Chapter 14

1. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08 \text{ g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00 \text{ g/cm}^3$$

where ρ_w is the density of the water. This implies

$$\rho_{\text{fish}} V = \rho_w (V + V_a) \text{ or } (V + V_a)/V = 1.08/1.00,$$

which gives $V_a/(V + V_a) = 0.074 = 7.4\%$.

2. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\text{N/m}^2 = 1$ Pa, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0\times10^5 \text{ Pa} - \frac{480 \text{ N}}{77\times10^{-4} \text{ m}^2} = 3.8\times10^4 \text{ Pa}.$$

3. **THINK** The increase in pressure is equal to the applied force divided by the area.

EXPRESS The change in pressure is given by $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston.

ANALYZE substituting the values given, we obtain

$$\Delta p = (42 \text{ N})/\pi(0.011 \text{ m})^2 = 1.1 \times 10^5 \text{ Pa}.$$

This is equivalent to 1.1 atm.

LEARN The increase in pressure is proportional to the force applied. In addition, since $\Delta p \sim 1/A$, the smaller the cross-sectional area of the syringe, the greater the pressure increase under the same applied force.

4. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that 1L = 1000 cm^3, we find the weight of the first liquid to be

$$W_1 = m_1 g = \rho_1 V_1 g = (2.6\ \text{g}/\text{cm}^3)(0.50\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 1.27\times10^6\,\text{g}\cdot\text{cm/s}^2$$
$$= 12.7\ \text{N}.$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0\ \text{g/cm}^3)(0.25\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 2.5\ \text{N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80\ \text{g/cm}^3)(0.40\ \text{L})(1000\ \text{cm}^3/\text{L})(980\ \text{cm/s}^2) = 3.1\ \text{N}.$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18$ N.

5. **THINK** The pressure difference between two sides of the window results in a net force acting on the window.

EXPRESS The air inside pushes outward with a force given by p_iA, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by p_oA, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$.

ANALYZE Since 1 atm = 1.013×10^5 Pa, the net force is

$$F = (p_i - p_o)A = (1.0\ \text{atm} - 0.96\ \text{atm})(1.013\times10^5\ \text{Pa/atm})(3.4\ \text{m})(2.1\ \text{m})$$
$$= 2.9\times10^4\ \text{N}.$$

LEARN The net force on the window vanishes when the pressure inside the office is equal to the pressure outside.

6. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

(a) $P = \left(28\ \text{lb/in.}^2\right)\left(\dfrac{1.01\times10^5\ \text{Pa}}{14.7\ \text{lb/in}^2}\right) = 190\ \text{kPa}$.

(b) $(120\ \text{mmHg})\left(\dfrac{1.01\times10^5\,\text{Pa}}{760\ \text{mmHg}}\right) = 15.9\ \text{kPa}, \quad (80\ \text{mmHg})\left(\dfrac{1.01\times10^5\ \text{Pa}}{760\ \text{mmHg}}\right) = 10.6\ \text{kPa}.$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle θ. Its leftward component is $\Delta p \cos\theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R \sin\theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\, d\theta$ and its area is $dA = 2\pi R^2 \sin\theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \ \sin^2\theta \Big|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = 1.01×10^5 Pa to show that Δp = 0.90 atm = 9.09×10^4 Pa. The sphere radius is R = 0.30 m, so

$$F_h = \pi(0.30 \text{ m})^2(9.09 \times 10^4 \text{ Pa}) = 2.6 \times 10^4 \text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. Using Eq. 14-7, we find the gauge pressure to be $p_{\text{gauge}} = \rho g h$, where ρ is the density of the fluid medium, and h is the vertical distance to the point where the pressure is equal to the atmospheric pressure.

The gauge pressure at a depth of 20 m in seawater is

$$p_1 = \rho_{\text{sw}} g d = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(20 \text{ m}) = 2.00 \times 10^5 \text{ Pa}.$$

On the other hand, the gauge pressure at an altitude of 7.6 km is

$$p_2 = \rho_{\text{air}} g h = (0.87 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(7600 \text{ m}) = 6.48 \times 10^4 \text{ Pa}.$$

Therefore, the change in pressure is

$$\Delta p = p_1 - p_2 = 2.00 \times 10^5 \text{ Pa} - 6.48 \times 10^4 \text{ Pa} \approx 1.4 \times 10^5 \text{ Pa}.$$

9. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the heart of the *Argentinosaurus* is

$$p_{\text{heart}} = p_{\text{brain}} + \rho g h = 80 \text{ torr} + (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m} - 9.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 1.0 \times 10^3 \text{ torr}.$$

(b) The gauge pressure at the feet of the *Argentinosaurus* is

$$p_{\text{feet}} = p_{\text{brain}} + \rho g h' = 80\text{ torr} + (1.06\times 10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(21\text{ m})\left(\frac{1\text{ torr}}{133.33\text{ Pa}}\right)$$
$$= 80\text{ torr} + 1642\text{ torr} = 1722\text{ torr} \approx 1.7\times 10^3\text{ torr.}$$

10. With A = 0.000500 m^2 and $F = pA$ (with p given by Eq. 14-9), then we have ρghA = 9.80 N. This gives $h \approx 2.0$ m, which means $d + h = 2.80$ m.

11. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the brain of the giraffe is

$$p_{\text{brain}} = p_{\text{heart}} - \rho g h = 250\text{ torr} - (1.06\times 10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(2.0\text{ m})\left(\frac{1\text{ torr}}{133.33\text{ Pa}}\right)$$
$$= 94\text{ torr.}$$

(b) The gauge pressure at the feet of the giraffe is

$$p_{\text{feet}} = p_{\text{heart}} + \rho g h = 250\text{ torr} + (1.06\times 10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(2.0\text{ m})\left(\frac{1\text{ torr}}{133.33\text{ Pa}}\right) = 406\text{ torr}$$
$$\approx 4.1\times 10^2\text{ torr.}$$

(c) The increase in the blood pressure at the brain as the giraffe lowers its head to the level of its feet is

$$\Delta p = p_{\text{feet}} - p_{\text{brain}} = 406\text{ torr} - 94\text{ torr} = 312\text{ torr} \approx 3.1\times 10^2\text{ torr.}$$

12. Note that 0.05 atm equals 5065 Pa. Application of Eq. 14-7 with the notation in this problem leads to

$$d_{\max} = \frac{p}{\rho_{\text{liquid}}\, g} = \frac{0.05\text{ atm}}{\rho_{\text{liquid}}\, g} = \frac{5065\text{ Pa}}{\rho_{\text{liquid}}\, g}\,.$$

Thus the difference of this quantity between fresh water (998 kg/m^3) and Dead Sea water (1500 kg/m^3) is

$$\Delta d_{\max} = \frac{5065\text{ Pa}}{g}\left(\frac{1}{\rho_{\text{fw}}} - \frac{1}{\rho_{\text{sw}}}\right) = \frac{5065\text{ Pa}}{9.8\text{ m/s}^2}\left(\frac{1}{998\text{ kg/m}^3} - \frac{1}{1500\text{ kg/m}^3}\right) = 0.17\text{ m.}$$

13. Recalling that 1 atm = 1.01×10^5 Pa, Eq. 14-8 leads to

$$\rho gh = (1024\ \text{kg/m}^3)\ (9.80\ \text{m/s}^2)\ (10.9 \times 10^3\ \text{m}) \left(\frac{1\ \text{atm}}{1.01 \times 10^5\ \text{Pa}} \right) \approx 1.08 \times 10^3\ \text{atm}.$$

14. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho gh = (1.06 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(1.83\ \text{m}) = 1.90 \times 10^4\ \text{Pa}.$$

15. In this case, Bernoulli's equation reduces to Eq. 14-10. Thus,

$$p_g = \rho g(-h) = -(1800\,\text{kg/m}^3)(9.8\ \text{m/s}^2)(1.5\ \text{m}) = -2.6 \times 10^4\ \text{Pa}\ .$$

16. At a depth h without the snorkel tube, the external pressure on the diver is $p = p_0 + \rho gh$, where p_0 is the atmospheric pressure. Thus, with a snorkel tube of length h, the pressure difference between the internal air pressure and the water pressure against the body is

$$\Delta p = p = p_0 = \rho gh\,.$$

(a) If $h = 0.20$ m, then

$$\Delta p = \rho gh = (998\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(0.20\,\text{m}) \frac{1\,\text{atm}}{1.01 \times 10^5\ \text{Pa}} = 0.019\,\text{atm}\ .$$

(b) Similarly, if $h = 4.0$ m, then

$$\Delta p = \rho gh = (998\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(4.0\,\text{m}) \frac{1\,\text{atm}}{1.01 \times 10^5\ \text{Pa}} \approx 0.39\,\text{atm}\ .$$

17. **THINK** The minimum force that must be applied to open the hatch is equal to the gauge pressure times the area of the hatch.

EXPRESS The pressure p at the depth d of the hatch cover is $p_0 + \rho gd$, where ρ is the density of ocean water and p_0 is atmospheric pressure. Thus, the gauge pressure is $p_{\text{gauge}} = \rho gd$, and the minimum force that must be applied by the crew to open the hatch has magnitude $F = p_{\text{gauge}} A = (\rho gd)A$, where A is the area of the hatch.

Substituting the values given, we find the force to be

$$\begin{aligned} F &= p_{\text{gauge}} A = (\rho gd)A = (1024\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(100\ \text{m})(1.2\ \text{m})(0.60\ \text{m}) \\ &= 7.2 \times 10^5\ \text{N}. \end{aligned}$$

LEARN The downward force of the water on the hatch cover is $(p_0 + \rho gd)A$, and the air in the submarine exerts an upward force of $p_0 A$. The greater the depth of the submarine, the greater the force required to open the hatch.

18. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

19. We can integrate the pressure (which varies linearly with depth according to Eq. 14-7) over the area of the wall to find out the net force on it, and the result turns out fairly intuitive (because of that linear dependence): the force is the "average" water pressure multiplied by the area of the wall (or at least the part of the wall that is exposed to the water), where "average" pressure is taken to mean $\frac{1}{2}$(pressure at surface + pressure at bottom). Assuming the pressure at the surface can be taken to be zero (in the gauge pressure sense explained in section 14-4), then this means the force on the wall is $\frac{1}{2}\rho gh$ multiplied by the appropriate area. In this problem the area is hw (where w is the 8.00 m width), so the force is $\frac{1}{2}\rho gh^2 w$, and the change in force (as h is changed) is

$$\tfrac{1}{2}\rho gw\,(h_f^{\,2} - h_i^{\,2}) = \tfrac{1}{2}(998\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(8.00\ \text{m})(4.00^2 - 2.00^2)\text{m}^2 = 4.69 \times 10^5\ \text{N}.$$

20. (a) The force on face A of area A_A due to the water pressure alone is

$$F_A = p_A A_A = \rho_w g h_A A_A = \rho_w g (2d) d^2 = 2\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(5.0\,\text{m}\right)^3$$
$$= 2.5\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0 \times 10^5\ \text{Pa})(5.0\ \text{m})^2 = 2.5 \times 10^6\ \text{N},$$

we have

$$F_A' = F_0 + F_A = 2.5\times10^6\ \text{N} + 2.5\times10^6\ \text{N} = 5.0\times10^6\ \text{N}.$$

(b) The force on face B due to water pressure alone is

$$F_B = p_{\text{avg}B} A_B = \rho_\omega g \left(\frac{5d}{2}\right) d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(5.0\,\text{m}\right)^3$$
$$= 3.1\times10^6\ \text{N}.$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0 \times 10^5\ \text{Pa})(5.0\ \text{m})^2 = 2.5 \times 10^6\ \text{N},$$

we obtain

$$F_B' = F_0 + F_B = 2.5\times10^6\ \text{N} + 3.1\times10^6\ \text{N} = 5.6\times10^6\ \text{N}.$$

21. **THINK** Work is done to remove liquid from one vessel to another.

EXPRESS When the levels are the same, the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid from the first vessel with volume $V = A(h_1 - h)$ and mass $m = \rho V = \rho A(h_1 - h)$, and lowering it a distance $\Delta y = h - h_2$. The work done by the force of gravity is

$$W_g = mg\Delta y = \rho A(h_1 - h)g(h - h_2).$$

ANALYZE We substitute $h = (h_1 + h_2)/2$ to obtain

$$W_g = \frac{1}{4}\rho gA\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\,\text{kg/m}^3)(9.80\text{ m/s}^2)(4.00\times10^{-4}\,\text{m}^2)(1.56\text{ m} - 0.854\text{ m})^2$$
$$= 0.635\text{ J}$$

LEARN Since gravitational force is conservative, the work done only depends on the initial and final heights of the vessels, and not on how the liquid is transferred.

22. To find the pressure at the brain of the pilot, we note that the inward acceleration can be treated from the pilot's reference frame as though it is an outward gravitational acceleration against which the heart must push the blood. Thus, with $a = 4g$, we have

$$p_{\text{brain}} = p_{\text{heart}} - \rho ar = 120\text{ torr} - (1.06\times10^3\text{ kg/m}^3)(4\times9.8\text{ m/s}^2)(0.30\text{ m})\left(\frac{1\text{ torr}}{133\text{ Pa}}\right)$$
$$= 120\text{ torr} - 94\text{ torr} = 26\text{ torr.}$$

23. Letting $p_a = p_b$, we find

$$\rho_c g(6.0\text{ km} + 32\text{ km} + D) + \rho_m(y - D) = \rho_c g(32\text{ km}) + \rho_m y$$

and obtain

$$D = \frac{(6.0\,\text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0\,\text{km})(2.9\,\text{g/cm}^3)}{3.3\,\text{g/cm}^3 - 2.9\,\text{g/cm}^3} = 44\,\text{km.}$$

24. (a) At depth y the gauge pressure of the water is $p = \rho gy$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho gyW\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho gyW\,dy = \frac{1}{2}\rho gWD^2 = \frac{1}{2}\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(314\,\text{m}\right)\left(35.0\,\text{m}\right)^2$$
$$= 1.88\times10^9\,\text{N.}$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is

$$d\tau = dF(D-y) = \rho g y W (D-y)dy$$

and the total torque of the water is

$$\tau = \int_0^D \rho g y W (D-y)\,dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3$$
$$= \frac{1}{6}\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(314\ \text{m}\right)\left(35.0\ \text{m}\right)^3 = 2.20\times10^{10}\ \text{N}\cdot\text{m}.$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} = \frac{35.0\ \text{m}}{3} = 11.7\ \text{m}.$$

25. As shown in Eq. 14-9, the atmospheric pressure p_0 bearing down on the barometer's mercury pool is equal to the pressure $\rho g h$ at the base of the mercury column: $p_0 = \rho g h$. Substituting the values given in the problem statement, we find the atmospheric pressure to be

$$p_0 = \rho g h = (1.3608\times10^4\ \text{kg/m}^3)(9.7835\ \text{m/s}^2)(0.74035\ \text{m})\left(\frac{1\ \text{torr}}{133.33\ \text{Pa}}\right)$$
$$= 739.26\ \text{torr}.$$

26. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)\left(4.0\times10^{-2}\ \text{m}\right)}{1.01\times10^5\ \text{Pa/atm}} = -3.9\times10^{-3}\,\text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

27. **THINK** The atmospheric pressure at a given height depends on the density distribution of air.

