_f/V_i = (0.800)/(0.200) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34 \text{ J/K}$.

(c) $V_f/V_i = (1.20)/(0.300) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34 \text{ J/K}$.

(d) $V_f/V_i = (1.20)/(0.300) = 4.00$, $\Delta S = (0.55)(8.31)\ln(4.00) = 6.34 \text{ J/K}$.

51. **THINK** Increasing temperature causes a shift of the probability distribution function $P(v)$ toward higher speed.

EXPRESS According to kinetic theory, the rms speed and the most probable speed are (see Eqs. 19-34 and 19-35) $v_{\text{rms}} = \sqrt{3RT/M}$, $v_p = \sqrt{2RT/M}$ and where T is the temperature and M is the molar mass. The rms speed is defined as $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v) dv$, with the Maxwell's speed distribution function given by

$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

Thus, the difference between the two speeds is

$$\Delta v = v_{\text{rms}} - v_p = \sqrt{\frac{3RT}{M}} - \sqrt{\frac{2RT}{M}} = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{RT}{M}}.$$

ANALYZE (a) With $M = 28 \text{ g/mol} = 0.028 \text{ kg/mol}$ (see Table 19-1), and $T_i = 250 \text{ K}$, we have

$$\Delta v_i = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{RT_i}{M}} = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{(8.31 \text{ J/mol} \cdot \text{K})(250 \text{ K})}{0.028 \text{ kg/mol}}} = 87 \text{ m/s}.$$

(b) Similarly, at $T_f = 500 \text{ K}$,

$$\Delta v_f = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{RT_f}{M}} = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{(8.31 \text{ J/mol} \cdot \text{K})(500 \text{ K})}{0.028 \text{ kg/mol}}} = 122 \text{ m/s} \approx 1.2 \times 10^2 \text{ m/s}.$$

(c) From Table 19-3 we have $C_V = 5R/2$ (see also Table 19-2). For $n = 1.5 \text{ mol}$, using Eq. 20-4, we find the change in entropy to be

$$\begin{aligned} \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) + nC_V \ln \left(\frac{T_f}{T_i} \right) = 0 + (1.5 \text{ mol})(5/2)(8.31 \text{ J/mol} \cdot \text{K}) \ln \left(\frac{500 \text{ K}}{250 \text{ K}} \right) \\ &= 22 \text{ J/K} \end{aligned}$$

LEARN Notice that the expression for Δv implies $T = \frac{M}{R(\sqrt{3}-\sqrt{2})^2}(\Delta v)^2$. Thus, one may also express ΔS as

$$\Delta S = n C_V \ln \left(\frac{T_f}{T_i} \right) = n C_V \ln \left(\frac{(\Delta v_f)^2}{(\Delta v_i)^2} \right) = 2n C_V \ln \left(\frac{\Delta v_f}{\Delta v_i} \right).$$

The entropy of the gas increases as the result of temperature increase.

52. (a) The most obvious input-heat step is the constant-volume process. Since the gas is monatomic, we know from Chapter 19 that $C_V = \frac{3}{2}R$. Therefore,

$$Q_V = n C_V \Delta T = (1.0 \text{ mol}) \left(\frac{3}{2} \right) \left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right) (600 \text{ K} - 300 \text{ K}) = 3740 \text{ J}.$$

Since the heat transfer during the isothermal step is positive, we may consider it also to be an input-heat step. The isothermal Q is equal to the isothermal work (calculated in the next part) because $\Delta E_{\text{int}} = 0$ for an ideal gas isothermal process (see Eq. 19-45). Borrowing from the part (b) computation, we have

$$Q_{\text{isotherm}} = n R T_H \ln 2 = (1 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right) (600 \text{ K}) \ln 2 = 3456 \text{ J}.$$

Therefore, $Q_H = Q_V + Q_{\text{isotherm}} = 7.2 \times 10^3 \text{ J}$.

