Chapter 33

1. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times10^8\text{ m/s})(0.0100\times10^{-9}\text{ m})}{(632.8\times10^{-9}\text{ m})^2} = 7.49\times10^9\text{ Hz}.$$

2. (a) The frequency of the radiation is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{(1.0\times10^5)(6.4\times10^6\text{ m})} = 4.7\times10^{-3}\text{ Hz}.$$

(b) The period of the radiation is

$$T = \frac{1}{f} = \frac{1}{4.7\times10^{-3}\text{ Hz}} = 212\text{ s} = 3\text{ min }32\text{ s}.$$

3. (a) From Fig. 33-2 we find the smaller wavelength in question to be about 515 nm.

(b) Similarly, the larger wavelength is approximately 610 nm.

(c) From Fig. 33-2 the wavelength at which the eye is most sensitive is about 555 nm.

(d) Using the result in (c), we have

$$f = \frac{c}{\lambda} = \frac{3.00\times10^8\text{ m/s}}{555\text{ nm}} = 5.41\times10^{14}\text{ Hz}.$$

(e) The period is $T = 1/f = (5.41 \times 10^{14}\text{ Hz})^{-1} = 1.85 \times 10^{-15}$ s.

4. In air, light travels at roughly $c = 3.0 \times 10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = (3.0\times10^8\text{ m/s})(1.0\times10^{-9}\text{ s}) = 0.30\text{ m}.$$

5. **THINK** The frequency of oscillation of the current in the LC circuit of the generator is $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. This frequency is the same as the frequency of an electromagnetic wave.

EXPRESS If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. Thus,

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

ANALYZE The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{(550\times10^{-9}\text{ m})^2}{4\pi^2(17\times10^{-12}\text{ F})(2.998\times10^8\text{ m/s})^2} = 5.00\times10^{-21}\text{ H}.$$

This is exceedingly small.

LEARN The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^8\text{ m/s}}{550\times10^{-9}\text{ m}} = 5.45\times10^{14}\text{ Hz}.$$

The EM wave is in the visible spectrum.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 2\pi\left(2.998\times10^8\text{ m/s}\right)\sqrt{\left(0.253\times10^{-6}\text{ H}\right)\left(25.0\times10^{-12}\text{ F}\right)} = 4.74\text{ m}.$$

7. The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{(3.0\times10^8\text{ m/s})(1.0\times10^{-4}\text{ T})^2}{2(1.26\times10^{-6}\text{ H/m})^2} = 1.2\times10^6\text{ W/m}^2.$$

8. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0\times10^6\text{ W}}{4\pi\,(4.3\text{ly})(9.46\times10^{15}\text{ m/ly})^{\,2}} = 4.8\times10^{-29}\text{ W/m}^2.$$

9. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\Delta t = (100\times10^{12}\text{ W})(1.0\times10^{-9}\text{ s}) = 1.0\times10^5\text{ J}.$$

10. (a) Setting $v = c$ in the wave relation $kv = \omega = 2\pi f$, we find $f = 1.91\times10^8$ Hz.

(b) $E_{\text{rms}} = E_m/\sqrt{2} = B_m/c\sqrt{2} = 18.2$ V/m.

(c) $I = (E_{\text{rms}})^2/c\mu_o = 0.878$ W/m^2.

11. (a) The amplitude of the magnetic field is

$$B_m = \frac{E_m}{c} = \frac{2.0\text{V/m}}{2.998\times10^8\,\text{m/s}} = 6.67\times10^{-9}\,\text{T} \approx 6.7\times10^{-9}\,\text{T}.$$

(b) Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. So, the oscillation of the magnetic field is parallel to the y axis.

(c) The direction ($+x$) of the electromagnetic wave propagation is determined by $\vec{E}\times\vec{B}$. If the electric field points in $+z$, then the magnetic field must point in the $-y$ direction.