EXPRESS If the air density were uniform, ρ =const., then the variation of pressure with height may be written as: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. On the other hand, if the density varies with altitude, then

$$p_2 = p_1 - \int_0^h \rho g\ dy\ .$$

For the case where the density decreases linearly with height, $\rho = \rho_0\,(1 - y/h)$, where ρ_0 is the density at Earth's surface and g = 9.8 m/s^2 for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g\left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 gh.$$

ANALYZE (a) For uniform density with ρ = 1.3 kg/m^3, we find the height of the atmosphere to be

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01\times 10^5\ \text{Pa}}{(1.3\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 7.9\times 10^3\ \text{m} = 7.9\ \text{km}.$$

(b) With density decreasing linearly with height, $p_2 = p_1 - \rho_0 gh/2$. The condition $p_2 = 0$ implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01\times 10^5\ \text{Pa})}{(1.3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 16\times 10^3\ \text{m} = 16\ \text{km}.$$

LEARN Actually the decrease in air density is approximately exponential, with pressure halved at a height of about 5.6 km.

28. (a) According to Pascal's principle, $F/A = f/a \to F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\ \text{cm})^2}{(53.0\ \text{cm})^2}(20.0\times 10^3\ \text{N}) = 103\ \text{N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

29. Equation 14-13 combined with Eq. 5-8 and Eq. 7-21 (in absolute value) gives

$$mg = kx\frac{A_1}{A_2}.$$

With $A_2 = 18A_1$ (and the other values given in the problem) we find m = 8.50 kg.

30. Taking "down" as the positive direction, then using Eq. 14-16 in Newton's second law, we have $(5.00\ \text{kg})g - (3.00\ \text{kg})g = 5a$. This gives $a = \frac{2}{5}g$ = 3.92 m/s^2, where g = 9.8 m/s^2. Then (see Eq. 2-15) $\frac{1}{2}at^2$ = 0.0784 m (in the downward direction).

31. **THINK** The block floats in both water and oil. We apply Archimedes' principle to analyze the problem.

EXPRESS Let V be the volume of the block. Then, the submerged volume in water is $V_s = 2V/3$. Since the block is floating, by Archimedes' principle the weight of the displaced water is equal to the weight of the block, i.e., $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block.

ANALYZE (a) We substitute $V_s = 2V/3$ to obtain the density of the block:

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) Now, if ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s' = \rho_b V$. Since the volume submerged in oil is $V_s' = 0.90V$, the density of the oil is

$$\rho_o = \rho_b\left(\frac{V}{V'}\right) = (6.7\times10^2 \text{ kg/m}^3)\frac{V}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

LEARN Another way to calculate the density of the oil is to note that the mass of the block can be written as

$$m = \rho_b V = \rho_o V_s' = \rho_w V_s.$$

Therefore,

$$\rho_o = \rho_w\left(\frac{V_s}{V_s'}\right) = (1000 \text{ kg/m}^3)\frac{2V/3}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

That is, by comparing the fraction submerged with that in water (or another liquid with known density), the density of the oil can be deduced.

32. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\text{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\text{top}} = p_{\text{atm}} + \rho g h_{\text{top}} = 1.01\times10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.300 \text{ m}) = 1.04\times10^5 \text{ Pa}$$

where the unit Pa (pascal) is equivalent to N/m^2. The force on the top surface (of area $A = L^2 = 0.36$ m^2) is

$$F_{\text{top}} = p_{\text{top}} A = 3.75\times10^4 \text{ N}.$$

(b) The pressure at a depth of $h_{\text{bot}} = 3L/2$ (that of the bottom of the block) is

$$\begin{aligned} p_{\text{bot}} &= p_{\text{atm}} + \rho g h_{\text{bot}} = 1.01\times10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.900 \text{ m}) \\ &= 1.10\times10^5 \text{ Pa} \end{aligned}$$

where we recall that the unit Pa (pascal) is equivalent to N/m^2. The force on the bottom surface is

$$F_{\text{bot}} = p_{\text{bot}} A = 3.96\times10^4 \text{ N}.$$

(c) Taking the difference $F_{\text{bot}} - F_{\text{top}}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{\text{bot}} - F_{\text{top}} = \rho g\,(h_{\text{bot}} - h_{\text{top}})\;A = \rho g L^3 = 2.18\times10^3\ \text{N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{\text{bot}} - F_{\text{top}}) = (450\ \text{kg})(9.80\ \text{m/s}^2) - 2.18\times10^3\ \text{N} = 2.23\times10^3\ \text{N}.$$

(d) This has already been noted in the previous part: $F_b = 2.18\times10^3\ \text{N}$, and $T + F_b = mg$.

33. **THINK** The iron anchor is submerged in water, so we apply Archimedes' principle to calculate its volume and weight in air.

EXPRESS The anchor is completely submerged in water of density ρ_w. Its apparent weight is $W_{\text{app}} = W - F_b$, where $W = mg$ is its actual weight and $F_b = \rho_w gV$ is the buoyant force.

ANALYZE (a) Substituting the values given, we find the volume of the anchor to be

$$V = \frac{W - W_{\text{app}}}{\rho_w g} = \frac{F_b}{\rho_w g} = \frac{200\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 2.04\times10^{-2}\ \text{m}^3.$$

(b) The mass of the anchor is $m = \rho_{\text{Fe}} g$, where ρ_{Fe} is the density of iron (found in Table 14-1). Therefore, its weight in air is

$$W = mg = \rho_{\text{Fe}} Vg = (7870\ \text{kg/m}^3)(2.04\times10^{-2}\ \text{m}^3)(9.80\ \text{m/s}^2) = 1.57\times10^3\ \text{N}.$$

LEARN In general, the apparent weight of an object of density ρ that is completely submerged in a fluid of density ρ_f can be written as $W_{\text{app}} = (\rho - \rho_f)Vg$.

34. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid that corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is W = 35.6 kN. In salt water of density ρ' = 1100 kg/m^3, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{3.56\times10^3\,\text{N}}{(1.10\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 3.30\ \text{m}^3.$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63 \text{ m}^3$, where $\rho = 1000 \text{ kg/m}^3$. The difference is $V - V' = 0.330 \text{ m}^3$.

35. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log:

$$V = \pi(0.15 \text{ m})^2 (1.80 \text{ m}) = 0.13 \text{ m}^3.$$

The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$, where we require $V_{\text{submerged}} \le NV$. The density of water is 1000 kg/m^3. To obtain the minimum value of N, we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \;\;\Rightarrow\;\; N = \frac{3(356\,\text{N})}{gV\left(\rho_{\text{water}} - \rho_{\text{wood}}\right)}$$

which yields $N = 4.28 \rightarrow 5$ logs.