(b) We consider the sum of works done during the processes (noting that no work is done during the constant-volume step). Using Eq. 19-14 and Eq. 19-16, we have

$$W = n R T_H \ln \left(\frac{V_{\text{max}}}{V_{\text{min}}} \right) + p_{\text{min}} (V_{\text{min}} - V_{\text{max}})$$

where, by the gas law in ratio form, the volume ratio is $\frac{V_{\text{max}}}{V_{\text{min}}} = \frac{T_H}{T_L} = \frac{600 \text{ K}}{300 \text{ K}} = 2$.

Thus, the net work is

$$\begin{aligned} W &= n R T_H \ln 2 + p_{\text{min}} V_{\text{min}} \left(1 - \frac{V_{\text{max}}}{V_{\text{min}}} \right) = n R T_H \ln 2 + n R T_L (1 - 2) = n R (T_H \ln 2 - T_L) \\ &= (1 \text{ mol}) \left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}} \right) ((600 \text{ K}) \ln 2 - (300 \text{ K})) \\ &= 9.6 \times 10^2 \text{ J}. \end{aligned}$$

(c) Equation 20-11 gives $\varepsilon = \frac{W}{Q_H} = 0.134 \approx 13\%$.

53. (a) If T_H is the temperature of the high-temperature reservoir and T_L is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(800 + 40) \text{ K}}{(800 + 273) \text{ K}} = 0.78 \text{ or } 78\%.$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_H|$, where W is the work done by the engine and Q_H is the heat input. W is positive. Over a complete cycle, $Q_H = W + |Q_L|$, where Q_L is the heat output, so $\varepsilon = W/(W + |Q_L|)$ and $|Q_L| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_H - T_L)/T_H$, where T_H is the temperature of the high-temperature heat reservoir and T_L is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_L}{T_H - T_L} \text{ and } |Q_L| = \frac{WT_L}{T_H - T_L}.$$

The heat output is used to melt ice at temperature $T_i = -40^\circ\text{C}$. The ice must be brought to 0°C , then melted, so

$$|Q_L| = mc(T_f - T_i) + mL_F,$$

where m is the mass of ice melted, T_f is the melting temperature (0°C), c is the specific heat of ice, and L_F is the heat of fusion of ice. Thus,

$$WT_L/(T_H - T_L) = mc(T_f - T_i) + mL_F.$$

We differentiate with respect to time and replace dW/dt with P , the power output of the engine, and obtain

$$PT_L/(T_H - T_L) = (dm/dt)[c(T_f - T_i) + L_F].$$

Therefore,

$$\frac{dm}{dt} = \left(\frac{PT_L}{T_H - T_L} \right) \left(\frac{1}{c(T_f - T_i) + L_F} \right).$$

Now, $P = 100 \times 10^6 \text{ W}$, $T_L = 0 + 273 = 273 \text{ K}$, $T_H = 800 + 273 = 1073 \text{ K}$, $T_i = -40 + 273 = 233 \text{ K}$, $T_f = 0 + 273 = 273 \text{ K}$, $c = 2220 \text{ J/kg} \cdot \text{K}$, and $L_F = 333 \times 10^3 \text{ J/kg}$, so

$$\begin{aligned} \frac{dm}{dt} &= \left[\frac{(100 \times 10^6 \text{ J/s})(273 \text{ K})}{1073 \text{ K} - 273 \text{ K}} \right] \left[\frac{1}{(2220 \text{ J/kg} \cdot \text{K})(273 \text{ K} - 233 \text{ K}) + 333 \times 10^3 \text{ J/kg}} \right] \\ &= 82 \text{ kg/s}. \end{aligned}$$

We note that the engine is now operated between 0°C and 800°C.

54. Equation 20-4 yields

$$\Delta S = nR \ln(V_f/V_i) + nC_V \ln(T_f/T_i) = 0 + nC_V \ln(425/380)$$

where $n = 3.20$ and $C_V = \frac{3}{2}R$ (Eq. 19-43). This gives 4.46 J/K.

55. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = 40.9^\circ\text{C},$$

which is equivalent to 314 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{copper}} = \int_{353}^{314} \frac{cm dT}{T} = (386)(0.600) \ln\left(\frac{314}{353}\right) = -27.1 \text{ J/K}.$$

(c) For water, the change in entropy is

$$\Delta S_{\text{water}} = \int_{283}^{314} \frac{cm dT}{T} = (4187)(0.0700) \ln\left(\frac{314}{283.15}\right) = 30.3 \text{ J/K}.$$

(d) The net result for the system is $(30.3 - 27.1) \text{ J/K} = 3.2 \text{ J/K}$. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

56. Using Hooke's law, we find the spring constant to be

$$k = \frac{F_s}{x_s} = \frac{1.50 \text{ N}}{0.0350 \text{ m}} = 42.86 \text{ N/m}.$$

To find the rate of change of entropy with a small additional stretch, we use Eq. 20-7 and obtain

$$\left| \frac{dS}{dx} \right| = \frac{k|x|}{T} = \frac{(42.86 \text{ N/m})(0.0170 \text{ m})}{275 \text{ K}} = 2.65 \times 10^{-3} \text{ J/K} \cdot \text{m}.$$

57. Since the volume of the monatomic ideal gas is kept constant, it does not do any work in the heating process. Therefore the heat Q it absorbs is equal to the change in its inertial

energy: $dQ = dE_{\text{int}} = \frac{3}{2}nR dT$. Thus,

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3nR/2)dT}{T} = \frac{3}{2}nR \ln\left(\frac{T_f}{T_i}\right) = \frac{3}{2}(1.00 \text{ mol})\left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) \ln\left(\frac{400 \text{ K}}{300 \text{ K}}\right) = 3.59 \text{ J/K}.$$

58. With the pressure kept constant,

$$dQ = nC_p dT = n(C_v + R)dT = \left(\frac{3}{2}nR + nR\right)dT = \frac{5}{2}nRdT,$$

so we need to replace the factor 3/2 in the last problem by 5/2. The rest is the same. Thus the answer now is

$$\Delta S = \frac{5}{2}nR \ln\left(\frac{T_f}{T_i}\right) = \frac{5}{2}(1.00 \text{ mol})\left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) \ln\left(\frac{400 \text{ K}}{300 \text{ K}}\right) = 5.98 \text{ J/K}.$$

59. **THINK** The temperature of the ice is first raised to 0°C, then the ice melts and the temperature of the resulting water is raised to 40°C. We want to calculate the entropy change in this process.

EXPRESS As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i ($= -20^\circ\text{C} = 253 \text{ K}$) is the initial temperature and T_f ($= 273 \text{ K}$) is the final temperature, then the change in its entropy is

$$\Delta S_1 = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\left(\frac{T_f}{T_i}\right) = (0.60 \text{ kg})(2220 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{273 \text{ K}}{253 \text{ K}}\right) = 101 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F , where L_F is the heat of fusion for ice. Thus,

$$\Delta S_2 = \frac{Q}{T} = \frac{mL_F}{T} = \frac{(0.60 \text{ kg})(333 \times 10^3 \text{ J/kg})}{273 \text{ K}} = 732 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S_3 = mc_w \ln\left(\frac{T_f}{T_i}\right) = (0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{313 \text{ K}}{273 \text{ K}}\right) = 344 \text{ J/K},$$

where $c_w = 4190 \text{ J/kg} \cdot \text{K}$ is the specific heat of water.

ANALYZE The total change in entropy for the ice and the water it becomes is

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = 101 \text{ J/K} + 732 \text{ J/K} + 344 \text{ J/K} = 1.18 \times 10^3 \text{ J/K}.$$

LEARN From the above, we readily see that the biggest increase in entropy comes from ΔS_2 , which accounts for the melting process.

