

Chapter 34

1. The bird is a distance d_2 in front of the mirror; the plane of its image is that same distance d_2 behind the mirror. The lateral distance between you and the bird is $d_3 = 5.00$ m. We denote the distance from the camera to the mirror as d_1 , and we construct a right triangle out of d_3 and the distance between the camera and the image plane ($d_1 + d_2$). Thus, the focus distance is

$$d = \sqrt{(d_1 + d_2)^2 + d_3^2} = \sqrt{(4.30 \text{ m} + 3.30 \text{ m})^2 + (5.00 \text{ m})^2} = 9.10 \text{ m}.$$

2. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of $10 \text{ cm} + 30 \text{ cm} = 40 \text{ cm}$.

3. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_P = A/d^2$, where A is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity I_P , as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance d behind the mirror and a distance $3d$ from the screen. Its contribution to the intensity at the center of the screen is

$$I_r = \frac{A}{(3d)^2} = \frac{A}{9d^2} = \frac{I_P}{9}.$$

The total intensity at the center of the screen is

$$I = I_P + I_r = I_P + \frac{I_P}{9} = \frac{10}{9} I_P.$$

The ratio of the new intensity to the original intensity is $I/I_P = 10/9 = 1.11$.

4. When S is barely able to see B , the light rays from B must reflect to S off the edge of the mirror. The angle of reflection in this case is 45° , since a line drawn from S to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

$$\frac{x}{d/2} = \tan 45^\circ = 1 \Rightarrow x = \frac{d}{2} = \frac{3.0 \text{ m}}{2} = 1.5 \text{ m}.$$

5. **THINK** This problem involves refraction at air–water interface and reflection from a plane mirror at the bottom of the pool.

EXPRESS We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin \theta}{\sin \theta'} = \frac{n_w}{n_{\text{air}}},$$

which in our case reduces to $\theta' \approx \theta/n_w$ (since both θ and θ' are small, and $n_{\text{air}} \approx 1$). We refer to our figure on the right.

The object O is a vertical distance d_1 above the water, and the water surface is a vertical distance d_2 above the mirror. We are looking for a distance d (treated as a positive number) below the mirror where the image I of the object is formed. In the triangle OAB

$$|AB| = d_1 \tan \theta \approx d_1 \theta,$$

and in the triangle CBD

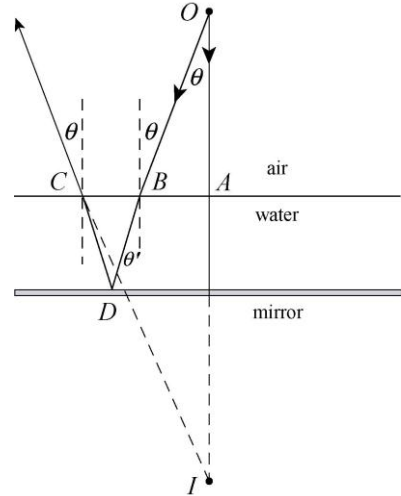
$$|BC| = 2d_2 \tan \theta' \approx 2d_2 \theta' \approx \frac{2d_2 \theta}{n_w}.$$

Finally, in the triangle ACI , we have $|AI| = d + d_2$.

ANALYZE Therefore,

$$\begin{aligned} d &= |AI| - d_2 = \frac{|AC|}{\tan \theta} - d_2 \approx \frac{|AB| + |BC|}{\theta} - d_2 = \left(d_1 \theta + \frac{2d_2 \theta}{n_w} \right) \frac{1}{\theta} - d_2 = d_1 + \frac{2d_2}{n_w} - d_2 \\ &= 250 \text{ cm} + \frac{2(200 \text{ cm})}{1.33} - 200 \text{ cm} = 351 \text{ cm}. \end{aligned}$$

LEARN If the pool were empty without water, then $\theta = \theta'$, and the distance would be $d = d_1 + 2d_2 - d_2 = d_1 + d_2$. This is precisely what we expect from a plane mirror.



6. We note from Fig. 34-34 that $m = \frac{1}{2}$ when $p = 5$ cm. Thus Eq. 34-7 (the magnification equation) gives us $i = -10$ cm in that case. Then, by Eq. 34-9 (which applies to mirrors and thin lenses) we find the focal length of the mirror is $f = 10$ cm. Next, the problem asks us to consider $p = 14$ cm. With the focal length value already determined, then Eq. 34-9 yields $i = 35$ cm for this new value of object distance. Then, using Eq. 34-7 again, we find $m = i/p = -2.5$.

7. We use Eqs. 34-3 and 34-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p} - \frac{1}{pm} = \frac{1}{f} = \frac{2}{r}.$$

We solve for p : $p = \frac{r}{2} \left(1 - \frac{1}{m} \right) = \frac{35.0 \text{ cm}}{2} \left(1 - \frac{1}{2.50} \right) = 10.5 \text{ cm}.$

8. The graph in Fig. 34-35 implies that $f = 20 \text{ cm}$, which we can plug into Eq. 34-9 (with $p = 70 \text{ cm}$) to obtain $i = +28 \text{ cm}$.

9. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With $f = +12 \text{ cm}$ and $p = +18 \text{ cm}$, the radius of curvature is $r = 2f = 2(12 \text{ cm}) = +24 \text{ cm}$.

(b) The image distance is $i = \frac{pf}{p-f} = \frac{(18 \text{ cm})(12 \text{ cm})}{18 \text{ cm} - 12 \text{ cm}} = 36 \text{ cm}.$

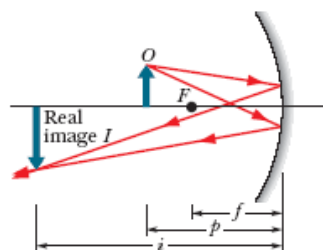
(c) The lateral magnification is $m = -i/p = -(36 \text{ cm})/(18 \text{ cm}) = -2.0.$

(d) Since the image distance i is positive, the image is real (R).

(e) Since the magnification m is negative, the image is inverted (I).

(f) A real image is formed on the same side as the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-10(c). The object is outside the focal point, and its image is real and inverted.



10. A concave mirror has a positive value of focal length.

(a) Then (with $f = +10$ cm and $p = +15$ cm), the radius of curvature is $r = 2f = +20$ cm.

(b) Equation 34-9 yields $i = pf/(p - f) = +30$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = -2.0$.

(d) Since the image distance computation produced a positive value, the image is real (R).

(e) The magnification computation produced a negative value, so it is inverted (I).

(f) A real image is formed on the same side as the object.

11. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is

$$m = -\frac{i}{p}.$$

The value of m is positive for upright (not inverted) images, and negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With $f = -10$ cm and $p = +8$ cm, the radius of curvature is $r = 2f = -20$ cm.

(b) The image distance is $i = \frac{pf}{p - f} = \frac{(8 \text{ cm})(-10 \text{ cm})}{8 \text{ cm} - (-10) \text{ cm}} = -4.44$ cm.

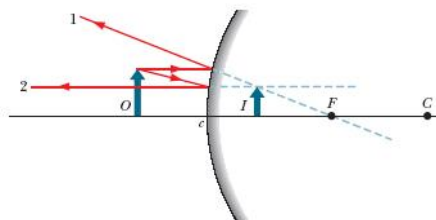
(c) The lateral magnification is $m = -i/p = -(-4.44 \text{ cm})/(8.0 \text{ cm}) = +0.56$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification m is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



12. A concave mirror has a positive value of focal length.

(a) Then (with $f = +36$ cm and $p = +24$ cm), the radius of curvature is $r = 2f = +72$ cm.

(b) Equation 34-9 yields $i = pf/(p - f) = -72$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

13. **THINK** A concave mirror has a positive value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$.

The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for real images and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE With $f = +18$ cm and $p = +12$ cm, the radius of curvature is $r = 2f = +36$ cm.

(b) Equation 34-9 yields $i = pf/(p - f) = -36$ cm.

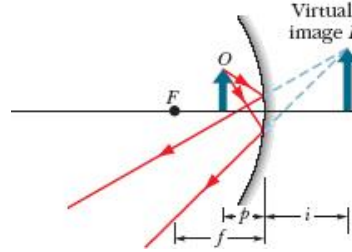
(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(a). The mirror is concave, and its image is virtual, enlarged, and upright.



14. A convex mirror has a negative value of focal length.

(a) Then (with $f = -35$ cm and $p = +22$ cm), the radius of curvature is $r = 2f = -70$ cm.

(b) Equation 34-9 yields $i = pf/(p - f) = -14$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.61$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) The side where a virtual image forms is opposite from the side where the object is.

15. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$.

The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images.

The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) With $f = -8$ cm and $p = +10$ cm, the radius of curvature is $r = 2f = 2(-8 \text{ cm}) = -16$ cm.

(b) The image distance is $i = \frac{pf}{p-f} = \frac{(10 \text{ cm})(-8 \text{ cm})}{10 \text{ cm} - (-8) \text{ cm}} = -4.44$ cm.

(c) The lateral magnification is $m = -i/p = -(-4.44 \text{ cm})/(10 \text{ cm}) = +0.44$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification m is positive, so the image is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.

16. A convex mirror has a negative value of focal length.

(a) Then (with $f = -14$ cm and $p = +17$ cm), the radius of curvature is $r = 2f = -28$ cm.

(b) Equation 34-9 yields $i = pf/(p-f) = -7.7$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.45$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) A virtual image is formed on the opposite side of the mirror from the object.

17. (a) The mirror is concave.

(b) $f = +20$ cm (positive, because the mirror is concave).

(c) $r = 2f = 2(+20 \text{ cm}) = +40$ cm.

(d) The object distance $p = +10$ cm, as given in the table.

(e) The image distance is $i = (1/f - 1/p)^{-1} = (1/20 \text{ cm} - 1/10 \text{ cm})^{-1} = -20$ cm.

(f) $m = -i/p = -(-20 \text{ cm}/10 \text{ cm}) = +2.0$.

(g) The image is virtual (V).

(h) The image is upright or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

18. (a) Since the image is inverted, we can scan Figs. 34-8, 34-10, and 34-11 in the textbook and find that the mirror must be concave.

(b) This also implies that we must put a minus sign in front of the “0.50” value given for m . To solve for f , we first find $i = -pm = +12$ cm from Eq. 34-6 and plug into Eq. 34-4; the result is $f = +8$ cm.

(c) Thus, $r = 2f = +16$ cm.

(d) $p = +24$ cm, as given in the table.

(e) As shown above, $i = -pm = +12$ cm.

(f) $m = -0.50$, with a minus sign.

(g) The image is real (R), since $i > 0$.

(h) The image is inverted (I), as noted above.

(i) A real image is formed on the same side as the object.

19. (a) Since $r < 0$ then (by Eq. 34-3) $f < 0$, which means the mirror is convex.

(b) The focal length is $f = r/2 = -20$ cm.

(c) $r = -40$ cm, as given in the table.

(d) Equation 34-4 leads to $p = +20$ cm.

(e) $i = -10$ cm, as given in the table.

(f) Equation 34-6 gives $m = +0.50$.

(g) The image is virtual (V).

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

20. (a) From Eq. 34-7, we get $i = -mp = +28$ cm, which implies the image is real (R) and on the same side as the object. Since $m < 0$, we know it was inverted (I). From Eq. 34-9,

we obtain $f = ip/(i + p) = +16$ cm, which tells us (among other things) that the mirror is concave.