36. From the "kink" in the graph it is clear that $d = 1.5$ cm. Also, the $h = 0$ point makes it clear that the (true) weight is 0.25 N. We now use Eq. 14-19 at $h = d = 1.5$ cm to obtain

$$F_b = (0.25 \text{ N} - 0.10 \text{ N}) = 0.15 \text{ N}.$$

Thus, $\rho_{\text{liquid}}\, g\, V = 0.15$, where

$$V = (1.5 \text{ cm})(5.67 \text{ cm}^2) = 8.5 \times 10^{-6} \text{ m}^3.$$

Thus, $\rho_{\text{liquid}} = 1800 \text{ kg/m}^3 = 1.8 \text{ g/cm}^3$.

37. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60 \text{ cm}.$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}}g \;\;\Rightarrow\;\; \rho_{\text{water}}gV_{\text{submerged}} = \rho_{\text{iron}}g\left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

where r_i is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm^3) and iron (7.87 g/cm^3), we obtain the inner diameter:

$$2r_i = 2r_o\left(1 - \frac{1.0 \text{ g/cm}^3}{7.87 \text{ g/cm}^3}\right)^{1/3} = 57.3 \text{ cm}.$$

38. (a) An object of the same density as the surrounding liquid (in which case the "object" could just be a packet of the liquid itself) is not going to accelerate up or down (and thus won't gain any kinetic energy). Thus, the point corresponding to zero K in the graph must correspond to the case where the density of the object equals ρ_{liquid}. Therefore, $\rho_{\text{ball}} = 1.5$ g/cm^3 (or 1500 kg/m^3).

(b) Consider the $\rho_{\text{liquid}} = 0$ point (where $K_{\text{gained}} = 1.6$ J). In this case, the ball is falling through perfect vacuum, so that $v^2 = 2gh$ (see Eq. 2-16) which means that $K = \frac{1}{2}mv^2 = 1.6$ J can be used to solve for the mass. We obtain $m_{\text{ball}} = 4.082$ kg. The volume of the ball is then given by

$$m_{\text{ball}}/\rho_{\text{ball}} = 2.72 \times 10^{-3} \text{ m}^3.$$

39. **THINK** The hollow sphere is half submerged in a fluid. We apply Archimedes' principle to calculate its mass and density.

EXPRESS The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where r_o is the outer radius.

ANALYZE (a) Substituting the values given, we find the mass of the sphere to be

$$m = \rho V_s = \rho\left(\frac{1}{2}\cdot\frac{4\pi}{3}r_o^3\right) = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)(800 \text{ kg/m}^3)(0.090 \text{ m})^3 = 1.22 \text{ kg}.$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090 \text{ m})^3 - (0.080 \text{ m})^3\right) = 9.09 \times 10^{-4} \text{ m}^3.$$

The density is

$$\rho_m = \frac{1.22 \text{ kg}}{9.09 \times 10^{-4} \text{ m}^3} = 1.3 \times 10^3 \text{ kg/m}^3.$$

LEARN Note that $\rho_m > \rho$, i.e., the density of the material is greater that of the fluid. However, the sphere floats (and displaces its own weight of fluid) because it's hollow.

40. If the alligator floats, by Archimedes' principle the buoyancy force is equal to the alligator's weight (see Eq. 14-17). Therefore,

$$F_b = F_g = m_{\mathrm{H_2O}} g = (\rho_{\mathrm{H_2O}} A h) g \,.$$

If the mass is to increase by a small amount $m \to m' = m + \Delta m$, then

$$F_b \to F_b' = \rho_{\mathrm{H_2O}} A (h + \Delta h) g \,.$$

With $\Delta F_b = F_b' - F_b = 0.010 mg$, the alligator sinks by

$$\Delta h = \frac{\Delta F_b}{\rho_{\mathrm{H_2O}} A g} = \frac{0.01 mg}{\rho_{\mathrm{H_2O}} A g} = \frac{0.010(130\ \mathrm{kg})}{(998\ \mathrm{kg/m^3})(0.20\ \mathrm{m^2})} = 6.5 \times 10^{-3}\ \mathrm{m} = 6.5\ \mathrm{mm}\,.$$

41. Let V_i be the total volume of the iceberg. The non-visible portion is below water, and thus the volume of this portion is equal to the volume V_f of the fluid displaced by the iceberg. The fraction of the iceberg that is visible is

$$\mathrm{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i}\,.$$

Since iceberg is floating, Eq. 14-18 applies:

$$F_g = m_i g = m_f g \quad \Rightarrow \quad m_i = m_f \,.$$

Since $m = \rho V$, the above equation implies

$$\rho_i V_i = \rho_f V_f \quad \Rightarrow \quad \frac{V_f}{V_i} = \frac{\rho_i}{\rho_f}\,.$$

Thus, the visible fraction is

$$\mathrm{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f}\,.$$

(a) If the iceberg ($\rho_i = 917\ \mathrm{kg/m^3}$) floats in salt water with $\rho_f = 1024\ \mathrm{kg/m^3}$, then the fraction would be

$$\mathrm{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \mathrm{kg/m^3}}{1024\ \mathrm{kg/m^3}} = 0.10 = 10\%\,.$$

(b) On the other hand, if the iceberg floats in fresh water ($\rho_f = 1000\ \mathrm{kg/m^3}$), then the fraction would be

$$\mathrm{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \mathrm{kg/m^3}}{1000\ \mathrm{kg/m^3}} = 0.083 = 8.3\%\,.$$

42. Work is the integral of the force over distance (see Eq. 7-32). Referring to the equation immediately preceding Eq. 14-7, we see the work can be written as

$$W = \int \rho_{\text{water}}\, gA(-y)\, dy$$

where we are using $y = 0$ to refer to the water surface (and the $+y$ direction is upward). Let $h = 0.500$ m. Then, the integral has a lower limit of $-h$ and an upper limit of y_f, with

$$y_f / h = -\rho_{\text{cylinder}} / \rho_{\text{water}} = -0.400.$$

The integral leads to

$$W = \tfrac{1}{2}\rho_{\text{water}} gAh^2(1 - 0.4^2) = 4.11 \text{ kJ}.$$

43. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$F_g - (F_g - F_b) = \Delta mg$$

which leads to $F_b = \Delta mg$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\ \text{kg/m}} \approx 6.378 \times 10^{-4}\ \text{m}^3.$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\ V_m = 5.102\ \text{m}^3\ .$$

(c) Using $\rho = \dfrac{m_{\text{dino}}}{V_{\text{dino}}} \approx \rho_w = 1000\,\text{kg/m}^3$, we find the mass of the *T. rex* to be

$$m_{\text{dino}} \approx \rho_w V_{\text{dino}} = (1000\,\text{kg/m}^3)\ (5.102\ \text{m}^3) = 5.102 \times 10^3\ \text{kg}\,.$$

44. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.900\,\rho_w V_{\text{wood}} g = 0.900\,\rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) = (m_{\text{wood}} + m_{\text{lead}})g\,.$$

Thus,

$$m_{\text{lead}} = 0.900\rho_w\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) - m_{\text{wood}} = \frac{(0.900)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\ \text{kg/m}^3} - 3.67\ \text{kg}$$
$$= 1.84\ \text{kg}.$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.900\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right)m_{\text{lead}}g = (m_{\text{wood}} + m_{\text{lead}})g,$$

which leads to

$$m_{\text{lead}} = \frac{0.900(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} = \frac{1.84\ \text{kg}}{1 - (1.00\times10^3\ \text{kg/m}^3/1.13\times10^4\ \text{kg/m}^3)}$$
$$= 2.01\ \text{kg}.$$

45. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$. The volume of the iron is given by $V_{\text{iron}} = W/g\rho_{\text{iron}}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{\text{eff}} = W - g\rho_w V_{\text{cast}}$. Thus, $V_{\text{cast}} = (W - W_{\text{eff}})/g\rho_w$ and

$$V_{\text{cav}} = \frac{W - W_{\text{eff}}}{g\rho_w} - \frac{W}{g\rho_{\text{iron}}} = \frac{6000\ \text{N} - 4000\ \text{N}}{(9.8\ \text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\ \text{N}}{(9.8\ \text{m/s}^2)(7.87\times10^3\ \text{kg/m}^3)}$$
$$= 0.126\ \text{m}^3\ .$$

46. Due to the buoyant force, the ball accelerates upward (while in the water) at rate a given by Newton's second law: $\rho_{\text{water}}Vg - \rho_{\text{ball}}Vg = \rho_{\text{ball}}Va$, which yields

$$\rho_{\text{water}} = \rho_{\text{ball}}(1 + a/g)\,.$$

With $\rho_{\text{ball}} = 0.300\ \rho_{\text{water}}$, we find that

$$a = g\left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1\right) = (9.80\ \text{m/s}^2)\left(\frac{1}{0.300} - 1\right) = 22.9\ \text{m/s}^2\,.$$

Using Eq. 2-16 with $\Delta y = 0.600$ m, the speed of the ball as it emerges from the water is

$$v = \sqrt{2a\Delta y} = \sqrt{2(22.9\ \text{m/s}^2)(0.600\ \text{m})} = 5.24\ \text{m/s}\,.$$

This causes the ball to reach a maximum height $h_{\max}$ (measured above the water surface) given by $h_{\max} = v^2/2g$ (see Eq. 2-16 again). Thus,

$$h_{\max} = \frac{v^2}{2g} = \frac{(5.24\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 1.40\ \text{m}.$$

47. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{\text{car}}$, which leads to

$$V_1 = \frac{W_{\text{car}}}{\rho_w g} = \frac{(1800\,\text{kg})(9.8\,\text{m/s}^2)}{(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.80\ \text{m}^3.$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{\text{car}} + \rho_w V_2 g$$

which gives

$$V_2 = V - \frac{W_{\text{car}}}{\rho_w g} = \left(0.750\,\text{m}^3 + 5.00\,\text{m}^3 + 0.800\,\text{m}^3\right) - \frac{1800\,\text{kg}}{1000\,\text{kg/m}^3} = 4.75\ \text{m}^3.$$

48. Let ρ be the density of the cylinder (0.30 g/cm^3 or 300 kg/m^3) and ρ_{Fe} be the density of the iron (7.9 g/cm^3 or 7900 kg/m^3). The volume of the cylinder is

$$V_c = (6\times 12)\ \text{cm}^3 = 72\ \text{cm}^3 = 0.000072\ \text{m}^3,$$

and that of the ball is denoted V_b . The part of the cylinder that is submerged has volume

$$V_s = (4 \times 12)\ \text{cm}^3 = 48\ \text{cm}^3 = 0.000048\ \text{m}^3.$$

Using the ideas of section 14-7, we write the equilibrium of forces as

$$\rho g V_c \ + \ \rho_{\text{Fe}}\, g V_b \ = \ \rho_{\text{w}}\, g V_s \ + \ \rho_{\text{w}}\, g V_b \quad \Rightarrow \quad V_b = 3.8\ \text{cm}^3$$

where we have used ρ_{w} = 998 kg/m^3 (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find r = 9.7 mm.

49. This problem involves use of continuity equation (Eq. 14-23): $A_1 v_1 = A_2 v_2$.

(a) Initially the flow speed is $v_i = 1.5$ m/s and the cross-sectional area is $A_i = HD$. At point a, as can be seen from the figure, the cross-sectional area is

$$A_a = (H-h)D - (b-h)d\,.$$

Thus, by continuity equation, the speed at point a is

$$v_a = \frac{A_i v_i}{A_a} = \frac{HDv_i}{(H-h)D-(b-h)d} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m}-0.80\text{ m})(55\text{ m})-(12\text{ m}-0.80\text{ m})(30\text{ m})}$$
$$= 2.96\text{ m/s} \approx 3.0\text{ m/s}.$$

(b) Similarly, at point b, the cross-sectional area is $A_b = HD - bd$, and therefore, by continuity equation, the speed at point b is

$$v_b = \frac{A_i v_i}{A_b} = \frac{HDv_i}{HD-bd} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m})(55\text{ m})-(12\text{ m})(30\text{ m})} = 2.8\text{ m/s}.$$

50. The left and right sections have a total length of 60.0 m, so (with a speed of 2.50 m/s) it takes 60.0/2.50 = 24.0 seconds to travel through those sections. Thus it takes (88.8 – 24.0) s = 64.8 s to travel through the middle section. This implies that the speed in the middle section is

$$v_{\text{mid}} = (50\text{ m})/(64.8\text{ s}) = 0.772\text{ m/s}.$$

Now Eq. 14-23 (plus that fact that $A = \pi r^2$) implies $r_{\text{mid}} = r_A\sqrt{(2.5\text{ m/s})/(0.772\text{ m/s})}$ where r_A = 2.00 cm. Therefore, $r_{\text{mid}} = 3.60\text{ cm}$.

51. **THINK** We use the equation of continuity to solve for the speed of water as it leaves the sprinkler hole.

EXPRESS Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. The cross-sectional area of the hose is $A_1 = \pi R^2$. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2\left(NA_2\right) \quad \Rightarrow \quad v_2 = \frac{A_1}{NA_2}v_1 = \frac{R^2}{Nr^2}v_1$$

where R is the radius of the hose and r is the radius of a hole.

ANALYZE Noting that $R/r = D/d$ (the ratio of diameters) we find the speed to be

$$v_2 = \frac{D^2}{Nd^2}v_1 = \frac{\left(1.9\text{ cm}\right)^2}{24\left(0.13\text{ cm}\right)^2}\left(0.91\text{ m/s}\right) = 8.1\text{ m/s}.$$

LEARN The equation of continuity implies that the smaller the cross-sectional area of the sprinkler hole, the greater the speed of water as it emerges from the hole.

52. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,\mathrm{m})(3.4\,\mathrm{m})(2.3\,\mathrm{m/s})+(6.8\,\mathrm{m})(3.2\,\mathrm{m})(2.6\,\mathrm{m/s})=h(10.5\,\mathrm{m})(2.9\,\mathrm{m/s})$$

which leads to $h = 4.0$ m.

53. **THINK** The power of the pump is the rate of work done in lifting the water.

EXPRESS Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by $\Delta U = (\Delta m)gh$, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\Delta K = \frac{1}{2}(\Delta m)v^2$, where v is its final speed. The work it does is

$$\Delta W = \Delta U + \Delta K = (\Delta m)gh + \frac{1}{2}(\Delta m)v^2$$

and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

The rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose.

ANALYZE The area of the hose is $A = \pi r^2 = \pi(0.010\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$ and

$$\rho_w A v = (1000\text{ kg/m}^3)\,(3.14 \times 10^{-4}\text{ m}^2)\,(5.00\text{ m/s}) = 1.57\text{ kg/s}.$$

Thus, the power of the pump is

$$P = \rho A v\left(gh + \frac{1}{2}v^2\right) = (1.57\,\mathrm{kg/s})\left((9.8\,\mathrm{m/s^2})(3.0\,\mathrm{m}) + \frac{(5.0\,\mathrm{m/s})^2}{2}\right) = 66\,\mathrm{W}.$$

LEARN The work done by the pump is converted into both the potential energy and kinetic energy of the water.

54. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

55. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = (1.0 \times 10^5 \text{ Pa})\ (1.4 \text{ m}^3) = 1.4 \times 10^5 \text{ J}.$$

56. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho gh$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1 \sqrt{2gh} A_1 = \rho_2 \sqrt{2gh} A_2 \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2.$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2}.$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$. Thus,

$$h_2 = h_1/4 = (12.0 \text{ cm})/4 = 3.00 \text{ cm}.$$

57. **THINK** We use the Bernoulli equation to solve for the flow rate, and the continuity equation to relate cross-sectional area to the vertical distance from the hole.

EXPRESS According to the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2,$$

where ρ is the density of water, h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then simplifies to $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$.

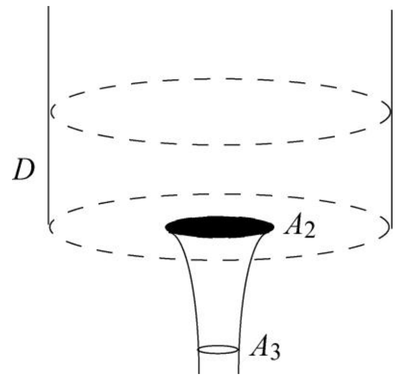
ANALYZE (a) With $D = h_1 - h_2 = 0.30$ m, the speed of water as it emerges from the hole is

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(0.30 \text{ m})} = 2.42 \text{ m/s}.$$

Thus, the flow rate is

$$A_2v_2 = (6.5 \times 10^{-4}\ \text{m}^2)(2.42\ \text{m/s}) = 1.6 \times 10^{-3}\ \text{m}^3/\text{s}.$$

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole (see diagram below).



Thus,

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\ \text{m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus,

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\,\text{m/s})^2 - (2.42\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.90\ \text{m}.$$

LEARN By combing the two expressions obtained from Bernoulli's equation and equation of continuity, the cross-sectional area of the stream may be related to the vertical height fallen as

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{v_2^2}{2g}\left[\left(\frac{A_2}{A_3}\right)^2 - 1\right] = \frac{v_3^2}{2g}\left[1 - \left(\frac{A_3}{A_2}\right)^2\right].$$

58. We use Bernoulli's equation:

$$p_2 - p_i = \rho g D + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000\ \text{kg/m}^3$, $D = 180$ m, $v_1 = 0.40$ m/s, and $v_2 = 9.5$ m/s. Therefore, we find $\Delta p = 1.7 \times 10^6$ Pa, or 1.7 MPa. The SI unit for pressure is the pascal (Pa) and is equivalent to N/m^2.

59. **THINK** The elevation and cross-sectional area of the pipe are changing, so we apply the Bernoulli equation and continuity equation to analyze the flow of water through the pipe.

EXPRESS To calculate the flow speed at the lower level, we use the equation of continuity: $A_1v_1 = A_2v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. As for the pressure at the lower level, we use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 ,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude.

ANALYZE (a) From the continuity equation, we find the speed at the lower level to be

$$v_2 = (A_1/A_2)v_1 = [(4.0 \text{ cm}^2)/(8.0 \text{ cm}^2)]\ (5.0 \text{ m/s}) = 2.5\text{m/s}.$$

(b) Similarly, from the Bernoulli equation, the pressure at the lower level is

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left[(5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right] + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa}. \end{aligned}$$

LEARN The water at the lower level has a smaller speed ($v_2 < v_1$) but higher pressure ($p_2 > p_1$).

60. (a) We use Av = const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s}) = 2.40\,\text{m/s}.$$

(b) Since $p + \tfrac{1}{2}\rho v^2 = \text{const.}$, the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}(1000\,\text{kg/m}^3)\left[(2.50\,\text{m/s})^2 - (2.40\,\text{m/s})^2\right] = 245\,\text{Pa}.$$

61. (a) The equation of continuity leads to

$$v_2A_2 = v_1A_1 \quad\Rightarrow\quad v_2 = v_1\left(\frac{r_1^2}{r_2^2}\right)$$

which gives v_2 = 3.9 m/s.

(b) With h = 7.6 m and $p_1 = 1.7\times10^5$ Pa, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho gh + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8\times10^4\,\text{Pa}.$$

62. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}}v^2 \cdot$ However, $\Delta p = p_A - p_B = \rho gh$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho gh = \frac{1}{2}\rho_{\text{air}}v^2$, or

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}} = \sqrt{\frac{2\left(810\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(0.260\,\text{m})}{1.03\,\text{kg/m}^3}} = 63.3\,\text{m/s}.$$

63. We use the formula for v obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\text{air}}}} = \sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}} = 1.1\times10^2\,\text{m/s}.$$

64. (a) The volume of water (during 10 minutes) is

$$V = \left(v_1 t\right)A_1 = \left(15\,\text{m/s}\right)\left(10\,\text{min}\right)\left(60\,\text{s/min}\right)\left(\frac{\pi}{4}\right)\left(0.03\,\text{m}\right)^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = \left(15\,\text{m/s}\right)\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho gh_2$$

and $h_1 = h_2, p_1 = p_0$, which is the atmospheric pressure,

$$\begin{aligned} p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) &= 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[\left(15\,\text{m/s}\right)^2 - \left(5.4\,\text{m/s}\right)^2\right] \\ &= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}. \end{aligned}$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8×10^4 Pa.

65. **THINK** The design principles of the Venturi meter, a device that measures the flow speed of a fluid in a pipe, involve both the continuity equation and Bernoulli's equation.

EXPRESS The continuity equation yields $AV = av$, and Bernoulli's equation yields $\frac{1}{2}\rho V^2 = \Delta p + \frac{1}{2}\rho v^2$, where $\Delta p = p_2 - p_1$ with p_2 equal to the pressure in the throat and p_1 the pressure in the pipe. The first equation gives $v = (A/a)V$. We use this to substitute for v in the second equation and obtain

$$\tfrac{1}{2}\rho V^2 = \Delta p + \tfrac{1}{2}\rho\left(A/a\right)^2 V^2 .$$

The equation can be used to solve for V.

ANALYZE (a) The above equation gives the following expression for V:

$$V = \sqrt{\frac{2\Delta p}{\rho\left(1-(A/a)^2\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}}.$$

(b) We substitute the values given to obtain

$$V = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}} = \sqrt{\frac{2(32\times 10^{-4}\,\mathrm{m}^2)^2(41\times 10^3\,\mathrm{Pa} - 55\times 10^3\,\mathrm{Pa})}{(1000\,\mathrm{kg/m}^3)\left((32\times 10^{-4}\,\mathrm{m}^2)^2 - (64\times 10^{-4}\,\mathrm{m}^2)^2\right)}} = 3.06\,\mathrm{m/s}.$$

Consequently, the flow rate is

$$R = AV = (64\times 10^{-4}\,\mathrm{m}^2)(3.06\ \mathrm{m/s}) = 2.0\times 10^{-2}\,\mathrm{m}^3/\mathrm{s}.$$

LEARN The pressure difference Δp between points 1 and 2 is what causes the height difference of the fluid in the two arms of the manometer. Note that $\Delta p = p_2 - p_1 < 0$ (pressure in throat less than that in the pipe), but $a < A$, so the expression inside the square root is positive.

66. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0$ atm. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{4(1.01\times 10^5\ \mathrm{Pa})}{(1000\ \mathrm{kg/m}^3)\ [(5a/a)^2 - 1]}} = 4.1\,\mathrm{m/s}.$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21$ m/s.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\,(5.0\times 10^{-4}\ \mathrm{m}^2)\ (4.1\ \mathrm{m/s}) = 8.0\times 10^{-3}\,\mathrm{m}^3/\mathrm{s}.$$

67. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3 \text{ kg/m}^3)\ (9.8 \text{ m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040 \text{ m})^2 = 74 \text{ N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}$. Thus, the volume of water flowing out of the pipe in t = 3.0 h is

$$V = Avt = \frac{\pi^2}{4}(0.040 \text{ m})^2\sqrt{2(9.8 \text{ m/s}^2)\ (6.0 \text{ m})}\ (3.0 \text{ h})\ (3600 \text{ s/h}) = 1.5\times10^2 \text{m}^3.$$

68. (a) We note (from the graph) that the pressures are equal when the value of inverse-area-squared is 16 (in SI units). This is the point at which the areas of the two pipe sections are equal. Thus, if $A_1 = 1/\sqrt{16}$ when the pressure difference is zero, then A_2 is 0.25 m^2.

(b) Using Bernoulli's equation (in the form Eq. 14-30) we find the pressure difference may be written in the form of a straight line: $mx + b$ where x is inverse-area-squared (the horizontal axis in the graph), m is the slope, and b is the intercept (seen to be –300 kN/m^2). Specifically, Eq. 14-30 predicts that b should be $-\frac{1}{2}\rho v_2{}^2$. Thus, with ρ = 1000 kg/m^3 we obtain $v_2 = \sqrt{600}$ m/s. Then the volume flow rate (see Eq. 14-24) is

$$R = A_2\, v_2 = (0.25 \text{ m}^2)(\sqrt{600} \text{ m/s}) = 6.12 \text{ m}^3/\text{s}.$$

If the more accurate value (see Table 14-1) ρ = 998 kg/m^3 is used, then the answer is 6.13 m^3/s.

69. (a) Combining Eq. 14-35 and Eq. 14-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2\sqrt{\frac{2\Delta p}{\rho\left(A_1^2 - A_2^2\right)}}$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66\times10^{-3} \text{ m}^4$, so that the square root is well defined. Therefore, we obtain R = 0.0776 m^3/s.

(b) The mass rate of flow is $\rho R = (900 \text{ kg/m}^3)(0.0776 \text{ m}^3/\text{s}) = 69.8 \text{ kg/s}$.

70. By Eq. 14-23, the speeds in the left and right sections are $\frac{1}{4}v_{\text{mid}}$ and $\frac{1}{9}v_{\text{mid}}$, respectively, where v_{mid} = 0.500 m/s. We also note that 0.400 m^3 of water has a mass of 399 kg (see Table 14-1). Then Eq. 14-31 (and the equation below it) gives

$$W = \frac{1}{2} m v_{\text{mid}}^2 \left(\frac{1}{9^2} - \frac{1}{4^2} \right) = \frac{1}{2} (399 \text{ kg})(0.50 \text{ m/s})^2 \left(\frac{1}{9^2} - \frac{1}{4^2} \right) = -2.50 \text{ J}.$$

71. (a) The stream of water emerges horizontally (θ_0 = 0° in the notation of Chapter 4) with $v_0 = \sqrt{2gh}$. Setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where $x_0 = 0$ by choice of coordinate origin, we find

$$x = v_0 t = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)} = 2\sqrt{(10 \text{ cm})(40 \text{ cm} - 10 \text{ cm})} = 35 \text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \quad \Rightarrow \quad h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H.$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$. Its numerical value is $h' = 40\text{cm} - 10 \text{ cm} = 30 \text{ cm}.$

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \Rightarrow h = \frac{H}{2}$$

or h = (40 cm)/2 = 20 cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.

72. We use Bernoulli's equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 \,.$$

When the water level rises to height h_2, just on the verge of flooding, v_2, the speed of water in pipe M is given by

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2 \;\Rightarrow\; v_2 = \sqrt{2g(h_1 - h_2)} = 13.86 \text{ m/s}.$$

By the continuity equation, the corresponding rainfall rate is

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \frac{\pi(0.030\text{ m})^2}{(30\text{ m})(60\text{ m})}(13.86\text{ m/s}) = 2.177\times 10^{-5}\text{ m/s} \approx 7.8\text{ cm/h}.$$

73. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\text{body}}\, g \Rightarrow \rho_{\text{liquid}}\, g V_{\text{submerged}} = \rho_{\text{body}} g V_{\text{total}}$$

which leads to

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}.$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with ρ_{body} = 0.98 g/cm^3, we find $\rho_{\text{liquid}} \approx$ 1.5 g/cm^3 — certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

74. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{\text{Hg}} g x$, which should be compensated for by the water pressure $p_w = \rho_w g h$, where h = 11.2 cm. In these units, ρ_w = 1.00 g/cm^3 and ρ_{Hg} = 13.6 g/cm^3 (see Table 14-1). We obtain

$$x = \frac{\rho_w g h}{2\rho_{\text{Hg}}\, g} = \frac{(1.00\text{ g/cm}^3)\,(11.2\text{ cm})}{2(13.6\text{ g/cm}^3)} = 0.412\text{ cm}.$$

75. Using $m = \rho V$, Newton's second law becomes

$$\rho_{\text{water}} V g - \rho_{\text{bubble}} V g = \rho_{\text{bubble}} V a,$$

or

$$\rho_{\text{water}} = \rho_{\text{bubble}}(1 + a/g)$$

With ρ_{water} = 998 kg/m^3 (see Table 14-1), we find

$$\rho_{\text{bubble}} = \frac{\rho_{\text{water}}}{1 + a/g} = \frac{998\ \text{kg/m}^3}{1 + (0.225\ \text{m/s}^2)/(9.80\ \text{m/s}^2)} = 975.6\ \text{kg/m}^3 .$$

Using volume $V = \frac{4}{3}\pi r^3$ with $r = 5.00 \times 10^{-4}$ m for the bubble, we then find its mass: $m_{\text{bubble}} = 5.11 \times 10^{-7}$ kg.

76. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \Rightarrow \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it that is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5.0% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

77. The normal force $\vec{F}_N$ exerted (upward) on the glass ball of mass m has magnitude 0.0948 N. The buoyant force exerted by the milk (upward) on the ball has magnitude

$$F_b = \rho_{\text{milk}}\, g\, V$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the ball. Its radius is $r = 0.0200$ m. The milk density is $\rho_{\text{milk}} = 1030\ \text{kg/m}^3$. The (actual) weight of the ball is, of course, downward, and has magnitude $F_g = m_{\text{glass}}\, g$. Application of Newton's second law (in the case of zero acceleration) yields

$$F_N + \rho_{\text{milk}}\, g\, V - m_{\text{glass}}\, g = 0$$

which leads to $m_{\text{glass}} = 0.0442$ kg.