60. The net work is figured from the (positive) isothermal expansion (Eq. 19-14) and the (negative) constant-pressure compression (Eq. 19-48). Thus,

$$W_{\text{net}} = nRT_H \ln(V_{\text{max}}/V_{\text{min}}) + nR(T_L - T_H)$$

where $n = 3.4$, $T_H = 500 \text{ K}$, $T_L = 200 \text{ K}$, and $V_{\text{max}}/V_{\text{min}} = 5/2$ (same as the ratio T_H/T_L). Therefore, $W_{\text{net}} = 4468 \text{ J}$. Now, we identify the “input heat” as that transferred in steps 1 and 2:

$$Q_{\text{in}} = Q_1 + Q_2 = nC_V(T_H - T_L) + nRT_H \ln(V_{\text{max}}/V_{\text{min}})$$

where $C_V = 5R/2$ (see Table 19-3). Consequently, $Q_{\text{in}} = 34135 \text{ J}$. Dividing these results gives the efficiency: $W_{\text{net}}/Q_{\text{in}} = 0.131$ (or about 13.1%).

61. Since the inventor’s claim implies that less heat (typically from burning fuel) is needed to operate his engine than, say, a Carnot engine (for the same magnitude of net work), then $Q_H' < Q_H$ (see Fig. 20-34(a)) which implies that the Carnot (ideal refrigerator) unit is delivering more heat to the high temperature reservoir than engine X draws from it. This (using also energy conservation) immediately implies Fig. 20-34(b), which violates the second law.

62. (a) From Eq. 20-1, we infer $Q = \int T dS$, which corresponds to the “area under the curve” in a T - S diagram. Thus, since the area of a rectangle is (height)×(width), we have

$$Q_{1 \rightarrow 2} = (350)(2.00) = 700 \text{ J}.$$

(b) With no “area under the curve” for process $2 \rightarrow 3$, we conclude $Q_{2 \rightarrow 3} = 0$.

(c) For the cycle, the (net) heat should be the “area inside the figure,” so using the fact that the area of a triangle is $\frac{1}{2}$ (base) × (height), we find

$$Q_{\text{net}} = \frac{1}{2} (2.00)(50) = 50 \text{ J}.$$

(d) Since we are dealing with an ideal gas (so that $\Delta E_{\text{int}} = 0$ in an isothermal process), then

$$W_{1 \rightarrow 2} = Q_{1 \rightarrow 2} = 700 \text{ J}.$$

(e) Using Eq. 19-14 for the isothermal work, we have

$$W_{1 \rightarrow 2} = nRT \ln \frac{V_2}{V_1}.$$

where $T = 350$ K. Thus, if $V_1 = 0.200$ m³, then we obtain

$$V_2 = V_1 \exp(W/nRT) = (0.200) e^{0.12} = 0.226 \text{ m}^3.$$

(f) Process $2 \rightarrow 3$ is adiabatic; Eq. 19-56 applies with $\gamma = 5/3$ (since only translational degrees of freedom are relevant here):

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}.$$

This yields $V_3 = 0.284$ m³.

(g) As remarked in part (d), $\Delta E_{\text{int}} = 0$ for process $1 \rightarrow 2$.

(h) We find the change in internal energy from Eq. 19-45 (with $C_V = \frac{3}{2}R$):

$$\Delta E_{\text{int}} = nC_V(T_3 - T_2) = -1.25 \times 10^3 \text{ J}.$$

(i) Clearly, the net change of internal energy for the entire cycle is zero. This feature of a closed cycle is as true for a T - S diagram as for a p - V diagram.

(j) For the adiabatic ($2 \rightarrow 3$) process, we have $W = -\Delta E_{\text{int}}$. Therefore, $W = 1.25 \times 10^3$ J. Its positive value indicates an expansion.

63. (a) It is a reversible set of processes returning the system to its initial state; clearly, $\Delta S_{\text{net}} = 0$.

(b) Process 1 is adiabatic and reversible (as opposed to, say, a free expansion) so that Eq. 20-1 applies with $dQ = 0$ and yields $\Delta S_1 = 0$.