(b) $f = ip/(i + p) = +16$ cm.

(c) $r = 2f = +32$ cm.

(d) $p = +40$ cm, as given in the table.

(e) $i = -mp = +28$ cm.

(f) $m = -0.70$, as given in the table.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the same side as the object.

21. (a) Since $f > 0$, the mirror is concave.

(b) $f = +20$ cm, as given in the table.

(c) Using Eq. 34-3, we obtain $r = 2f = +40$ cm.

(d) $p = +10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p - f) = +60$ cm.

(f) Equation 34-6 gives $m = -i/p = -2.0$.

(g) Since $i > 0$, the image is real (R).

(h) Since $m < 0$, the image is inverted (I).

(i) A real image is formed on the same side as the object.

22. (a) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex.

(b) Thus, we must put a minus sign in front of the “20” value given for f , that is, $f = -20$ cm.

(c) Equation 34-3 then gives $r = 2f = -40$ cm.

(d) To solve for i and p we must set up Eq. 34-4 and Eq. 34-6 as a simultaneous set and solve for the two unknowns. The results are $p = +180 \text{ cm} = +1.8 \text{ m}$, and

(e) $i = -18 \text{ cm}$.

(f) $m = 0.10$, as given in the table.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI), as already noted.

(i) A virtual image is formed on the opposite side of the mirror from the object.

23. **THINK** A positive value for the magnification means that the image is upright (not inverted).

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The magnification is given by $m = -i/p$. Since $p > 0$, a positive value for m means that the image distance (i) is negative, implying a virtual image. A positive magnification of magnitude less than unity is only possible for convex mirrors.

(b) With $i = -mp$, we may write $p = f(1 - 1/m)$. For $0 < m < 1$, a positive value for p can be obtained only if $f < 0$. Thus, with a minus sign, we have $f = -30 \text{ cm}$.

(c) The radius of curvature is $r = 2f = -60 \text{ cm}$.

(d) The object distance is $p = f(1 - 1/m) = (-30 \text{ cm})(1 - 1/0.20) = +120 \text{ cm} = 1.2 \text{ m}$.

(e) The image distance is $i = -mp = -(0.20)(120 \text{ cm}) = -24 \text{ cm}$.

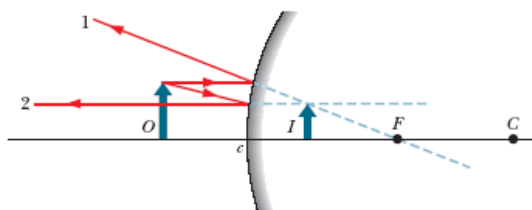
(f) The magnification is $m = +0.20$, as given in the Table.

(g) As discussed in (a), the image is virtual (V).

(h) As discussed in (a), the image is upright, or not inverted (NI).

- (i) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



24. (a) Since $m = -1/2 < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 34-8, 34-10, and 34-11, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$).

(b) Next, we find i from Eq. 34-6 (which yields $i = mp = 30$ cm) and then use this value (and Eq. 34-4) to compute the focal length; we obtain $f = +20$ cm.

(c) Then, Eq. 34-3 gives $r = 2f = +40$ cm.

(d) $p = 60$ cm, as given in the table.

(e) As already noted, $i = +30$ cm.

(f) $m = -1/2$, as given.

(g) Since $i > 0$, the image is real (R).

(h) As already noted, the image is inverted (I).

(i) A real image is formed on the same side as the object.

25. (a) As stated in the problem, the image is inverted (I), which implies that it is real (R). It also (more directly) tells us that the magnification is equal to a negative value: $m = -0.40$. By Eq. 34-7, the image distance is consequently found to be $i = +12$ cm. Real images don't arise (under normal circumstances) from convex mirrors, so we conclude that this mirror is concave.

(b) The focal length is $f = +8.6$ cm, using Eq. 34-9, $f = +8.6$ cm.

(c) The radius of curvature is $r = 2f = +17.2$ cm ≈ 17 cm.

(d) $p = +30$ cm, as given in the table.

(e) As noted above, $i = +12$ cm.

(f) Similarly, $m = -0.40$, with a minus sign.

(g) The image is real (R).

(h) The image is inverted (I).

(i) A real image is formed on the same side as the object.

26. (a) We are told that the image is on the same side as the object; this means the image is real (R) and further implies that the mirror is concave.

(b) The focal distance is $f = +20$ cm.

(c) The radius of curvature is $r = 2f = +40$ cm.

(d) $p = +60$ cm, as given in the table.

(e) Equation 34-9 gives $i = pf/(p - f) = +30$ cm.

(f) Equation 34-7 gives $m = -i/p = -0.50$.

(g) As noted above, the image is real (R).

(h) The image is inverted (I) since $m < 0$.

(i) A real image is formed on the same side as the object.

27. (a) The fact that the focal length is given as a negative value means the mirror is convex.

(b) $f = -30$ cm, as given in the Table.

(c) The radius of curvature is $r = 2f = -60$ cm.

(d) Equation 34-9 gives $p = if/(i - f) = +30$ cm.

(e) $i = -15$, as given in the table.

(f) From Eq. 34-7, we get $m = +1/2 = 0.50$.

(g) The image distance is given as a negative value (as it would have to be, since the mirror is convex), which means the image is virtual (V).

(h) Since $m > 0$, the image is upright (not inverted: NI).

(i) The image is on the opposite side of the mirror as the object.

28. (a) The fact that the magnification is 1 means that the mirror is flat (plane).

(b) Flat mirrors (and flat “lenses” such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case).

(c) The radius of curvature is $r = 2f = \infty$ (or $r = -\infty$) by Eq. 34-3.

(d) $p = +10$ cm, as given in the table.

(e) Equation 34-4 readily yields $i = pf/(p - f) = -10$ cm.

(f) The magnification is $m = -i/p = +1.0$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

29. **THINK** A convex mirror has a negative value of focal length.

EXPRESS For spherical mirrors, the focal length f is related to the radius of curvature r by $f = r/2$. The object distance p , the image distance i , and the focal length f are related by Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images. Real images are formed on the same side as the object, while virtual images are formed on the opposite side of the mirror.

ANALYZE (a) The mirror is convex, as given.

(b) Since the mirror is convex, the radius of curvature is negative, so $r = -40$ cm. Then, the focal length is $f = r/2 = (-40 \text{ cm})/2 = -20$ cm.

(c) The radius of curvature is $r = -40$ cm.

(d) The fact that the mirror is convex also means that we need to insert a minus sign in front of the “4.0” value given for i , since the image in this case must be virtual. Eq. 34-4 leads to

$$p = \frac{if}{i - f} = \frac{(-4.0 \text{ cm})(-20 \text{ cm})}{-4.0 \text{ cm} - (-20 \text{ cm})} = 5.0 \text{ cm}$$

(e) As noted above, $i = -4.0$ cm.

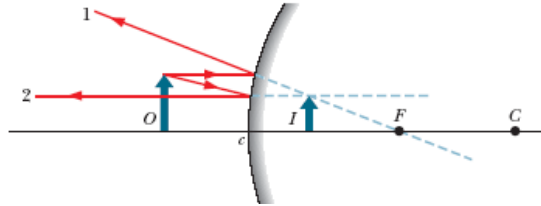
(f) The magnification is $m = -i/p = -(-4.0 \text{ cm})/(5.0 \text{ cm}) = +0.80$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) A virtual image is formed on the opposite side of the mirror from the object.

LEARN The situation in this problem is similar to that illustrated in Fig. 34-11(c). The mirror is convex, and its image is virtual and upright.



30. We note that there is “singularity” in this graph (Fig. 34-36) like there was in Fig. 34-35), which tells us that there is no point where $p = f$ (which causes Eq. 34-9 to “blow up”). Since $p > 0$, as usual, then this means that the focal length is not positive. We know it is not a flat mirror since the curve shown does decrease with p , so we conclude it is a convex mirror. We examine the point where $m = 0.50$ and $p = 10 \text{ cm}$. Combining Eq. 34-7 and Eq. 34-9 we obtain

$$m = -\frac{i}{p} = -\frac{f}{p-f}.$$

This yields $f = -10 \text{ cm}$ (verifying our expectation that the mirror is convex). Now, for $p = 21 \text{ cm}$, we find $m = -f/(p-f) = +0.32$.

31. (a) From Eqs. 34-3 and 34-4, we obtain

$$i = \frac{pf}{p-f} = \frac{pr}{2p-r}.$$

Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt} \left(\frac{pr}{2p-r} \right) = \frac{-rv_O(2p-r) + 2v_O pr}{(2p-r)^2} = \left(\frac{r}{2p-r} \right)^2 v_O.$$

(b) If $p = 30 \text{ cm}$, we obtain $v_I = \left[\frac{15 \text{ cm}}{2(30 \text{ cm}) - 15 \text{ cm}} \right]^2 (5.0 \text{ cm/s}) = 0.56 \text{ cm/s}$.

(c) If $p = 8.0 \text{ cm}$, we obtain $v_I = \left[\frac{15 \text{ cm}}{2(8.0 \text{ cm}) - 15 \text{ cm}} \right]^2 (5.0 \text{ cm/s}) = 1.1 \times 10^3 \text{ cm/s}$.

(d) If $p = 1.0$ cm, we obtain $v_I = \left[\frac{15 \text{ cm}}{2(1.0 \text{ cm}) - 15 \text{ cm}} \right]^2 (5.0 \text{ cm/s}) = 6.7 \text{ cm/s}$.

32. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (b) $p = +10$ cm, and (c) $r = +30$ cm.

(d) Equation 34-8 yields $i = n_2 \left(\frac{n_2 - n_1}{r} - \frac{n_1}{p} \right)^{-1} = 1.5 \left(\frac{1.5 - 1.0}{30 \text{ cm}} - \frac{1.0}{10 \text{ cm}} \right)^{-1} = -18 \text{ cm}$.

(e) The image is virtual (V) and upright since $i < 0$.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(c) in the textbook.

33. **THINK** An image is formed by refraction through a spherical surface. A negative value for the image distance implies that the image is virtual.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (b) $p = +10$ cm, and (d) $i = -13$ cm.

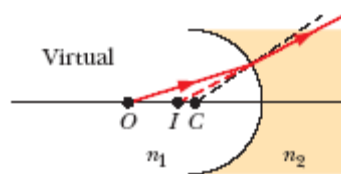
(c) Eq. 34-8 yields

$$r = (n_2 - n_1) \left(\frac{n_1}{p} + \frac{n_2}{i} \right)^{-1} = (1.5 - 1.0) \left(\frac{1.0}{10 \text{ cm}} + \frac{1.5}{-13 \text{ cm}} \right)^{-1} = -32.5 \text{ cm} \approx -33 \text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(e). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



34. In addition to $n_1 = 1.5$, we are given (b) $p = +100$, (c) $r = -30$ cm, and (d) $i = +600$ cm.

(a) We manipulate Eq. 34-8 to separate the indices:

$$n_2 \left(\frac{1}{r} - \frac{1}{i} \right) = \left(\frac{n_1}{p} + \frac{n_1}{r} \right) \Rightarrow n_2 \left(\frac{1}{-30} - \frac{1}{600} \right) = \left(\frac{1.5}{100} + \frac{1.5}{-30} \right) \Rightarrow n_2 (-0.035) = -0.035$$

which implies $n_2 = 1.0$.