With SI units understood, we may write

$$B_y = B_m \cos\left[\pi\times10^{15}\left(t-\frac{x}{c}\right)\right] = \frac{2.0\cos\left[10^{15}\pi\left(t-x/c\right)\right]}{3.0\times10^8}$$
$$= \left(6.7\times10^{-9}\right)\cos\left[10^{15}\pi\left(t-\frac{x}{c}\right)\right]$$

12. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.67\times10^{-8}\,\text{T}.$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{\left(5.00\,\text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.998\times10^8\,\text{m/s}\right)} = 3.31\times10^{-2}\,\text{W/m}^2.$$

13. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(1.40\times10^3\,\text{W/m}^2\right)\left(2.998\times10^8\,\text{m/s}\right)}$$
$$= 1.03\times10^3\,\text{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03\times10^4\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 3.43\times10^{-6}\,\text{T}.$$

14. From the equation immediately preceding Eq. 33-12, we see that the maximum value of $\partial B/\partial t$ is ωB_m. We can relate B_m to the intensity:

$$B_m = \frac{E_m}{c} = \frac{\sqrt{2c\mu_0 I}}{c},$$

and relate the intensity to the power P (and distance r) using Eq. 33-27. Finally, we relate ω to wavelength λ using $\omega = kc = 2\pi c/\lambda$. Putting all this together, we obtain

$$\left(\frac{\partial B}{\partial t}\right)_{\max} = \sqrt{\frac{2\mu_0 P}{4\pi c}}\,\frac{2\pi c}{\lambda r} = 3.44\times10^6\ \text{T/s}.$$

15. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2\left(4\pi\times10^{-7}\ \text{H/m}\right)\left(2.998\times10^8\ \text{m/s}\right)\left(10\times10^{-6}\ \text{W/m}^2\right)}$$
$$= 8.7\times10^{-2}\ \text{V/m}.$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7\times10^{-2}\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 2.9\times10^{-10}\ \text{T}.$$

(c) At a distance r from the transmitter, the intensity is $I = P/2\pi r^2$, where P is the power of the transmitter over the hemisphere having a surface area $2\pi r^2$. Thus

$$P = 2\pi r^2 I = 2\pi\left(10\times10^3\ \text{m}\right)^2\left(10\times10^{-6}\ \text{W/m}^2\right) = 6.3\times10^3\ \text{W}.$$

16. (a) The power received is

$$P_r = \left(1.0\times10^{-12}\ \text{W}\right)\frac{\pi(300\ \text{m})^2/4}{4\pi\left(6.37\times10^6\ \text{m}\right)^2} = 1.4\times10^{-22}\ \text{W}.$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi\left[\left(2.2\times10^4\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^2\left[\frac{1.0\times10^{-12}\ \text{W}}{4\pi\left(6.37\times10^6\ \text{m}\right)^2}\right] = 1.1\times10^{15}\ \text{W}.$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 6.7\times10^{-9}\,\text{T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.998\times10^8\,\text{m/s})} = 5.3\times10^{-3}\,\text{W/m}^2.$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\text{avg}} = 4\pi(10\,\text{m})^2\left(5.3\times10^{-3}\,\text{W/m}^2\right) = 6.7\,\text{W}.$$

18. Equation 33-27 suggests that the slope in an intensity versus inverse-square-distance graph (I plotted versus r^{-2}) is $P/4\pi$. We estimate the slope to be about 20 (in SI units), which means the power is $P = 4\pi(30) \approx 2.5\times10^2$ W.

19. **THINK** The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity.

EXPRESS The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation.

ANALYZE Thus, the radiation pressure is

$$p_r = \frac{2I}{c} = \frac{2P}{Ac} = \frac{2\left(1.5\times10^9\,\text{W}\right)}{\left(1.00\times10^{-6}\,\text{m}^2\right)\left(2.998\times10^8\,\text{m/s}\right)} = 1.0\times10^7\,\text{Pa}.$$

LEARN In the case of total absorption, the radiation pressure would be $p_r = I/c$, a factor of 2 smaller than the case of total reflection.

20. (a) The radiation pressure produces a force equal to

$$F_r = p_r\left(\pi R_e^2\right) = \left(\frac{I}{c}\right)\left(\pi R_e^2\right) = \frac{\pi\left(1.4\times10^3\,\text{W/m}^2\right)\left(6.37\times10^6\,\text{m}\right)^2}{2.998\times10^8\,\text{m/s}} = 6.0\times10^8\,\text{N}.$$

(b) The gravitational pull of the Sun on the Earth is

$$\begin{aligned}F_{\text{grav}} &= \frac{GM_sM_e}{d_{es}^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\times10^{30}\,\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(1.5\times10^{11}\,\text{m}\right)^2}\\ &= 3.6\times10^{22}\,\text{N},\end{aligned}$$

which is much greater than F_r.

21. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\ \text{m})^2\,(2.998\times10^8\ \text{m/s})} = 5.9\times10^{-8}\ \text{Pa}.$$

22. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998\times10^8\ \text{m/s}} = 3.3\times10^{-8}\ \text{Pa}.$$

23. (a) The upward force supplied by radiation pressure in this case (Eq. 33-32) must be equal to the magnitude of the pull of gravity (mg). For a sphere, the "projected" area (which is a factor in Eq. 33-32) is that of a circle $A = \pi r^2$ (not the entire surface area of the sphere) and the volume (needed because the mass is given by the density multiplied by the volume: $m = \rho V$) is $V = 4\pi r^3/3$. Finally, the intensity is related to the power P of the light source and another area factor $4\pi R^2$, given by Eq. 33-27. In this way, with $\rho = 1.9\times10^4\ \text{kg/m}^3$, equating the forces leads to

$$P = 4\pi R^2 c\left(\rho\frac{4\pi r^3 g}{3}\right)\frac{1}{\pi r^2} = 4.68\times10^{11}\ \text{W}.$$

(b) Any chance disturbance could move the sphere from being directly above the source, and then the two force vectors would no longer be along the same axis.

24. We require $F_{\text{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c},$$

and solve for the area A:

$$A = \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1500\ \text{kg})(1.99\times10^{30}\ \text{kg})(2.998\times10^8\ \text{m/s})}{2(1.40\times10^3\ \text{W/m}^2)(1.50\times10^{11}\ \text{m})^2}$$
$$= 9.5\times10^5\ \text{m}^2 = 0.95\ \text{km}^2.$$

25. **THINK** In this problem we relate radiation pressure to energy density in the incident beam.

EXPRESS Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1 - f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c},$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c}.$$

ANALYZE To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell / c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc.$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is

$$I = I_0 + fI_0 = (1+f)I_0,$$

where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c},$$

the same as radiation pressure.

LEARN In the case of total reflection, $f = 1$, and $p_{\text{total}} = p_r = 2I_0/c$. On the other hand, the energy density is $u = I/c = 2I_0/c$, which is the same as p_{total}. Similarly, for total absorption, $f = 0$, $p_{\text{total}} = p_a = I_0/c$, and since $I = I_0$, we have $u = I/c = I_0/c$, which again is the same as p_{total}.

26. The mass of the cylinder is $m = \rho(\pi D^2/4)H$, where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g\rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$\begin{aligned} H &= \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho} \\ &= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)} \\ &= 4.91\times10^{-7}\,\text{m}. \end{aligned}$$

27. **THINK** Electromagnetic waves travel at speed of light, and carry both linear momentum and energy.

EXPRESS The speed of the electromagnetic wave is $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave. The angular frequency is $\omega = 2\pi f$, and the angular wave number is $k = 2\pi/\lambda$. The magnetic field amplitude is related to the electric field amplitude by $B_m = E_m/c$. The intensity of the wave is given by Eq. 33-26:

$$I = \frac{1}{c\mu_0}E_{\text{rms}}^2 = \frac{1}{2c\mu_0}E_m^2.$$

ANALYZE (a) With λ = 3.0 m, the frequency of the wave is

$$f = \frac{c}{\lambda} = \frac{2.998\times10^8\ \text{m/s}}{3.0\ \text{m}} = 1.0\times10^8\ \text{Hz}.$$

(b) From the value of f obtained in (a), we find the angular frequency to be

$$\omega = 2\pi f = 2\pi(1.0\times10^8\ \text{Hz}) = 6.3\times10^8\ \text{rad/s}.$$

(c) The corresponding angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\ \text{m}} = 2.1\ \text{rad/m}.$$

(d) With E_m = 300 V/m, the magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.0\times10^{-6}\,\text{T}.$$

(e) Since $\vec{E}$ is in the positive y direction, $\vec{B}$ must be in the positive z direction so that their cross product $\vec{E}\times\vec{B}$ points in the positive x direction (the direction of propagation).