(c) Since the working substance is an ideal gas, then an isothermal process implies $Q = W$, which further implies (regarding Eq. 20-1) $dQ = p dV$. Therefore,

$$\int \frac{dQ}{T} = \int \frac{p dV}{\left(\frac{pV}{nR}\right)} = nR \int \frac{dV}{V}$$

which leads to $\Delta S_3 = nR \ln(1/2) = -23.0$ J/K.

(d) By part (a), $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$. Then, part (b) implies $\Delta S_2 = -\Delta S_3$. Therefore, $\Delta S_2 = 23.0$ J/K.

64. (a) Combining Eq. 20-11 with Eq. 20-13, we obtain

$$|W| = |Q_H| \left(1 - \frac{T_L}{T_H} \right) = (500 \text{ J}) \left(1 - \frac{260 \text{ K}}{320 \text{ K}} \right) = 93.8 \text{ J}.$$

(b) Combining Eq. 20-14 with Eq. 20-16, we find

$$|W| = \frac{|Q_L|}{\left(\frac{T_L}{T_H - T_L} \right)} = \frac{1000 \text{ J}}{\left(\frac{260 \text{ K}}{320 \text{ K} - 260 \text{ K}} \right)} = 231 \text{ J}.$$

65. (a) Processes 1 and 2 both require the input of heat, which is denoted Q_H . Noting that rotational degrees of freedom are not involved, then, from the discussion in Chapter 19, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$. We further note that since the working substance is an ideal gas, process 2 (being isothermal) implies $Q_2 = W_2$. Finally, we note that the volume ratio in process 2 is simply $8/3$. Therefore,

$$Q_H = Q_1 + Q_2 = nC_V(T' - T) + nRT' \ln \frac{8}{3}$$

which yields (for $T = 300 \text{ K}$ and $T' = 800 \text{ K}$) the result $Q_H = 25.5 \times 10^3 \text{ J}$.

(b) The net work is the net heat ($Q_1 + Q_2 + Q_3$). We find Q_3 from

$$nC_p(T - T') = -20.8 \times 10^3 \text{ J}.$$

Thus, $W = 4.73 \times 10^3 \text{ J}$.

(c) Using Eq. 20-11, we find that the efficiency is

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{4.73 \times 10^3}{25.5 \times 10^3} = 0.185 \text{ or } 18.5\%.$$

66. (a) Equation 20-14 gives $K = 560/150 = 3.73$.

(b) Energy conservation requires the exhaust heat to be $560 + 150 = 710 \text{ J}$.

67. The change in entropy in transferring a certain amount of heat Q from a heat reservoir at T_1 to another one at T_2 is $\Delta S = \Delta S_1 + \Delta S_2 = Q(1/T_2 - 1/T_1)$.

(a) $\Delta S = (260 \text{ J})(1/100 \text{ K} - 1/400 \text{ K}) = 1.95 \text{ J/K}$.

(b) $\Delta S = (260 \text{ J})(1/200 \text{ K} - 1/400 \text{ K}) = 0.650 \text{ J/K}$.

(c) $\Delta S = (260 \text{ J})(1/300 \text{ K} - 1/400 \text{ K}) = 0.217 \text{ J/K}$.

(d) $\Delta S = (260 \text{ J})(1/360 \text{ K} - 1/400 \text{ K}) = 0.072 \text{ J/K}$.

(e) We see that as the temperature difference between the two reservoirs decreases, so does the change in entropy.

68. Equation 20-10 gives

$$\left| \frac{Q_{\text{to}}}{Q_{\text{from}}} \right| = \frac{T_{\text{to}}}{T_{\text{from}}} = \frac{300 \text{ K}}{4.0 \text{ K}} = 75.$$

69. (a) Equation 20-2 gives the entropy change for each reservoir (each of which, by definition, is able to maintain constant temperature conditions within itself). The net entropy change is therefore

$$\Delta S = \frac{+|Q|}{273 + 24} + \frac{-|Q|}{273 + 130} = 4.45 \text{ J/K}$$

where we set $|Q| = 5030 \text{ J}$.