(e) The image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(b) in the textbook.

35. **THINK** An image is formed by refraction through a spherical surface. Whether the image is real or virtual depends on the relative values of n_1 and n_2 , and on the geometry.

EXPRESS Let n_1 be the index of refraction of the material where the object is located, n_2 be the index of refraction of the material on the other side of the refracting surface, and r be the radius of curvature of the surface. The image distance i is related to the object distance p by Eq. 34-8:

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}.$$

The value of i is positive for a real images, and negative for virtual images.

ANALYZE In addition to $n_1 = 1.5$, we are also given (a) $n_2 = 1.0$, (b) $p = +70$ cm, and (c) $r = +30$ cm. Notice that $n_2 < n_1$.

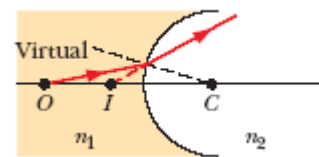
(d) We manipulate Eq. 34-8 to find the image distance:

$$i = n_2 \left(\frac{n_2 - n_1}{r} - \frac{n_1}{p} \right)^{-1} = 1.0 \left(\frac{1.0 - 1.5}{30 \text{ cm}} - \frac{1.5}{70 \text{ cm}} \right)^{-1} = -26 \text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-12(f). Here refraction always directs the ray away from the central axis; the images are always virtual, regardless of the object distance.



36. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (c) $r = -30$ cm and (d) $i = -7.5$ cm.

(b) We manipulate Eq. 34-8 to find p :

$$p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.5}{\frac{1.0 - 1.5}{-30 \text{ cm}} - \frac{1.0}{-7.5 \text{ cm}}} = 10 \text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(d) in the textbook.

37. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (b) $p = +10$ cm, and (d) $i = -6.0$ cm.

(c) We manipulate Eq. 34-8 to find r :

$$r = (n_2 - n_1) \left(\frac{n_1}{p} + \frac{n_2}{i} \right)^{-1} = (1.0 - 1.5) \left(\frac{1.5}{10 \text{ cm}} + \frac{1.0}{-6.0 \text{ cm}} \right)^{-1} = 30 \text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are on the same side. The ray diagram would be similar to Fig. 34-12(f) in the textbook, but with the object and the image located closer to the surface.

38. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (c) $r = +30$ cm, and (d) $i = +600$.

(b) Equation 34-8 gives $p = \frac{n_1}{\frac{n_2 - n_1}{r} - \frac{n_2}{i}} = \frac{1.0}{\frac{1.5 - 1.0}{30 \text{ cm}} - \frac{1.5}{600 \text{ cm}}} = 71 \text{ cm}.$

(e) With $i > 0$, the image is real (R) and inverted.

(f) The object and its image are on the opposite side. The ray diagram would be similar to Fig. 34-12(a) in the textbook.

39. (a) We use Eq. 34-8 and note that $n_1 = n_{\text{air}} = 1.00$, $n_2 = n$, $p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n - 1}{r}.$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 34-8 becomes

$$\frac{n}{r} = \frac{n-1}{r},$$

which is not valid unless $n \rightarrow \infty$ or $r \rightarrow \infty$. It is impossible to focus at the center of the sphere.

40. We use Eq. 34-8 (and Fig. 34-11(d) is useful), with $n_1 = 1.6$ and $n_2 = 1$ (using the rounded-off value for air):

$$\frac{1.6}{p} + \frac{1}{i} = \frac{1-1.6}{r}.$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = -5.0$ cm), we obtain $i = -2.4$ cm for objects at $p = 3.0$ cm. Returning to Fig. 34-38 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

41. (a) We use Eq. 34-10:

$$f = \left[(n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right]^{-1} = \left[(1.5-1) \left(\frac{1}{\infty} - \frac{1}{-20 \text{ cm}} \right) \right]^{-1} = +40 \text{ cm}.$$

(b) From Eq. 34-9,

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \left(\frac{1}{40 \text{ cm}} - \frac{1}{40 \text{ cm}} \right)^{-1} = \infty.$$

42. Combining Eq. 34-7 and Eq. 34-9, we have $m(p-f) = -f$. The graph in Fig. 34-39 indicates that $m = 0.5$ where $p = 15$ cm, so our expression yields $f = -15$ cm. Plugging this back into our expression and evaluating at $p = 35$ cm yields $m = +0.30$.

43. We solve Eq. 34-9 for the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \frac{fp}{p-f}.$$

The height of the image is

$$h_i = mh_p = \left(\frac{i}{p} \right) h_p = \frac{fh_p}{p-f} = \frac{(75 \text{ mm})(1.80 \text{ m})}{27 \text{ m} - 0.075 \text{ m}} = 5.0 \text{ mm}.$$

44. The singularity the graph (where the curve goes to $\pm\infty$) is at $p = 30$ cm, which implies (by Eq. 34-9) that $f = 30$ cm > 0 (converging type lens). For $p = 100$ cm, Eq. 34-9 leads to $i = +43$ cm.

45. Let the diameter of the Sun be d_s and that of the image be d_i . Then, Eq. 34-5 leads to

$$d_i = |m|d_s = \left(\frac{i}{p}\right)d_s \approx \left(\frac{f}{p}\right)d_s = \frac{(20.0 \times 10^{-2} \text{ m})(2)(6.96 \times 10^8 \text{ m})}{1.50 \times 10^{11} \text{ m}} = 1.86 \times 10^{-3} \text{ m} = 1.86 \text{ mm}.$$

46. Since the focal length is a constant for the whole graph, then $1/p + 1/i = \text{constant}$. Consider the value of the graph at $p = 20 \text{ cm}$; we estimate its value there to be -10 cm . Therefore, $1/20 + 1/(-10) = 1/70 + 1/i_{\text{new}}$. Thus, $i_{\text{new}} = -16 \text{ cm}$.

47. **THINK** Our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1) \left(\frac{1}{r_1} + \frac{1}{2r_1} \right) = \frac{3(n-1)}{2r_1}.$$

ANALYZE (a) We solve for the smaller radius r_1 :

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60 \text{ mm})}{2} = 45 \text{ mm}.$$

(b) The magnitude of the larger radius is $|r_2| = 2r_1 = 90 \text{ mm}$.

LEARN An image of an object can be formed with a lens because it can bend the light rays, but the bending is possible only if the index of refraction of the lens is different from that of its surrounding medium.

48. Combining Eq. 34-7 and Eq. 34-9, we have $m(p-f) = -f$. The graph in Fig. 34-42 indicates that $m = 2$ where $p = 5 \text{ cm}$, so our expression yields $f = 10 \text{ cm}$. Plugging this back into our expression and evaluating at $p = 14 \text{ cm}$ yields $m = -2.5$.

49. **THINK** The image is formed on the screen, so the sum of the object distance and the image distance is equal to the distance between the slide and the screen.

EXPRESS Using Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}$$

and noting that $p + i = d = 44$ cm, we obtain $p^2 - dp + df = 0$.

ANALYZE The focal length is $f = 11$ cm. Solving the quadratic equation, we find the solution to p to be

$$p = \frac{1}{2}(d \pm \sqrt{d^2 - 4df}) = 22 \text{ cm} \pm \frac{1}{2}\sqrt{(44 \text{ cm})^2 - 4(44 \text{ cm})(11 \text{ cm})} = 22 \text{ cm}.$$

LEARN Since $p > f$, the object is outside the focal length. The image distance is $i = d - p = 44 - 22 = 22$ cm.

50. We recall that for a converging (C) lens, the focal length value should be positive ($f = +4$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = +5.3$ cm.

(b) Equation 34-7 gives $m = -i/p = -0.33$.

(c) The fact that the image distance i is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the opposite side of the object (see Fig. 34-16(a)).

51. We recall that for a converging (C) lens, the focal length value should be positive ($f = +16$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -48$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.0$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

52. We recall that for a converging (C) lens, the focal length value should be positive ($f = +35$ cm).

(a) Equation 34-9 gives $i = pf/(p - f) = -88$ cm.

(b) Equation 34-7 give $m = -i/p = +3.5$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(b)).

53. **THINK** For a diverging (D) lens, the focal length value is negative.

EXPRESS The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = -12$ cm and $p = +8.0$ cm.

(a) The image distance is $i = \frac{pf}{p-f} = \frac{(8.0 \text{ cm})(-12 \text{ cm})}{8.0 \text{ cm} - (-12 \text{ cm})} = -4.8$ cm.

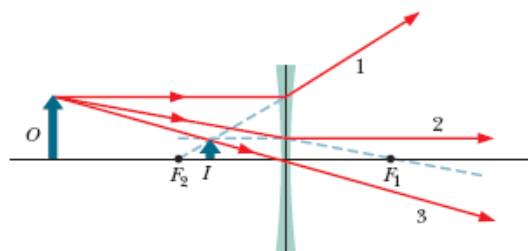
(b) The magnification is $m = -i/p = -(-4.8 \text{ cm})/(8.0 \text{ cm}) = +0.60$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.



54. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -6$ cm).

(a) Equation 34-9 gives $i = pf/(p-f) = -3.8$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.38$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

55. **THINK** For a diverging (D) lens, the value of the focal length is negative.

EXPRESS The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = -14$ cm and $p = +22.0$ cm.

(a) The image distance is $i = \frac{pf}{p-f} = \frac{(22 \text{ cm})(-14 \text{ cm})}{22 \text{ cm} - (-14) \text{ cm}} = -8.6$ cm.

(b) The magnification is $m = -i/p = -(-8.6 \text{ cm})/(22 \text{ cm}) = +0.39$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

56. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -31$ cm).

(a) Equation 34-9 gives $i = pf/(p-f) = -8.7$ cm.

(b) Equation 34-7 gives $m = -i/p = +0.72$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-16(c)).

57. **THINK** For a converging (C) lens, the focal length value is positive.

EXPRESS The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE For this lens, we have $f = +20$ cm and $p = +45.0$ cm.

(a) The image distance is $i = \frac{pf}{p-f} = \frac{(45 \text{ cm})(20 \text{ cm})}{45 \text{ cm} - 20 \text{ cm}} = +36$ cm.

(b) The magnification is $m = -i/p = -(+36 \text{ cm})/(45 \text{ cm}) = -0.80$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) A negative value of magnification means the image is inverted (I).

(e) The image is on the opposite side of the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.

58. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -63$ cm.

(b) Equation 34-7 gives $m = -i/p = +2.2$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

59. **THINK** Since r_1 is positive and r_2 is negative, our lens is of double-convex type. We apply lens maker's equation to analyze the problem.

EXPRESS The lens maker's equation is given by Eq. 34-10:

$$\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

ANALYZE For this lens, we have $r_1 = +30$ cm, $r_2 = -42$ cm, $n = 1.55$ and $p = +75$ cm.

(a) The focal length is

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = \frac{(+30 \text{ cm})(-42 \text{ cm})}{(1.55-1)(-42 \text{ cm} - 30 \text{ cm})} = +31.8 \text{ cm}.$$

Thus, the image distance is $i = \frac{pf}{p-f} = \frac{(75 \text{ cm})(31.8 \text{ cm})}{75 \text{ cm} - 31.8 \text{ cm}} = +55 \text{ cm}$.