(f) The intensity of the wave is

$$I=\frac{E_m^2}{2\mu_0 c}=\frac{(300\,\mathrm{V/m})^2}{2(4\pi\times10^{-7}\,\mathrm{H/m})(2.998\times10^8\,\mathrm{m/s})}=119\,\mathrm{W/m^2}\approx1.2\times10^2\,\mathrm{W/m^2}.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt}=\frac{IA}{c}=\frac{(119\ \mathrm{W/m^2})(2.0\ \mathrm{m^2})}{2.998\times10^8\ \mathrm{m/s}}=8.0\times10^{-7}\ \mathrm{N}.$$

(h) The radiation pressure is

$$p_r=\frac{dp/dt}{A}=\frac{8.0\times10^{-7}\ \mathrm{N}}{2.0\ \mathrm{m^2}}=4.0\times10^{-7}\ \mathrm{Pa}.$$

LEARN The energy density is given by

$$u=\frac{I}{c}=\frac{119\ \mathrm{W/m^2}}{2.998\times10^8\ \mathrm{m/s}}=4.0\times10^{-7}\ \mathrm{J/m^3}$$

which is the same as the radiation pressure p_r.

28. (a) Assuming complete absorption, the radiation pressure is

$$p_r=\frac{I}{c}=\frac{1.4\times10^3\ \mathrm{W/m^2}}{3.0\times10^8\ \mathrm{m/s}}=4.7\times10^{-6}\ \mathrm{N/m^2}.$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0}=\frac{4.7\times10^{-6}\ \mathrm{N/m^2}}{1.0\times10^5\ \mathrm{N/m^2}}=4.7\times10^{-11}.$$

29. **THINK** The laser beam carries both energy and momentum. The total momentum of the spaceship and light is conserved.

EXPRESS If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. By momentum conservation, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$.

ANALYZE We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10\times10^3 \text{ W})(86400 \text{ s})}{(1.5\times10^3 \text{ kg})(2.998\times10^8 \text{ m/s})} = 1.9\times10^{-3} \text{ m/s}.$$

LEARN As expected, the speed of the spaceship is proportional to the power of the laser beam.

30. (a) We note that the cross-section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00\times10^{-3}\text{ W}}{\pi\left[(2.00)\left(633\times10^{-9}\text{ m}\right)\right]^2/4} = 3.97\times10^9 \text{ W/m}^2.$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97\times10^9 \text{ W/m}^2}{2.998\times10^8 \text{ m/s}} = 13.2 \text{ Pa}.$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right)p_r = \left(\frac{P}{I}\right)p_r = \frac{\left(5.00\times10^{-3}\text{ W}\right)\left(13.2\text{ Pa}\right)}{3.97\times10^9\text{ W/m}^2} = 1.67\times10^{-11}\text{ N}.$$

(d) The acceleration of the sphere is

$$a = \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67\times10^{-11}\text{ N})}{\pi(5.00\times10^3 \text{ kg/m}^3)[(2.00)(633\times10^{-9}\text{ m})]^3}$$
$$= 3.14\times10^3 \text{ m/s}^2.$$

31. We shall assume that the Sun is far enough from the particle to act as an isotropic point source of light.

(a) The forces that act on the dust particle are the radially outward radiation force $\vec{F}_r$ and the radially inward (toward the Sun) gravitational force $\vec{F}_g$. Using Eqs. 33-32 and 33-27, the radiation force can be written as

$$F_r = \frac{IA}{c} = \frac{P_S}{4\pi r^2}\frac{\pi R^2}{c} = \frac{P_S R^2}{4r^2 c},$$

where R is the radius of the particle, and $A = \pi R^2$ is the cross-sectional area. On the other hand, the gravitational force on the particle is given by Newton's law of gravitation (Eq. 13-1):

$$F_g = \frac{GM_S m}{r^2} = \frac{GM_S \rho(4\pi R^3/3)}{r^2} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

where $m = \rho(4\pi R^3/3)$ is the mass of the particle. When the two forces balance, the particle travels in a straight path. The condition that $F_r = F_g$ implies

$$\frac{P_S R^2}{4r^2 c} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

which can be solved to give

$$R = \frac{3P_S}{16\pi c\rho GM_S} = \frac{3(3.9\times10^{26}\ \text{W})}{16\pi(3\times10^8\ \text{m/s})(3.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99\times10^{30}\ \text{kg})}$$
$$= 1.7\times10^{-7}\ \text{m}.$$

(b) Since F_g varies with R^3 and F_r varies with R^2, if the radius R is larger, then $F_g > F_r$, and the path will be curved toward the Sun (like path 3).

32. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1+\theta_2)\cos^2(\theta_2+\theta_3) = \frac{1}{2}\cos^2(50°+50°)\cos^2(50°+50°)$$
$$= 4.5\times10^{-4}.$$

Thus, 0.045% of the light's initial intensity is transmitted.

33. **THINK** Unpolarized light becomes polarized when it is sent through a polarizing sheet. In this problem, three polarizing sheets are involved, we work through the system sheet by sheet, applying either the one-half rule or the cosine-squared rule.

EXPRESS Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is $\theta_1 = 40°$ *counterclockwise* from the y axis in the diagram. For the second sheet (and the third one as well), we apply the cosine-squared rule:

$$I_2 = I_1 \cos^2\theta_2'$$

where θ_2' is the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet.

ANALYZE The polarizing direction of the second sheet is θ_2 = 20° *clockwise* from the *y* axis, so $\theta_2' = 40° + 20° = 60°$. The transmitted intensity is

$$I_2 = I_1 \cos^2 60° = \frac{1}{2} I_0 \cos^2 60°,$$

and the direction of polarization of the transmitted light is 20° clockwise from the *y* axis. The polarizing direction of the third sheet is θ_3 = 40° *counterclockwise* from the *y* axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60° = \frac{1}{2} I_0 \cos^4 60° = 3.1\times10^{-2} I_0.$$

Thus, 3.1% of the light's initial intensity is transmitted.

LEARN When two polarizing sheets are crossed ($\theta = 90°$), no light passes through and the transmitted intensity is zero.

34. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2} I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2(90°–70°) = \frac{1}{2}(43\ \mathrm{W/m^2})(\cos^2 20°) = 19\ \mathrm{W/m^2}.$$

35. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is θ_1 = 70°. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0 \cos^2\theta_1 = (43\ \mathrm{W/m^2})\cos^2 70° = 5.03\ \mathrm{W/m^2}.$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of θ_2 = 20° with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1 \cos^2\theta_2 = (5.03\ \mathrm{W/m^2})\cos^2 20° = 4.4\ \mathrm{W/m^2}.$$

36. (a) The fraction of light that is transmitted by the glasses is

$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84.$$

37. **THINK** A polarizing sheet can change the direction of polarization of the incident beam since it allows only the component that is parallel to its polarization direction to pass.

EXPRESS The 90° rotation of the polarization direction cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

ANALYZE (a) The 90° rotation of the polarization direction can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is

$$I = I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta,$$

where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated $90°/n$ in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is

$$I = I_0 \cos^{2n}(90°/n).$$

We want the smallest integer value of n for which this is greater than $0.60I_0$. We start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

LEARN The intensities associated with $n = 1$ to 5 are:

$$I_{n=1} = I_0 \cos^2(90°) = 0$$
$$I_{n=2} = I_0 \cos^4(45°) = I_0 / 4 = 0.25 I_0$$
$$I_{n=3} = I_0 \cos^6(30°) = 0.422 I_0$$
$$I_{n=4} = I_0 \cos^8(22.5°) = 0.531 I_0$$
$$I_{n=5} = I_0 \cos^{10}(18°) = 0.605 I_0$$

38. We note the points at which the curve is zero (θ_2 = 0° and 90°) in Fig. 33-43. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 0°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 90°. Without loss of generality, we choose θ_1 = 0°, θ_3 = 90°. Now, when θ_2 = 30°, it will be $\Delta\theta$ = 30° relative to sheet 1 and $\Delta\theta'$ = 60° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 9.4\% .$$

39. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is I = 5.0 mW/m^2. The intensity and the electric field amplitude are related by $I = E_m^2 / 2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\ \text{H/m})(3.00\times10^{8}\ \text{m/s})(5.0\times10^{-3}\ \text{W/m}^2)}$$
$$= 1.9\ \text{V/m}.$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0\times10^{-3}\ \text{W/m}^2}{3.00\times10^{8}\ \text{m/s}} = 1.7\times10^{-11}\ \text{Pa}.$$

40. We note the points at which the curve is zero (θ_2 = 60° and 140°) in Fig. 33-44. We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 60°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 140°. Without loss of generality, we choose θ_1 = 150°, θ_3 = 50°. Now, when θ_2 = 90°, it will be $|\Delta\theta|$ = 60° relative to sheet 1 and $|\Delta\theta'|$ = 40° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 7.3\% .$$

41. As the polarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $I_0 \cos^2\theta$. After passing through the second polarizer, which makes a 90° angle with the first filter, the intensity is

$$I = (I_0 \cos^2\theta)\sin^2\theta = I_0/10$$

which implies $\sin^2\theta\cos^2\theta = 1/10$, or $\sin\theta\cos\theta = \sin 2\theta/2 = 1/\sqrt{10}$. This leads to $\theta = 70°$ or 20°.

42. We examine the point where the graph reaches zero: $\theta_2 = 160°$. Since the polarizers must be "crossed" for the intensity to vanish, then $\theta_1 = 160° - 90° = 70°$. Now we consider the case $\theta_2 = 90°$ (which is hard to judge from the graph). Since θ_1 is still equal to 70º, then the angle between the polarizers is now $\Delta\theta = 20°$. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}\cos^2(\Delta\theta) = 0.442 \approx 44\%.$$

43. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0\cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0\cos^2\theta + \frac{1}{2}(1-f)I_0.$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\min} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\max} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0.$$

The ratio of $I_{\max}$ to $I_{\min}$ is

$$\frac{I_{\max}}{I_{\min}} = \frac{1+f}{1-f}.$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

44. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta_2\cos^2(90° - \theta_2).$$

Using trig identities, we rewrite this as $\frac{I}{I_0} = \frac{1}{8}\sin^2(2\theta_2)$.

(a) Therefore we find $\theta_2 = \frac{1}{2}\sin^{-1}\sqrt{0.40} = 19.6°$.

(b) Since the first expression we wrote is symmetric under the exchange $\theta_2 \leftrightarrow 90° - \theta_2$, we see that the angle's complement, 70.4°, is also a solution.

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by $\tan\theta_1 = L/D$, where D is the height of the tank and L is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{1.10 \text{ m}}{0.850 \text{ m}}\right) = 52.31°.$$

The law of refraction yields

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26,$$

where the index of refraction of air was taken to be unity.

46. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-47(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the topmost point of each curve. With θ_2 = 90º and θ_1 = ½(90º), and with n_2 = 1.33 (Table 33-1), we find n_1 = 1.9 from Snell's law.

(d) Similarly, with θ_2 = 90º and θ_1 = ¾(90º), we obtain n_1 = 1.4.

47. The law of refraction states

$$n_1\sin\theta_1 = n_2\sin\theta_2.$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48.$$

48. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-48(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the right end-point of each curve. With θ_1 = 90º and θ_2 = ¾(90º), and with n_1 = 1.33 (Table 33-1) we find, from Snell's law, n_2 = 1.4 for material 1.

(d) Similarly, with θ_1 = 90º and θ_2 = ½(90º), we obtain n_2 = 1.9.

49. The angle of incidence for the light ray on mirror B is 90° – θ. So the outgoing ray r' makes an angle 90° – (90° – θ) = θ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore 180°.

50. (a) From $n_1\sin\theta_1 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_3\sin\theta_3$, we find $n_1\sin\theta_1 = n_3\sin\theta_3$. This has a simple implication: that $\theta_1 = \theta_3$ when $n_1 = n_3$. Since we are given θ_1 = 40º in Fig. 33-50(a), then we look for a point in Fig. 33-50(b) where θ_3 = 40º. This seems to occur at n_3 = 1.6, so we infer that n_1 = 1.6.