(b) We have assumed that the conductive heat flow in the rod is “steady-state”; that is, the situation described by the problem has existed and will exist for “long times.” Thus there are no entropy change terms included in the calculation for elements of the rod itself.

70. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = -44.2^\circ\text{C},$$

which is equivalent to 229 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{tungsten}} = \int_{303}^{229} \frac{cm dT}{T} = (134)(0.045) \ln\left(\frac{229}{303}\right) = -1.69 \text{ J/K}.$$

(c) Also,

$$\Delta S_{\text{silver}} = \int_{153}^{229} \frac{cm dT}{T} = (236)(0.0250) \ln\left(\frac{229}{153}\right) = 2.38 \text{ J/K}.$$

(d) The net result for the system is $(2.38 - 1.69) \text{ J/K} = 0.69 \text{ J/K}$. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

71. (a) We use Eq. 20-16. For configuration A

$$W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{50!}{(25!)(25!)} = 1.26 \times 10^{14}.$$

(b) For configuration B

$$W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{50!}{[0.6(50)]![0.4(50)]!} = 4.71 \times 10^{13}.$$

(c) Since all microstates are equally probable,

$$f = \frac{W_B}{W_A} = \frac{1265}{3393} \approx 0.37.$$

We use these formulas for $N = 100$. The results are

$$(d) W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{100!}{(50!)(50!)} = 1.01 \times 10^{29},$$

$$(e) W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{100!}{[0.6(100)]![0.4(100)]!} = 1.37 \times 10^{28},$$

(f) and $f = W_B/W_A \approx 0.14$.

Similarly, using the same formulas for $N = 200$, we obtain

$$(g) W_A = 9.05 \times 10^{58},$$

$$(h) W_B = 1.64 \times 10^{57},$$

(i) and $f = 0.018$.

(j) We see from the calculation above that f decreases as N increases, as expected.

72. A metric ton is 1000 kg, so that the heat generated by burning 380 metric tons during one hour is $(380000 \text{ kg})(28 \text{ MJ/kg}) = 10.6 \times 10^6 \text{ MJ}$. The work done in one hour is

$$W = (750 \text{ MJ/s})(3600 \text{ s}) = 2.7 \times 10^6 \text{ MJ}$$

where we use the fact that a watt is a joule-per-second. By Eq. 20-11, the efficiency is

$$\varepsilon = \frac{2.7 \times 10^6 \text{ MJ}}{10.6 \times 10^6 \text{ MJ}} = 0.253 = 25\%.$$

73. **THINK** The performance of the Carnot refrigerator is related to its rate of doing work.

EXPRESS The coefficient of performance for a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where Q_L is the energy absorbed from the cold reservoir (interior of refrigerator) as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected as heat to the hot reservoir (the room). Thus, $Q_L = W - Q_H$. Q_H is negative and greater in magnitude than W , so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|Q_H| = |W|(1 + K) = |Q_L|(1 + K)/K$.

ANALYZE (a) From the expression above, the energy per cycle transferred as heat to the room is

$$|Q_H| = |Q_L| \frac{1 + K}{K} = (35.0 \text{ kJ}) \frac{1 + 4.60}{4.60} = 42.6 \text{ kJ}.$$

(b) Similarly, the work done per cycle is $|W| = \frac{|Q_L|}{K} = \frac{35.0 \text{ kJ}}{4.60} = 7.61 \text{ kJ}$.

LEARN A Carnot refrigerator is a Carnot engine operating in reverse. Its coefficient of performance can also be written as

$$K = \frac{T_L}{T_H - T_L}$$

The value of K is higher when the temperatures of the two reservoirs are close to each other.

74. The Carnot efficiency (Eq. 20-13) depends linearly on T_L so that we can take a derivative

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow \frac{d\varepsilon}{dT_L} = -\frac{1}{T_H}$$

and quickly get to the result. With $d\varepsilon \rightarrow \Delta\varepsilon = 0.100$ and $T_H = 400 \text{ K}$, we find $dT_L \rightarrow \Delta T_L = -40 \text{ K}$.