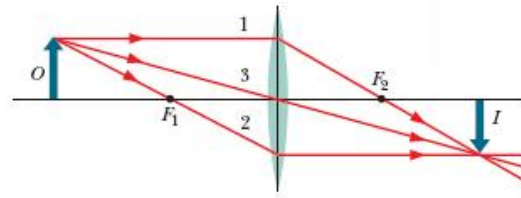
(b) Eq. 34-7 give $m = -i/p = -(55 \text{ cm})/(75 \text{ cm}) = -0.74$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(a). The lens is converging, forming a real, inverted image on the opposite side of the object.



60. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -26$ cm.

(b) Equation 34-7 gives $m = -i/p = +4.3$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

61. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -18$ cm.

(b) Equation 34-7 gives $m = -i / p = +0.76$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

62. (a) Equation 34-10 yields

$$f = \frac{r_1 r_2}{(n-1)(r_2 - r_1)} = +30 \text{ cm}$$

Since $f > 0$, this must be a converging (“C”) lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{30 \text{ cm}} - \frac{1}{10 \text{ cm}}} = -15 \text{ cm}.$$

(b) Equation 34-6 yields $m = -i / p = -(-15 \text{ cm}) / (10 \text{ cm}) = +1.5$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(b) of the textbook.

63. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -30$ cm.

(b) Equation 34-7 gives $m = -i / p = +0.86$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

64. (a) Equation 34-10 yields

$$f = \frac{1}{n-1} (1/r_1 - 1/r_2)^{-1} = -120 \text{ cm}.$$

Since $f < 0$, this must be a diverging (“D”) lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-120 \text{ cm}} - \frac{1}{10 \text{ cm}}} = -9.2 \text{ cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-9.2 \text{ cm})/(10 \text{ cm}) = +0.92$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

65. (a) Equation 34-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -30 \text{ cm}$. Since $f < 0$, this must be a diverging (“D”) lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-30 \text{ cm}} - \frac{1}{10 \text{ cm}}} = -7.5 \text{ cm}.$$

(b) Equation 34-6 yields $m = -i/p = -(-7.5 \text{ cm})/(10 \text{ cm}) = +0.75$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image is on the same side as the object. The ray diagram is similar to Fig. 34-16(c) of the textbook.

66. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -9.7 \text{ cm}$.

(b) Equation 34-7 gives $m = -i/p = +0.54$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object.

67. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = +84 \text{ cm}$.

(b) Equation 34-7 gives $m = -i/p = -1.4$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object.

68. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0 \text{ cm})}{3} = 26.7 \text{ cm.}$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p} \right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3} \right)^{-1} = \frac{2d}{9} = \frac{2(40.0 \text{ cm})}{9} = 8.89 \text{ cm.}$$

69. (a) Since $f > 0$, this is a converging lens ("C").

(d) Equation 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10 \text{ cm}} - \frac{1}{5.0 \text{ cm}}} = -10 \text{ cm.}$$

(e) From Eq. 34-6, $m = -(-10 \text{ cm})/(5.0 \text{ cm}) = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

70. (a) The fact that $m < 1$ and that the image is upright (not inverted: NI) means the lens is of the diverging type (D) (it may help to look at Fig. 34-16 to illustrate this).

(b) A diverging lens implies that $f = -20 \text{ cm}$, with a minus sign.

(d) Equation 34-9 gives $i = -5.7 \text{ cm}$.

(e) Equation 34-7 gives $m = -i/p = +0.71$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image is on the same side as the object.

71. (a) Eq. 34-7 yields $i = -mp = -(0.25)(16 \text{ cm}) = -4.0 \text{ cm}$. Equation 34-9 gives $f = -5.3 \text{ cm}$, which implies the lens is of the diverging type (D).

(b) From (a), we have $f = -5.3 \text{ cm}$.

(d) Similarly, $i = -4.0 \text{ cm}$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

72. (a) Equation 34-7 readily yields $i = +4.0 \text{ cm}$. Then Eq. 34-9 gives $f = +3.2 \text{ cm}$, which implies the lens is of the converging type (C).

(b) From (a), we have $f = +3.2 \text{ cm}$.

(d) Similarly, $i = +4.0 \text{ cm}$.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the opposite side of the object.

73. (a) Using Eq. 34-6 (which implies the image is inverted) and the given value of p , we find $i = -mp = +5.0 \text{ cm}$; it is a real image. Equation 34-9 then yields the focal length: $f = +3.3 \text{ cm}$. Therefore, the lens is of the converging ("C") type.

(b) From (a), we have $f = +3.3 \text{ cm}$.

(d) Similarly, $i = -mp = +5.0 \text{ cm}$.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object. The ray diagram is similar to Fig. 34-16(a) of the textbook.

74. (b) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length.

(d) Equation 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10 \text{ cm}} - \frac{1}{20 \text{ cm}}} = +20 \text{ cm}.$$

(e) From Eq. 34-6, $m = -20/20 = -1.0$.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object.

75. **THINK** Since the image is on the same side as the object, it must be a virtual image.

EXPRESS The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Since the image is virtual (on the same side as the object), the image distance i is negative. By substituting $i = fp/(p - f)$ into $m = -i/p$, we obtain

$$m = -\frac{i}{p} = -\frac{f}{p - f}.$$

The fact that the magnification is less than 1.0 implies that f must be negative. This means that the lens is of the diverging (“D”) type.

(b) Thus, the focal length is $f = -10 \text{ cm}$.

(d) The image distance is $i = \frac{pf}{p - f} = \frac{(5.0 \text{ cm})(-10 \text{ cm})}{5.0 \text{ cm} - (-10 \text{ cm})} = -3.3 \text{ cm}$.

(e) The magnification is $m = -i/p = -(-3.3 \text{ cm})/(5.0 \text{ cm}) = +0.67$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation as the object, and on the same side as the object.

76. (a) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 34-16, 34-17, and 34-19), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging (“C”) type (and if $p < f$).

(b) We should put a plus sign in front of the “10” value given for the focal length.

(d) Equation 34-9 gives $i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10 \text{ cm}} - \frac{1}{5.0 \text{ cm}}} = -10 \text{ cm}$.

(e) $m = -i/p = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image is on the same side as the object.

77. **THINK** A positive value for the magnification m means that the image is upright (not inverted). In addition, $m > 1$ indicates that the image is enlarged.

EXPRESS The object distance p , the image distance i , and the focal length f are related by Eq. 34-9:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}.$$

The value of i is positive for a real images, and negative for virtual images. The corresponding lateral magnification is $m = -i/p$. The value of m is positive for upright (not inverted) images, and is negative for inverted images.

ANALYZE (a) Combining Eqs. 34-7 and 34-9, we find the focal length to be

$$f = \frac{p}{1 - 1/m} = \frac{16 \text{ cm}}{1 - 1/1.25} = 80 \text{ cm}.$$

Since the value of f is positive, the lens is of the converging type (C).

(b) From (a), we have $f = +80 \text{ cm}$.

(d) The image distance is $i = -mp = -(1.25)(16 \text{ cm}) = -20 \text{ cm}$.

- (e) The magnification is $m = +1.25$, as given.
- (f) The fact that the image distance i is a negative value means the image is virtual (V).
- (g) A positive value of magnification means the image is not inverted (NI).
- (h) The image is on the same side as the object.

LEARN The ray diagram for this problem is similar to the one shown in Fig. 34-16(b). The lens is converging. With the object placed inside the focal point ($p < f$), we have a virtual image with the same orientation as the object, and on the same side as the object.

78. (a) We are told the absolute value of the magnification is 0.5 and that the image was upright (NI). Thus, $m = +0.5$. Using Eq. 34-6 and the given value of p , we find $i = -5.0$ cm; it is a virtual image. Equation 34-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging (“D”) type.

- (b) From (a), we have $f = -10$ cm.
- (d) Similarly, $i = -5.0$ cm.
- (e) $m = +0.5$, with a plus sign.
- (f) The fact that the image distance i is a negative value means the image is virtual (V).
- (h) The image is on the same side as the object.

79. (a) The fact that $m > 1$ means the lens is of the converging type (C) (it may help to look at Fig. 34-16 to illustrate this).

- (b) A converging lens implies $f = +20$ cm, with a plus sign.
- (d) Equation 34-9 then gives $i = -13$ cm.
- (e) Equation 34-7 gives $m = -i/p = +1.7$.
- (f) The fact that the image distance i is a negative value means the image is virtual (V).
- (g) A positive value of magnification means the image is not inverted (NI).
- (h) The image is on the same side as the object.

80. (a) The image from lens 1 (which has $f_1 = +15$ cm) is at $i_1 = -30$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = +8$ cm) with $p_2 = d - i_1 = 40$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +10$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -0.75$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

81. (a) The image from lens 1 (which has $f_1 = +8$ cm) is at $i_1 = 24$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 8$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +24$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = +6.0$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the side opposite from the object (relative to lens 2).

82. (a) The image from lens 1 (which has $f_1 = -6$ cm) is at $i_1 = -3.4$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 15.4$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +9.8$ cm.

(b) Equation 34-11 yields $M = -0.27$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

83. **THINK** In a system with two lenses, the image formed by lens 1 serves the “object” for lens 2.

EXPRESS To analyze two-lens systems, we first ignore lens 2, and apply the standard procedure used for a single-lens system. The object distance p_1 , the image distance i_1 , and the focal length f_1 are related by:

$$\frac{1}{f_1} = \frac{1}{p_1} + \frac{1}{i_1}.$$

Next, we ignore the lens 1 but treat the image formed by lens 1 as the object for lens 2. The object distance p_2 is the distance between lens 2 and the location of the first image. The location of the final image, i_2 , is obtained by solving

$$\frac{1}{f_2} = \frac{1}{p_2} + \frac{1}{i_2}$$

where f_2 is the focal length of lens 2.

ANALYZE (a) Since lens 1 is converging, $f_1 = +9$ cm, and we find the image distance to be

$$i_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(20 \text{ cm})(9 \text{ cm})}{20 \text{ cm} - 9 \text{ cm}} = 16.4 \text{ cm}.$$

This serves as an “object” for lens 2 (which has $f_2 = +5$ cm) with an object distance given by $p_2 = d - i_1 = -8.4$ cm. The negative sign means that the “object” is behind lens 2. Solving the lens equation, we obtain

$$i_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-8.4 \text{ cm})(5.0 \text{ cm})}{-8.4 \text{ cm} - 5.0 \text{ cm}} = 3.13 \text{ cm}.$$

(b) The overall magnification is $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -0.31$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (relative to lens 2).

LEARN Since this result involves a negative value for p_2 (and perhaps other “non-intuitive” features), we offer a few words of explanation: lens 1 is converging the rays towards an image (that never gets a chance to form due to the intervening presence of lens 2) that would be real and inverted (and 8.4 cm beyond lens 2’s location). Lens 2, in a sense, just causes these rays to converge a little more rapidly, and causes the image to form a little closer (to the lens system) than if lens 2 were not present.

84. (a) The image from lens 1 (which has $f_1 = +12$ cm) is at $i_1 = +60$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = +10$ cm) with $p_2 = d - i_1 = 7$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -23$ cm.