78. Since $F_g = mg = \rho_{\text{skier}}\, g\, V$ and the buoyant force is $F_b = \rho_{\text{snow}}\, g\, V$, then their ratio is

$$\frac{F_b}{F_g} = \frac{\rho_{\text{snow}}\, g\, V}{\rho_{\text{skier}}\, g\, V} = \frac{\rho_{\text{snow}}}{\rho_{\text{skier}}} = \frac{96}{1020} = 0.094 \text{ (or 9.4\%).}$$

79. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight W of the object. The buoyant force of the water on the object is therefore (30 – 20) N = 10 N, which means

$$F_b = \rho_w V g \;\Rightarrow\; V = \frac{10\,\text{N}}{(1000\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.02 \times 10^{-3}\ \text{m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is (30 – 24) N = 6.0 N, which implies

$$\rho_2 = \frac{6.0\ \text{N}}{(9.8\ \text{m/s}^2)(1.02 \times 10^{-3}\ \text{m}^3)} = 6.0\times10^2\ \text{kg/m}^3\ .$$

80. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where V_{sub} is the portion of the object that is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{\text{liquid}}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A, $\dfrac{\rho}{\rho_A} = \dfrac{1}{2}$, so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*, $\dfrac{\rho}{\rho_B} = \dfrac{1}{3}$, so that $\rho_B/\rho_w = 3$.

(c) For liquid *C*, noting that one-fourth *above* means three-fourths *below*, $\dfrac{\rho}{\rho_C} = \dfrac{3}{4}$, so that $\rho_C/\rho_w = 4/3$.

81. **THINK** The U-tube contains two types of liquid in static equilibrium. The pressures at the interface level on both sides of the tube must be the same.

EXPRESS If we examine both sides of the U-tube at the level where the low-density liquid (with $\rho = 0.800\ \text{g/cm}^3 = 800\ \text{kg/m}^3$) meets the water (with $\rho_w = 0.998\ \text{g/cm}^3 = 998\ \text{kg/m}^3$), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where h = 8.00 cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is h_w = (800/998)(8.00 cm) = 6.41 cm.

ANALYZE The volume of water in that column is

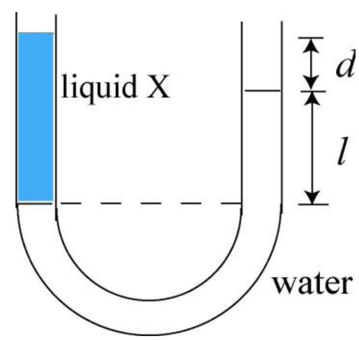
$$V = \pi r^2 h_w = \pi(1.50\ \text{cm})^2(6.41\ \text{cm}) = 45.3\ \text{cm}^3.$$

This is the amount of water that flows out of the right arm.

LEARN As discussed in the Sample Problem 14.3 – Balancing of pressure in a U-tube, the relationship between the densities of the two liquids can be written as

$$\rho_X = \rho_w \frac{l}{l+d}$$

The liquid in the left arm is higher than the water in the right because the liquid is less dense than water $\rho_X < \rho_w$.



82. The downward force on the balloon is mg and the upward force is $F_b = \rho_{\text{out}} Vg$. Newton's second law (with $m = \rho_{\text{in}} V$) leads to

$$\rho_{\text{out}} Vg - \rho_{\text{in}} Vg = \rho_{\text{in}} Va \quad \Rightarrow \left(\frac{\rho_{\text{out}}}{\rho_{\text{in}}} - 1\right) g = a.$$

The problem specifies $\rho_{\text{out}} / \rho_{\text{in}}$ = 1.39 (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is

$$a = (1.39 - 1.00)(9.80\ \text{m/s}^2) = 3.82\ \text{m/s}^2.$$

83. (a) We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B, and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_C = \sqrt{2(9.8\ \text{m/s}^2)(0.40\ \text{m} + 0.12\ \text{m})} = 3.2\ \text{m/s}.$$

(b) We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C\ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g(h_1 + h_2 + d) \\ &= 1.0\times10^5\ \text{Pa} - (1.0\times10^3\text{kg/m}^3)(9.8\ \text{m/s}^2)(0.25\ \text{m} + 0.40\ \text{m} + 0.12\ \text{m}) \\ &= 9.2\times10^4\ \text{Pa}. \end{aligned}$$

(c) Since $p_B \geq 0$, we must let

$$p_{\text{air}} - \rho g(h_1 + d + h_2) \geq 0,$$

which yields

$$h_1 \leq h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \leq \frac{p_{\text{air}}}{\rho} = 10.3\ \text{m}.$$

84. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2}.$$

(a) With $R = 7.0 \times 10^{-3}\ \text{m}^3/\text{s}$ and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where v_s = 343 m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

85. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be V = 1200 cm^3. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\, g = \rho_w V g \Rightarrow m_\ell = (1\text{g/cm}^3)\ (1200\ \text{cm}^3) - 130\ \text{g}$$

which yields 1.07×10^3 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

86. Before undergoing acceleration, the net force exerted on the block is zero, and Newton's second law gives

$$F_b - mg - T_0 = 0 \quad \Rightarrow \quad T_0 = F_b - mg$$

where $F_b = \rho V g$ is the buoyant force from the fluid of density ρ. When the container is given an upward acceleration a, the apparent weight of the block becomes $m(g+a)$, and the corresponding buoyant force is $F_b' = \rho V(g+a)$. In this case, Newton's second-law equation is

$$F_b' - m(g+a) - T = 0$$

which gives

$$T = F_b' - m(g+a) = \rho V(g+a) - m(g+a) = (\rho V - m)g(1 + a/g) = T_0(1 + a/g)\,.$$

With $a = 0.25g$, we have $T/T_0 = 1 + a/g = 1.25$.

87. We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let M be the mass of the automobile, ρ_i be the density of ice, and ρ_w be the density of water. Suppose the ice slab has area A and thickness h. Since the volume of ice is Ah, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{938\,\text{kg}}{\left(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3\right)(0.441\text{m})} = 26.3\ \text{m}^2.$$

88. (a) Using Eq. 14-10, we have

$$p_g = \rho g h = (1025\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(2.22\times10^3\,\text{m}) = 2.23\times10^7\,\text{Pa}\,.$$

(b) By definition, the total pressure is

$$p = p_0 + p_g = 1.01\times10^5\,\text{Pa} + 2.23\times10^7\,\text{Pa} = 2.24\times10^7\,\text{Pa}\,.$$

(c) The net force compressing the sphere's surface is

$$F = pA = p(4\pi R^2) = (2.24\times10^7\,\text{Pa})4\pi(6.22\times10^{-2}\,\text{m})^2 = 1.09\times10^6\,\text{N}.$$

(d) The upward buoyant force exerted on the sphere by the seawater is

$$F_b = \rho g V = \rho g\left(\frac{4\pi}{3}R^3\right) = (1025\,\text{kg/m}^3)(9.8\,\text{m/s}^2)\frac{4\pi}{3}(6.22\times10^{-2}\,\text{m})^3 = 10.1\,\text{N}.$$

(e) Newton's second law applied to the sphere of mass m = 6.80 kg yields

$$F_b - mg = ma \quad\Rightarrow\quad a = \frac{F_b}{m} - g = \frac{10.1\,\text{N}}{8.60\,\text{kg}} - 9.8\,\text{m/s}^2 = -8.62\,\text{m/s}^2.$$

The acceleration vector has a magnitude of 8.62 m/s^2 and the direction is downward.

89. (a) The total weight is

$$W = \rho g V = \rho g h A = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})(2200\,\text{m}^2) = 5.66\times10^9\,\text{N}.$$

(b) The gauge pressure at this depth is

$$p_g = \rho g h = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})\left(\frac{1\,\text{atm}}{1.01\times10^5\,\text{Pa}}\right) = 25.5\,\text{atm}.$$

90. Using Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2,$$

we find the minimum pressure to be (setting $v_1 = v_2$)

$$\Delta p = p_2 - p_1 = \rho g(y_1 - y_2) = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(6.59\,\text{m} - 2.16\,\text{m}) = 4.34\times10^4\ \text{Pa}.$$