75. THINK The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

EXPRESS In general, if there are N molecules and if the box is divided into two halves, with n_L molecules in the left half and n_R in the right half, such that $n_L + n_R = N$. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the left half, and $n_R!$ are rearrangements of the n_R molecules in the right half. These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_L!n_R!}.$$

The entropy is given by $S = k \ln W$.

ANALYZE (a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, for system A with $N = 3$, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box B, with $N = 5$, $W = 5!/(5!0!) = 1$ in the “least” case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Therefore,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k \ln W = (1.38 \times 10^{-23}) \ln 3 = 1.5 \times 10^{-23} \text{ J/K}.$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k \ln 10 = 3.2 \times 10^{-23} \text{ J/K}.$$

LEARN The least multiplicity is $W = 1$; this happens when $n_L = N$ or $n_L = 0$. On the other hand, the greatest multiplicity occurs when $n_L = (N-1)/2$ or $n_L = (N+1)/2$.

76. (a) Using $Q = T\Delta S$, we note that heat enters the cycle along the top path at 400 K, and leaves along the bottom path at 250 K. Thus,

$$\begin{aligned} Q_{\text{in}} &= (400 \text{ K})(0.60 \text{ J/K} - 0.10 \text{ J/K}) = 200 \text{ J} \\ Q_{\text{out}} &= (250 \text{ K})(0.10 \text{ J/K} - 0.60 \text{ J/K}) = -125 \text{ J} \end{aligned}$$

and the net heat transfer is $Q = Q_{\text{in}} + Q_{\text{out}} = 200 \text{ J} - 125 \text{ J} = 75 \text{ J}$.

(b) For cyclic path, $\Delta E_{\text{int}} = Q - W = 0$. Therefore, the work done by the system is $W = Q = 75 \text{ J}$.

77. The efficiency of an ideal heat engine and coefficient of performance of a reversible refrigerator are

$$\varepsilon = \frac{|W|}{|Q_{\text{H}}|}, \quad K = \frac{|Q_{\text{H}}| - |W|}{|W|}.$$

Thus,

$$K = \frac{|Q_{\text{H}}| - |W|}{|W|} = \frac{|Q_{\text{H}}|}{|W|} - 1 = \frac{1}{\varepsilon} - 1 \Rightarrow \varepsilon = \frac{1}{K + 1}$$

78. (a) The efficiency is defined by $\varepsilon = |W|/|Q_{\text{H}}|$, where W is the work done by the engine and Q_{H} is the heat input. In our case, the temperatures of the hot and cold reservoirs are $T_{\text{H}} = 100^\circ\text{C} = 373 \text{ K}$ and $T_{\text{L}} = 60^\circ\text{C} = 333 \text{ K}$, respectively. The maximum efficiency of the engine is

$$\varepsilon = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{333 \text{ K}}{373 \text{ K}} = 0.107.$$

Thus, the rate of heat input is

$$\frac{dQ_{\text{H}}}{dt} = \frac{1}{\varepsilon} \frac{dW}{dt} = \frac{1}{0.107} (500 \text{ W}) = 4.66 \times 10^3 \text{ W}.$$

(b) The rate of exhaust heat output is

$$\frac{dQ_{\text{L}}}{dt} = \frac{dQ_{\text{H}}}{dt} - \frac{dW}{dt} = 4.66 \times 10^3 \text{ W} - 500 \text{ W} = 4.16 \times 10^3 \text{ W}.$$

79. The temperatures of the hot and cold reservoirs are $T_{\text{H}} = 26^\circ\text{C} = 299 \text{ K}$ and $T_{\text{L}} = -13^\circ\text{C} = 260 \text{ K}$, respectively. Therefore, the theoretical coefficient of performance of the refrigerator is

$$K = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}} = \frac{260 \text{ K}}{299 \text{ K} - 260 \text{ K}} = 6.67.$$