(b) Equation 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -13$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

85. (a) The image from lens 1 (which has $f_1 = +6$ cm) is at $i_1 = -12$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = -6$ cm) with $p_2 = d - i_1 = 20$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -4.6$ cm.

(b) Equation 34-11 yields $M = +0.69$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

86. (a) The image from lens 1 (which has $f_1 = +8$ cm) is at $i_1 = +24$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = -8$ cm) with $p_2 = d - i_1 = 6$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -3.4$ cm.

(b) Equation 34-11 yields $M = -1.1$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the same side as the object (relative to lens 2).

87. (a) The image from lens 1 (which has $f_1 = -12$ cm) is at $i_1 = -7.5$ cm (by Eq. 34-9). This serves as an “object” for lens 2 (which has $f_2 = -8$ cm) with

$$p_2 = d - i_1 = 17.5 \text{ cm.}$$

Then Eq. 34-9 (applied to lens 2) yields $i_2 = -5.5$ cm.

(b) Equation 34-11 yields $M = +0.12$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image is on the same side as the object (relative to lens 2).

88. The minimum diameter of the eyepiece is given by

$$d_{\text{ey}} = \frac{d_{\text{ob}}}{m_{\theta}} = \frac{75 \text{ mm}}{36} = 2.1 \text{ mm.}$$

89. **THINK** The compound microscope shown in Fig. 34-20 consists of an objective and an eyepiece. It's used for viewing small objects that are very close to the objective.

EXPRESS Let f_{ob} be the focal length of the objective, and f_{ey} be the focal length of the eyepiece. The distance between the two lenses is

$$L = s + f_{\text{ob}} + f_{\text{ey}},$$

where s is the tube length. The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{s}{f_{\text{ob}}}$$

and the angular magnification produced by the eyepiece is $m_{\theta} = (25 \text{ cm}) / f_{\text{ey}}$.

ANALYZE (a) The tube length is

$$s = L - f_{\text{ob}} - f_{\text{ey}} = 25.0 \text{ cm} - 4.00 \text{ cm} - 8.00 \text{ cm} = 13.0 \text{ cm}.$$

(b) We solve $(1/p) + (1/i) = (1/f_{\text{ob}})$ for p . The image distance is

$$i = f_{\text{ob}} + s = 4.00 \text{ cm} + 13.0 \text{ cm} = 17.0 \text{ cm},$$

so

$$p = \frac{if_{\text{ob}}}{i - f_{\text{ob}}} = \frac{(17.0 \text{ cm})(4.00 \text{ cm})}{17.0 \text{ cm} - 4.00 \text{ cm}} = 5.23 \text{ cm}.$$

(c) The magnification of the objective is $m = -\frac{i}{p} = -\frac{17.0 \text{ cm}}{5.23 \text{ cm}} = -3.25$.

(d) The angular magnification of the eyepiece is $m_{\theta} = \frac{25 \text{ cm}}{f_{\text{ey}}} = \frac{25 \text{ cm}}{8.00 \text{ cm}} = 3.13$.

(e) The overall magnification of the microscope is

$$M = mm_{\theta} = (-3.25)(3.13) = -10.2.$$

LEARN The objective produces a real image I of the object inside the focal point of the eyepiece ($i > f_{\text{ey}}$). Image I then serves as the object for the eyepiece, which produces a virtual image I' seen by the observer.

90. (a) Now, the lens-film distance is $i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \left(\frac{1}{5.0 \text{ cm}} - \frac{1}{100 \text{ cm}} \right)^{-1} = 5.3 \text{ cm}$.

(b) The change in the lens-film distance is $5.3 \text{ cm} - 5.0 \text{ cm} = 0.30 \text{ cm}$.

91. **THINK** This problem is about human eyes. We model the cornea and eye lens as a single effective thin lens, with image formed at the retina.

EXPRESS When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus, $i = f = 2.50$ cm. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change.

ANALYZE (a) If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'}.$$

We substitute $i = f$ and solve for f' : $f' = \frac{pf}{f + p} = \frac{(40.0 \text{ cm})(2.50 \text{ cm})}{40.0 \text{ cm} + 2.50 \text{ cm}} = 2.35 \text{ cm}.$

(b) Consider the lens maker's equation

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34-46, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

LEARN When focusing on an object near the eye, the lens bulges a bit (smaller radius of curvature), and its focal length decreases.

92. We refer to Fig. 34-20. For the intermediate image, $p = 10$ mm and

$$i = (f_{\text{ob}} + s + f_{\text{ey}}) - f_{\text{ey}} = 300 \text{ mm} - 50 \text{ mm} = 250 \text{ mm},$$

so

$$\frac{1}{f_{\text{ob}}} = \frac{1}{i} + \frac{1}{p} = \frac{1}{250 \text{ mm}} + \frac{1}{10 \text{ mm}} \Rightarrow f_{\text{ob}} = 9.62 \text{ mm},$$

and

$$s = (f_{\text{ob}} + s + f_{\text{ey}}) - f_{\text{ob}} - f_{\text{ey}} = 300 \text{ mm} - 9.62 \text{ mm} - 50 \text{ mm} = 240 \text{ mm}.$$

Then from Eq. 34-14,

$$M = -\frac{s}{f_{\text{ob}}} \frac{25 \text{ cm}}{f_{\text{ey}}} = -\left(\frac{240 \text{ mm}}{9.62 \text{ mm}} \right) \left(\frac{150 \text{ mm}}{50 \text{ mm}} \right) = -125.$$

93. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 34-19). With the magnifier, letting

$$i = -|i| = -P_n,$$

we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25 \text{ cm}}{f}.$$

With $f = 10 \text{ cm}$, $m_\theta = 1 + \frac{25 \text{ cm}}{10 \text{ cm}} = 3.5$.

(b) In the case where the image appears at infinity, let $i = -|i| \rightarrow -\infty$, so that $1/p + 1/i = 1/p = 1/f$, we have

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25 \text{ cm}}{f}.$$

With $f = 10 \text{ cm}$, $m_\theta = \frac{25 \text{ cm}}{10 \text{ cm}} = 2.5$.

94. By Eq. 34-9, $1/i + 1/p$ is equal to constant ($1/f$). Thus,

$$1/(-10) + 1/(15) = 1/i_{\text{new}} + 1/(70).$$

This leads to $i_{\text{new}} = -21 \text{ cm}$.

95. A converging lens has a positive-valued focal length, so $f_1 = +8 \text{ cm}$, $f_2 = +6 \text{ cm}$, and $f_3 = +6 \text{ cm}$. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 24 \text{ cm}$ and $i_2 = -12 \text{ cm}$. Our final results are as follows:

(a) $i_3 = +8.6 \text{ cm}$.

(b) $m = +2.6$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

96. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +4.0$ cm. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -2.4$ cm and $i_2 = 12$ cm. Our final results are as follows:

- (a) $i_3 = -4.0$ cm.
- (b) $m = -1.2$.
- (c) The image is virtual (V).
- (d) The image is inverted (I).
- (e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

97. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +3.0$ cm, and $f_3 = +3.0$ cm. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 9.0$ cm and $i_2 = 6.0$ cm. Our final results are as follows:

- (a) $i_3 = +7.5$ cm.
- (b) $m = -0.75$.
- (c) The image is real (R).
- (d) The image is inverted (I).
- (e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

98. A converging lens has a positive-valued focal length, so $f_1 = +6.0$ cm, $f_2 = +6.0$ cm, and $f_3 = +5.0$ cm. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -3.0$ cm and $i_2 = 9.0$ cm. Our final results are as follows:

- (a) $i_3 = +10$ cm.
- (b) $m = +0.75$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

99. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -8.0$ cm, $f_2 = -16$ cm, and $f_3 = +8.0$ cm. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -4.0$ cm and $i_2 = -6.86$ cm. Our final results are as follows:

(a) $i_3 = +24.2$ cm.

(b) $m = -0.58$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (as expected for a real image).

100. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = +6.0$ cm, $f_2 = -4.0$ cm, and $f_3 = -12$ cm. We use Eq. 34-9 for each lens separately, “bridging the gap” between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 , etc.), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -12$ cm and $i_2 = -3.33$ cm. Our final results are as follows:

(a) $i_3 = -5.15$ cm ≈ -5.2 cm.

(b) $m = +0.285 \approx +0.29$.

(c) The image is virtual (V).

(d) The image is not inverted (NI).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

101. **THINK** In this problem we convert the Gaussian form of the thin-lens formula to the Newtonian form.

EXPRESS For a thin lens, the Gaussian form of the thin-lens formula gives $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. To convert the formula to the Newtonian form, let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. In addition, let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside.

ANALYZE From the Gaussian form, we solve for I and obtain:

$$i = \frac{fp}{p-f}.$$

Substituting $p = f + x$ gives

$$i = \frac{f(f+x)}{x}.$$

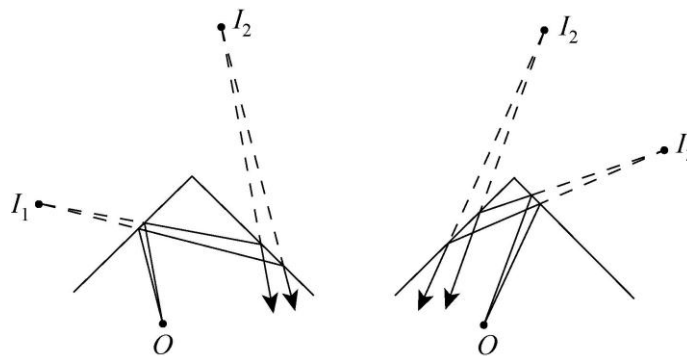
With $i = f + x'$, we have

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

which leads to $xx' = f^2$.

LEARN The Newtonian form is equivalent to the Gaussian form, and it provides another convenient way to analyze problems involving thin lenses.

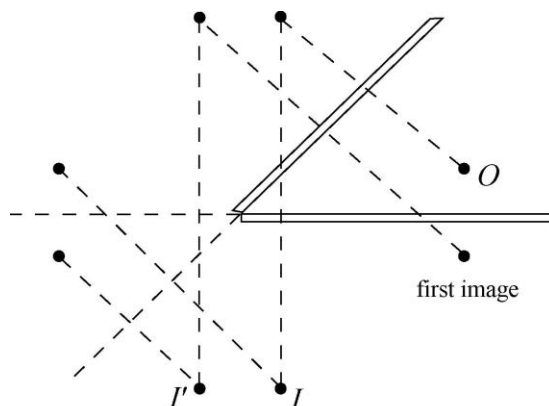
102. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors. The positions of the images are shown on the two diagrams that follow. The diagram on the left shows the image I_1 , formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object O is in front, and lies on the line perpendicular to the mirror and through the object. Image I_2 is formed by light that is reflected from both mirrors.



We may consider I_2 to be the image of I_1 formed by the right-hand mirror, extended. I_2 is the same distance behind the line of the right-hand mirror as I_1 is in front, and it is on the line that is perpendicular to the line of the mirror. The diagram on the right shows image I_3 , formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and

through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at I_2 .

(b) For $\theta = 45^\circ$, we have two images in the second mirror caused by the object and its “first” image, and from these one can construct two new images I and I' behind the first mirror plane. Extending the second mirror plane, we can find two further images of I and I' that are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other’s “twins.” We show this construction in the figure below. Summarizing, we find $1 + 2 + 2 + 2 = 7$ images in this case.



(c) For $\theta = 60^\circ$, we have two images in the second mirror caused by the object and its “first” image, and from these one can construct two new images I and I' behind the first mirror plane. The images I and I' are each other’s “twins” in the sense that they are each other’s reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find $1 + 2 + 2 = 5$ images in this case.

For $\theta = 120^\circ$, we have two images I'_1 and I_2 behind the extension of the second mirror plane, caused by the object and its “first” image (which we refer to here as I_1). No further images can be constructed from I'_1 and I_2 , since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer.

(d) Thus, the smallest number of images that can be seen is 1. For example, if the observer’s eye is collinear with I_1 and I'_1 , then the observer can only see one image (I_1 and not the one behind it). Note that an observer who stands close to the second mirror would probably be able to see two images, I_1 and I_2 .

(e) Similarly, the largest number would be 3. This happens if the observer moves further back from the vertex of the two mirrors. He or she should also be able to see the third image, I'_1 , which is essentially the “twin” image formed from I_1 relative to the extension of the second mirror plane.

103. **THINK** Two lenses in contact can be treated as one single lens with an effective focal length.

EXPRESS We place an object far away from the composite lens and find the image distance i . Since the image is at a focal point, $i = f$, where f equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thickness of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive.

ANALYZE In the thin lens equation, we replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2}.$$

Thus, the effective focal length of the system is $f = \frac{f_1 f_2}{f_1 + f_2}$.

LEARN The reciprocal of the focal length, $1/f$, is known as the power of the lens, a quantity used by the optometrists to specify the strength of eyeglasses. From the derivation above, we see that when two lenses are in contact, the power of the effective lens is the sum of the two powers.

104. (a) In the closest mirror M_1 , the “first” image I_1 is 10 cm behind M_1 and therefore 20 cm from the object O . This is the smallest distance between the object and an image of the object.

(b) There are images from both O and I_1 in the more distant mirror, M_2 : an image I_2 located at 30 cm behind M_2 . Since O is 30 cm in front of it, I_2 is 60 cm from O . This is the second smallest distance between the object and an image of the object.

(c) There is also an image I_3 that is 50 cm behind M_2 (since I_1 is 50 cm in front of it). Thus, I_3 is 80 cm from O . In addition, we have another image I_4 that is 70 cm behind M_1 (since I_2 is 70 cm in front of it). The distance from I_4 to O for is 80 cm.

(d) Returning to the closer mirror M_1 , there is an image I_5 that is 90 cm behind the mirror (since I_3 is 90 cm in front of it). The distances (measured from O) for I_5 is 100 cm = 1.0 m.

105. (a) The “object” for the mirror that results in that box image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to $i_1 = 10 - 4 = 6$ cm. Thus, with $f_1 = 2$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow p_1 = 3.00 \text{ cm.}$$

(b) The previously mentioned box image (4 cm behind the mirror) serves as an “object” (at $p_3 = 14$ cm) for the return trip of light through the lens ($f_3 = f_1 = 2$ cm). This time, Eq. 34-9 leads to

$$\frac{1}{p_3} + \frac{1}{i_3} = \frac{1}{f_3} \Rightarrow i_3 = 2.33 \text{ cm.}$$

106. (a) First, the lens forms a real image of the object located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1} \right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1} \right)^{-1} = 2f_1$$

to the right of the lens, or at

$$p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$$

in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2} \right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2} \right)^{-1} = 2f_2$$

to the left of the mirror, or at

$$p'_1 = 2(f_1 + f_2) - 2f_2 = 2f_1$$

to the right of the lens. The final image formed by the lens is at a distance i'_1 to the left of the lens, where

$$i'_1 = \left(\frac{1}{f_1} - \frac{1}{p'_1} \right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1} \right)^{-1} = 2f_1.$$

This turns out to be the same as the location of the original object.

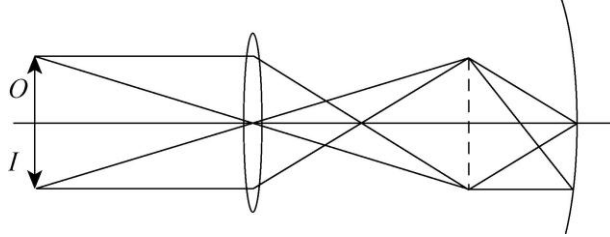
(b) The lateral magnification is

$$m = \left(-\frac{i_1}{p_1} \right) \left(-\frac{i_2}{p_2} \right) \left(-\frac{i'_1}{p'_1} \right) = \left(-\frac{2f_1}{2f_1} \right) \left(-\frac{2f_2}{2f_2} \right) \left(-\frac{2f_1}{2f_1} \right) = -1.0.$$

(c) The final image is real (R).

(d) It is at a distance i'_1 to the left of the lens,

(e) and inverted (I), as shown in the figure below.



107. **THINK** The nature of the lenses, whether converging or diverging, can be determined from the magnification and orientation of the images they produce.

EXPRESS By examining the ray diagrams shown in Fig. 34-16(a) – (c), we see that only a converging lens can produce an enlarged, upright image, while the image produced by a diverging lens is always virtual, reduced in size, and not inverted.

ANALYZE (a) In this case $m > +1$ and we know that lens 1 is converging (producing a virtual image), so that our result for focal length should be positive. Since $|p + i_1| = 20$ cm and $i_1 = -2p_1$, we find $p_1 = 20$ cm and $i_1 = -40$ cm. Substituting these into Eq. 34-9,

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1}$$

leads to

$$f_1 = \frac{p_1 i_1}{p_1 + i_1} = \frac{(20 \text{ cm})(-40 \text{ cm})}{20 \text{ cm} + (-40 \text{ cm})} = +40 \text{ cm},$$

which is positive as we expected.

(b) The object distance is $p_1 = 20$ cm, as shown in part (a).

(c) In this case $0 < m < 1$ and we know that lens 2 is diverging (producing a virtual image), so that our result for focal length should be negative. Since $|p + i_2| = 20$ cm and $i_2 = -p_2/2$, we find $p_2 = 40$ cm and $i_2 = -20$ cm. Substituting these into Eq. 34-9 leads to

$$f_2 = \frac{p_2 i_2}{p_2 + i_2} = \frac{(40 \text{ cm})(-20 \text{ cm})}{40 \text{ cm} + (-20 \text{ cm})} = -40 \text{ cm},$$

which is negative as we expected.

(d) The object distance is $p_2 = 40$ cm, as shown in part (c).

LEARN The ray diagram for lens 1 is similar to the one shown in Fig. 34-16(b). The lens is converging. With the fly inside the focal point ($p_1 < f_1$), we have a virtual image with the same orientation, and on the same side as the object. On the other hand, the ray diagram for lens 2 is similar to the one shown in Fig. 34-16(c). The lens is diverging, forming a virtual image with the same orientation but smaller in size as the object, and on the same side as the object.

108. We use Eq. 34-10, with the conventions for signs discussed in the text.

(a) For lens 1, the biconvex (or double convex) case, we have

$$f = \left[(n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right]^{-1} = \left[(1.5-1) \left(\frac{1}{40 \text{ cm}} - \frac{1}{-40 \text{ cm}} \right) \right]^{-1} = 40 \text{ cm}.$$

(b) Since $f > 0$ the lens forms a real image of the Sun.

(c) For lens 2, of the planar convex type, we find

$$f = \left[(1.5-1) \left(\frac{1}{\infty} - \frac{1}{-40 \text{ cm}} \right) \right]^{-1} = 80 \text{ cm}.$$

(d) The image formed is real (since $f > 0$).

(e) Now for lens 3, of the meniscus convex type, we have

$$f = \left[(1.5-1) \left(\frac{1}{40 \text{ cm}} - \frac{1}{60 \text{ cm}} \right) \right]^{-1} = 240 \text{ cm} = 2.4 \text{ m}.$$

(f) The image formed is real (since $f > 0$).

(g) For lens 4, of the biconcave type, the focal length is

$$f = \left[(1.5-1) \left(\frac{1}{-40 \text{ cm}} - \frac{1}{40 \text{ cm}} \right) \right]^{-1} = -40 \text{ cm}.$$

(h) The image formed is virtual (since $f < 0$).

(i) For lens 5 (plane-concave), we have $f = \left[(1.5-1) \left(\frac{1}{\infty} - \frac{1}{40 \text{ cm}} \right) \right]^{-1} = -80 \text{ cm}.$

(j) The image formed is virtual (since $f < 0$).

(k) For lens 6 (meniscus concave), $f = \left[(1.5 - 1) \left(\frac{1}{60\text{cm}} - \frac{1}{40\text{cm}} \right) \right]^{-1} = -240\text{cm} = -2.4\text{ m}.$

(l) The image formed is virtual (since $f < 0$).

109. (a) The first image is figured using Eq. 34-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p} + \frac{8}{5i} = \frac{1.6 - 1}{r}$$

For a “flat lens” $r = \infty$, so we obtain

$$i = -8p/5 = -64/5$$

(with the unit cm understood) for that object at $p = 10\text{ cm}$. Relative to the second surface, this image is at a distance of $3 + 64/5 = 79/5$. This serves as an object in order to find the final image, using Eq. 34-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'} + \frac{4}{3i'} = 0$$

which produces (for $p' = 79/5$)

$$i' = -5p'/6 = -79/6 \approx -13.2.$$

This means the observer appears $13.2 + 6.8 = 20\text{ cm}$ from the fish.

(b) It is straightforward to “reverse” the above reasoning, the result being that the final fish image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

110. Setting $n_{\text{air}} = 1$, $n_{\text{water}} = n$, and $p = r/2$ in Eq. 34-8 (and being careful with the sign convention for r in that equation), we obtain $i = -r/(1 + n)$, or $|i| = r/(1 + n)$. Then we use similar triangles (where h is the size of the fish and h' is that of the “virtual fish”) to set up the ratio

$$\frac{h'}{r - |i|} = \frac{h}{r/2}.$$

Using our previous result for $|i|$, this gives $h'/h = 2(1 - 1/(1 + n)) = 1.14$.

111. (a) Parallel rays are bent by positive- f lenses to their focal points F_1 , and rays that come from the focal point positions F_2 in front of positive- f lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point F_1 of the first lens coincide with the front focal point F_2 of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are

vertex angles), then $W_f/f_2 = W_i/f_1$ follows immediately. Substituting the values given, we have

$$W_f = \frac{f_2}{f_1} W_i = \frac{30.0 \text{ cm}}{12.5 \text{ cm}} (2.5 \text{ mm}) = 6.0 \text{ mm}.$$

(b) The area is proportional to W^2 . Since intensity is defined as power P divided by area, we have

$$\frac{I_f}{I_i} = \frac{P/W_f^2}{P/W_i^2} = \frac{W_i^2}{W_f^2} = \frac{f_1^2}{f_2^2} \Rightarrow I_f = \left(\frac{f_1}{f_2} \right)^2 I_i = 1.6 \text{ kW/m}^2.$$

(c) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is

$$f_2 - |f_1| = 30.0 \text{ cm} - 26.0 \text{ cm} = 4.0 \text{ cm}.$$

112. The water is medium 1, so $n_1 = n_w$, which we simply write as n . The air is medium 2, for which $n_2 \approx 1$. We refer to points where the light rays strike the water surface as A (on the left side of Fig. 34-56) and B (on the right side of the picture). The point midway between A and B (the center point in the picture) is C . The penny P is directly below C , and the location of the “apparent” or virtual penny is V . We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to θ_2 , and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to θ_1 . The triangles CVB and CPB share a common side, the horizontal distance from C to B (which we refer to as x). Therefore,

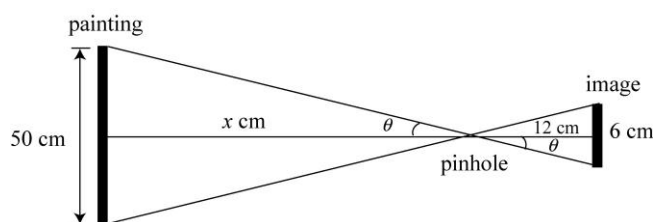
$$\tan \theta_2 = \frac{x}{d_a} \quad \text{and} \quad \tan \theta_1 = \frac{x}{d}.$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan \theta_2}{\tan \theta_1} \approx \frac{\sin \theta_2}{\sin \theta_1} \Rightarrow \frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2} \Rightarrow \frac{d}{d_a} \approx n$$

which yields the desired relation: $d_a = d/n$.

113. The top view of the arrangement is depicted in the figure below.

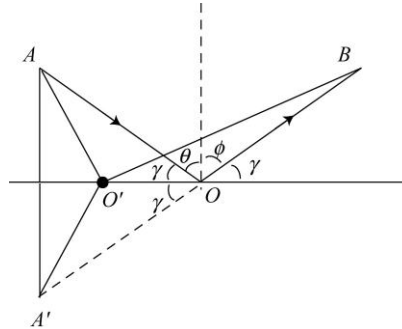


From the figure, we obtain

$$\tan \theta = \frac{25}{x} = \frac{3}{12}$$

which gives $x = 100$ cm.

114. Consider the ray diagram below.



Since $\theta + \gamma = \phi + \gamma = \pi/2$, we readily see that $\theta = \phi$, i.e., the angle of incidence is equal to the angle of reflection. To show that AOB is the shortest path, consider an incident ray AO' with a reflected ray $O'B$, where the angle of incidence is not equal to the angle of reflection. From the figure, we have

$$AO'B = AO' + O'B = A'O' + O'B > A'B = A'O + OB = AO + OB = AOB$$

The inequality comes from the fact that the sum of the two sides of a triangle is always greater than the hypotenuse.

115. We refer to Fig. 34-2 in the textbook. Consider the two light rays, r and r' , which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B , the distance between A and B (call it x) is

$$x = 2d_o \tan \theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 34-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{ey} + d_o$, and the small angle in this triangle is again θ . Thus,

$$\tan \theta = \frac{R}{d_{ey} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{ey} + d_o} = 2(100 \text{ mm}) \frac{2.5 \text{ mm}}{300 \text{ mm} + 100 \text{ mm}}$$

which yields $x = 1.67 \text{ mm}$. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4} \pi x^2 = \frac{\pi}{4} (1.67 \text{ mm})^2 = 2.2 \text{ mm}^2 .$$

116. For an object in front of a thin lens, the object distance p and the image distance i are related by $(1/p) + (1/i) = (1/f)$, where f is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance x between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for x :

$$x = \frac{p^2}{p - f} .$$

To find the minimum value of x , we set $dx/dp = 0$ and solve for p . Since

$$\frac{dx}{dp} = \frac{p(p - 2f)}{(p - f)^2} ,$$

the result is $p = 2f$. The minimum distance is

$$x_{\min} = \frac{p^2}{p - f} = \frac{(2f)^2}{2f - f} = 4f .$$

This is a minimum, rather than a maximum, since the image distance i becomes large without bound as the object approaches the focal point.

117. (a) If the object distance is x , then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D - x} = \frac{1}{f} .$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x , we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D - 4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D - 4f)}}{2} .$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1}.$$

Now $x_1 = \frac{1}{2}(D-d)$, where $d = \sqrt{D(D-f)}$, so

$$|m_1| = \frac{D - (D-d)/2}{(D-d)/2} = \frac{D+d}{D-d}.$$

Similarly, when the object is at x_2 , the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D-d}{D+d}.$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2.$$

118. (a) Our first step is to form the image from the first lens. With $p_1 = 10$ cm and $f_1 = -15$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = -6.0 \text{ cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.60$. This image serves the role of “object” for the second lens, with $p_2 = 12 + 6.0 = 18$ cm, and $f_2 = 12$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = 36 \text{ cm}.$$

(b) The corresponding magnification is $m_2 = -i_2/p_2 = -2.0$, which results in a net magnification of $m = m_1 m_2 = -1.2$. The height of the final image is (in absolute value) $(1.2)(1.0 \text{ cm}) = 1.2 \text{ cm}$.

(c) The fact that i_2 is positive means that the final image is real.

(d) The fact that m is negative means that the orientation of the final image is inverted with respect to the (original) object.

119. (a) Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1} \right)^{-1} = \left(\frac{1}{20 \text{ cm}} - \frac{1}{40 \text{ cm}} \right)^{-1} = 40 \text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40 \text{ cm} - 10 \text{ cm}) = -30 \text{ cm}$. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2} \right)^{-1} = \left(\frac{1}{-15 \text{ cm}} - \frac{1}{-30 \text{ cm}} \right)^{-1} = -30 \text{ cm}.$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2.

(b) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0 > 0$, so the image is not inverted.

(c) The image is virtual since $i_2 < 0$.

(d) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0$, so the image has the same size as the object.

120. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1} \right)^{-1} = \left(\frac{1}{10 \text{ cm}} - \frac{1}{20 \text{ cm}} \right)^{-1} = 20 \text{ cm}.$$

For the subsequent image formed by the second lens $p_2 = 30 \text{ cm} - 20 \text{ cm} = 10 \text{ cm}$, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2} \right)^{-1} = \left(\frac{1}{12.5 \text{ cm}} - \frac{1}{10 \text{ cm}} \right)^{-1} = -50 \text{ cm}.$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object.

(b) The magnification is

$$m = \left(\frac{i_1}{p_1} \right) \left(\frac{i_2}{p_2} \right) = \left(\frac{20 \text{ cm}}{20 \text{ cm}} \right) \left(\frac{-50 \text{ cm}}{10 \text{ cm}} \right) = -5.0,$$

which means that the final image is five times larger than the original object.

(c) The image is virtual since $i_2 < 0$.

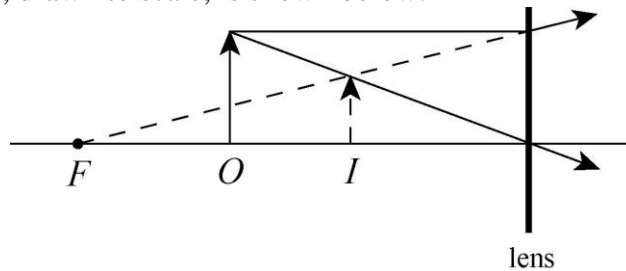
(d) The image is inverted since $m < 0$.

121. (a) We solve Eq. 34-9 for the image distance i : $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30$ cm. The object distance is $p = 20$ cm. Thus,

$$i = \frac{(20 \text{ cm})(-30 \text{ cm})}{(20 \text{ cm}) - (-30 \text{ cm})} = -12 \text{ cm}.$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object.

(b) The ray diagram, drawn to scale, is shown below.



122. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \left(\frac{1}{0.50 \text{ m}} - \frac{1}{1.0 \text{ m}} \right)^{-1} = 1.0 \text{ m}$$

to the right of the lens, or $2.0 \text{ m} - 1.0 \text{ m} = 1.0 \text{ m}$ in front of the mirror. The image formed by the mirror for this real image is then at 1.0 m to the right of the mirror, or $2.0 \text{ m} + 1.0 \text{ m} = 3.0 \text{ m}$ to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'} \right)^{-1} = \left(\frac{1}{0.50 \text{ m}} - \frac{1}{3.0 \text{ m}} \right)^{-1} = 0.60 \text{ m}$$

to the left of the lens (that is, 2.6 cm from the mirror).

(b) The lateral magnification is

$$m = \left(-\frac{i}{p} \right) \left(-\frac{i'}{p'} \right) = \left(-\frac{1.0 \text{ m}}{1.0 \text{ m}} \right) \left(-\frac{0.60 \text{ m}}{3.0 \text{ m}} \right) = +0.20.$$

(c) The final image is real since $i' > 0$.

(d) The image is to the left of the lens.

(e) It also has the same orientation as the object since $m > 0$. Therefore, the image is not inverted.

123. (a) We use Eq. 34-8 (and Fig. 34-12(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$.

$$\frac{1}{p} + \frac{1.5}{i} = \frac{1.5 - 1}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance i is negative with increasing magnitude as p increases from very small values to some value p_0 at which point $i \rightarrow -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0} = \frac{1.5 - 1}{r} \Rightarrow p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \leq 12$ cm.

124. (a) Suppose one end of the object is a distance p from the mirror and the other end is a distance $p + L$. The position i_1 of the image of the first end is given by

$$\frac{1}{p} + \frac{1}{i_1} = \frac{1}{f}$$

where f is the focal length of the mirror. Thus, $i_1 = \frac{fp}{p-f}$. The image of the other end is located at

$$i_2 = \frac{f(p+L)}{p+L-f},$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p-f} - \frac{f(p+L)}{p+L-f} = \frac{f^2 L}{(p-f)(p+L-f)}.$$

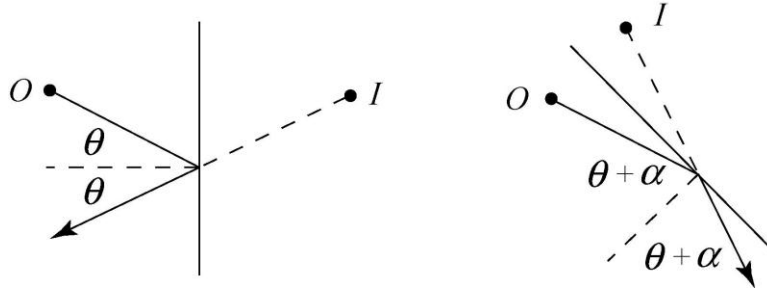
Since the object is short compared to $p - f$, we may neglect the L in the denominator and write

$$L' = L \left(\frac{f}{p-f} \right)^2.$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p - f)$, this can be written $m = -f/(p - f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p - f} \right)^2 = m^2.$$

125. Consider a single ray from the source to the mirror and let θ be the angle of incidence. The angle of reflection is also θ and the reflected ray makes an angle of 2θ with the incident ray.



Now we rotate the mirror through the angle α so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of 2α . If the mirror is rotated so the angle of incidence is decreased by α , then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through 2α . The diagrams below show the situation for $\alpha = 45^\circ$. The ray from the object to the mirror is the same in both cases and the reflected rays are 90° apart.

126. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{2}{p} = \frac{1}{f}$$

or

$$\frac{3}{30.0 \text{ cm}} = \frac{1}{f}.$$

Consequently, we find $f = (30.0 \text{ cm})/3 = 10.0 \text{ cm}$. The fact that $f > 0$ implies the mirror is concave.

127. (a) The mirror has focal length $f = 12.0 \text{ cm}$. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{1}{-3p} = \frac{1}{12 \text{ cm}}$$

or

$$\frac{2}{3p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find $p = 2(12 \text{ cm})/3 = 8.0 \text{ cm}$.

(b) With $m = -3$, we have $i = +3p$, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{1}{3p} = \frac{1}{12}$$

or

$$\frac{4}{3p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find $p = 4(12 \text{ cm})/3 = 16 \text{ cm}$.

(c) With $m = -1/3$, we have $i = p/3$. Thus, Eq. 34-4 leads to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow \frac{1}{p} + \frac{3}{p} = \frac{1}{12 \text{ cm}}$$

or

$$\frac{4}{p} = \frac{1}{12 \text{ cm}}.$$

Consequently, we find $p = 4(12 \text{ cm}) = 48 \text{ cm}$.

128. Since $0 < m < 1$, we conclude the lens is of the diverging type (so $f = -40 \text{ cm}$). Thus, substituting $i = -3p/10$ into Eq. 34-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f}.$$

Therefore, we find $p = 93.3 \text{ cm}$ and $i = -28.0 \text{ cm}$, or $|i| = 28.0 \text{ cm}$.

129. (a) We show the $\alpha = 0.500 \text{ rad}$, $r = 12 \text{ cm}$, $p = 20 \text{ cm}$ calculation in detail. The understood length unit is the centimeter:

The distance from the object to point x :

$$\begin{aligned} d &= p - r + x = 8 + x \\ y &= d \tan \alpha = 4.3704 + 0.54630x \end{aligned}$$

From the solution of $x^2 + y^2 = r^2$ we get $x = 8.1398$.

$$\beta = \tan^{-1}(y/x) = 0.8253 \text{ rad}$$

$$\gamma = 2\beta - \alpha = 1.151 \text{ rad}$$

From the solution of $\tan(\gamma) = y/(x + i - r)$ we get $i = 7.799$. The other results are shown without the intermediate steps:

For $\alpha = 0.100$ rad, we get $i = 8.544$ cm; for $\alpha = 0.0100$ rad, we get $i = 8.571$ cm. Eq. 34-3 and Eq. 34-4 (the mirror equation) yield $i = 8.571$ cm.

(b) Here the results are: ($\alpha = 0.500$ rad, $i = -13.56$ cm), ($\alpha = 0.100$ rad, $i = -12.05$ cm), ($\alpha = 0.0100$ rad, $i = -12.00$ cm). The mirror equation gives $i = -12.00$ cm.

130. (a) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm, or $|i| = 1.50$ cm.

(b) The focal length is negative.

(c) As shown in (a), the image is virtual.

131. First, we note that — *relative to the water* — the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with Problem 34-122). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40 \text{ mm}/1.1 = 36.4$ mm. This “apparent penny” serves as an “object” for the rays propagating upward through the 20 mm layer of water, where this “object” should be thought of as being $20 \text{ mm} + 36.4 \text{ mm} = 56.4$ mm from the top surface. Using the result of Problem 34-122 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be $56.4 \text{ mm}/1.33 = 42$ mm below the water surface.

132. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 34-4 to be applicable.

(a) With $p = 1.0$ m, the equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 0.15 m from the front surface, appearing to be *inside* the sphere.

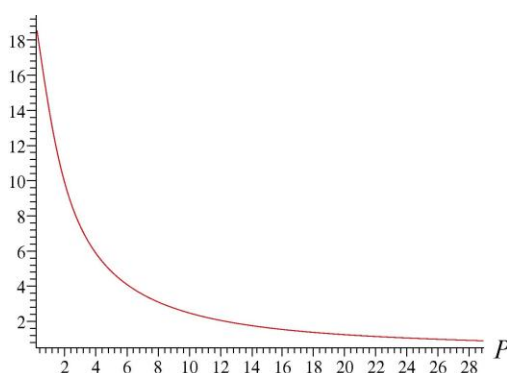
(b) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is $(0.15)(2.0 \text{ m}) = 0.30$ m.

(c) Since $m > 0$, the image is upright, or not inverted (NI).

133. (a) In this case $i < 0$ so $i = -|i|$, and Eq. 34-9 becomes $1/f = 1/p - 1/|i|$. We differentiate this with respect to time (t) to obtain

$$\frac{d|i|}{dt} = \left(\frac{|i|}{p}\right)^2 \frac{dp}{dt} .$$

As the object is moved toward the lens, p is decreasing, so $dp/dt < 0$. Consequently, the above expression shows that $d|i|/dt < 0$; that is, the image moves in from infinity. The angular magnification $m_\theta = \theta'/\theta$ also increases as the following graph shows (“read” the graph from left to right since we are considering decreasing p from near the focal length to near 0). To obtain this graph of m_θ , we chose $f = 30$ cm and $h = 2$ cm.



(b) When the image appears to be at the near point (that is, $|i| = P_n$), m_θ is at its maximum usable value. Since one generally takes P_n to be equal to 25 cm (this value, too, was used in making the above graph).

(c) In this case,

$$p = \frac{if}{i - f} = \frac{|i|f}{|i| + f} = \frac{P_n f}{P_n + f} .$$

If we use the small angle approximation, we have $\theta' \approx h'/|i|$ and $\theta \approx h/P_n$ (note: this approximation was not used in obtaining the graph, above). We therefore find

$$m_\theta \approx (h'/|i|)/(h/P_n)$$

which (using Eq. 34-7 relating the ratio of heights to the ratio of distances) becomes

$$m_\theta \approx \frac{h'}{h} \cdot \frac{P_n}{|i|} = \frac{|i|}{p} \cdot \frac{P_n}{|i|} = \frac{P_n}{p} = \frac{P_n}{P_n f / (P_n + f)} = \frac{P_n + f}{f}$$

which readily simplifies to the desired result.

(d) The linear magnification (Eq. 34-7) is given by $(h'/h) \approx m_\theta (|i|/P_n)$ (see the first in the chain of equalities, above). Once we set $|i| = P_n$ (see part (b)) then this shows the equality in the magnifications.

134. (a) The discussion in the textbook of the refracting telescope applies to the Newtonian arrangement if we replace the objective lens of Fig. 34-21 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 34-21, which is why Newton added the mirror M' in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive f device with another positive f device. Thus, the converging lens serving as the objective of Fig. 34-21 must be replaced (as Newton has done in Fig. 34-58) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 34-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{\text{ob}}/f_{\text{ey}}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{\text{stick}} \approx \frac{1 \text{ m}}{2000 \text{ m}} = 0.0005 \text{ rad.}$$

multiplying this by the mirror focal length gives $(16.8 \text{ m})(0.0005) = 8.4 \text{ mm}$ for the size of the image.

(c) With $r = 10 \text{ m}$, Eq. 34-3 gives $f_{\text{ob}} = 5 \text{ m}$. Plugging this into (the absolute value of) Eq. 34-15 leads to $f_{\text{ey}} = 5/200 = 2.5 \text{ cm}$.

135. (a) If we let $p \rightarrow \infty$ in Eq. 34-8, we get $i = n_2 r / (n_2 - n_1)$. If we set $n_1 = 1$ (for air) and restrict n_2 so that $1 < n_2 < 2$, then this suggests that $i > 2r$ (so this image does form before the rays strike the opposite side of the sphere). We can still consider this as a sort of “virtual” object for the second imaging event, where this “virtual” object distance is

$$2r - i = (n - 2) r / (n - 1),$$

where we have simplified the notation by writing $n_2 = n$. Putting this in for p in Eq. 34-8 and being careful with the sign convention for r in that equation, we arrive at the final image location: $i' = (0.5)(2 - n)r/(n - 1)$.

(b) The image is to the right of the right side of the sphere.

136. We set up an xyz coordinate system where the individual planes (xy , yz , xz) serve as the mirror surfaces. Suppose an incident ray of light A first strikes the mirror in the xy plane. If the unit vector denoting the direction of A is given by

$$\cos(\alpha)\hat{i} + \cos(\beta)\hat{j} + \cos(\gamma)\hat{k}$$

where α, β, γ are the angles A makes with the axes, then after reflection off the xy plane the unit vector becomes $\cos(\alpha)\hat{i} + \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$ (one way to rationalize this is to think of the reflection as causing the angle γ to become $\pi - \gamma$). Next suppose it strikes the mirror in the xz plane. The unit vector of the reflected ray is now $\cos(\alpha)\hat{i} - \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$. Finally as it reflects off the mirror in the yz plane α becomes $\pi - \alpha$, so the unit vector in the direction of the reflected ray is given by $-\cos(\alpha)\hat{i} - \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$, exactly reversed from A 's original direction. A further observation may be made: this argument would fail if the ray could strike any given surface twice and some consideration (perhaps an illustration) should convince the student that such an occurrence is not possible.

137. Since $m = -2$ and $p = 4.00$ cm, then $i = 8.00$ cm (and is real). Eq. 34-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

138. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f}.$$

Therefore, we find $p = -4f = -4(-40.0 \text{ cm}) = 160 \text{ cm}$.

139. (a) Our first step is to form the image from the first lens. With $p_1 = 3.00$ cm and $f_1 = +4.00$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(4.00 \text{ cm})(3.00 \text{ cm})}{3.00 \text{ cm} - 4.00 \text{ cm}} = -12.0 \text{ cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8.00 + 12.0 = 20.0$ cm, and $f_2 = -4.00$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = \frac{f_2 p_2}{p_2 - f_2} = \frac{(-4.00 \text{ cm})(20.0 \text{ cm})}{20.0 \text{ cm} - (-4.00 \text{ cm})} = -3.33 \text{ cm},$$

or $|i_2| = 3.33 \text{ cm}$.

(b) The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

(c) The image is virtual.

(d) With $m_2 = -i_2/p_2 = 1/6$, the net magnification is $m = m_1 m_2 = 2/3 > 0$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. Therefore, the image is not inverted.

140. The far point of the person is 50 cm = 0.50 m from the eye. The object distance is taken to be at infinity, and the corrected lens will allow the image to be formed at the near point. Thus,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} = \frac{1}{\infty} + \frac{1}{-0.50 \text{ m}}$$

and we find the focal length of the lens to $f = -0.50 \text{ m}$.

(b) Since $f < 0$, the lens is diverging.

(c) The power of the lens is $P = \frac{1}{f} = \frac{1}{-0.50 \text{ m}} = -2.0 \text{ diopters}$.

141. (a) Without the magnifier, $\theta = h/P_n$. With the magnifier, letting $p = p_n$ and $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25 \text{ cm}}{f}.$$

(b) Now $i = -|i| \rightarrow -\infty$, so $1/p + 1/i = 1/p = 1/f$ and

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25 \text{ cm}}{f}.$$

(c) For $f = 10 \text{ cm}$, we find the magnifications to be $m_\theta = 1 + \frac{25 \text{ cm}}{10 \text{ cm}} = 3.5$ for cases (a), and

$$m_\theta = \frac{25 \text{ cm}}{10 \text{ cm}} = 2.5 \text{ for case (b).}$$