(b) Our first step in our solution to part (a) shows that information concerning n_2 disappears (cancels) in the manipulation. Thus, we cannot tell; we need more information.

(c) From 1.6sin70° = 2.4sinθ_3 we obtain θ_3 = 39°.

51. (a) Approximating n = 1 for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \quad \Rightarrow \quad 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 33-1, we obtain 56.8°.

(b) Equation 33-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = \sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3°.$$

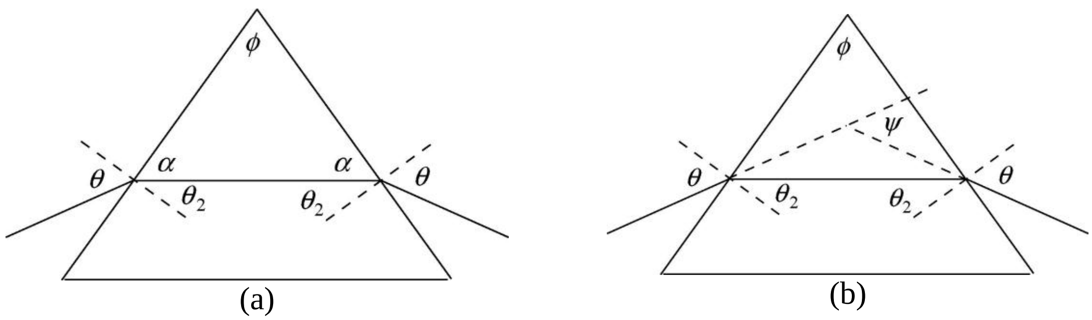
52. (a) A simple implication of Snell's law is that $\theta_2 = \theta_1$ when $n_1 = n_2$. Since the angle of incidence is shown in Fig. 33-52(a) to be 30º, we look for a point in Fig. 33-52(b) where θ_2 = 30º. This seems to occur when n_2 = 1.7. By inference, then, n_1 = 1.7.

(b) From 1.7sin(60º) = 2.4sin(θ_2) we get θ_2 = 38°.

53. **THINK** The angle with which the light beam emerges from the triangular prism depends on the index of refraction of the prism.

EXPRESS Consider diagram (a) shown next. The incident angle is θ and the angle of refraction is θ_2. Since $\theta_2 + \alpha = 90°$ and $\phi + 2\alpha = 180°$, we have

$$\theta_2 = 90° - \alpha = 90° - \frac{1}{2}\left(180° - \phi\right) = \frac{\phi}{2}.$$



ANALYZE Next, examine diagram (b) and consider the triangle formed by the two normals and the ray in the interior. One can show that ψ is given by

$$\psi = 2(\theta - \theta_2) .$$

Upon substituting $\phi/2$ for θ_2, we obtain $\psi = 2(\theta - \phi/2)$ which yields $\theta = (\phi + \psi)/2$. Thus, using the law of refraction, we find the index of refraction of the prism to be

$$n = \frac{\sin\theta}{\sin\theta_2} = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi}.$$

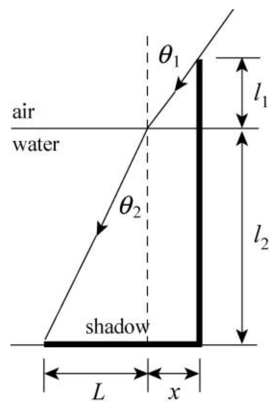
LEARN The angle ψ is called the deviation angle. Physically, it represents the total angle through which the beam has turned while passing through the prism. This angle is minimum when the beam passes through the prism "symmetrically," as it does in this case. Knowing the value of ϕ and ψ allows us to determine the value of n for the prism material.

54. (a) Snell's law gives n_{air} sin(50º) = n_{2b} sin θ_{2b} and n_{air} sin(50º) = n_{2r} sin θ_{2r} where we use subscripts b and r for the blue and red light rays. Using the common approximation for air's index (n_{air} = 1.0) we find the two angles of refraction to be 30.176° and 30.507°. Therefore, $\Delta\theta$ = 0.33°.

(b) Both of the refracted rays emerge from the other side with the same angle (50°) with which they were incident on the first side (generally speaking, light comes into a block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no dispersion in this case.

55. **THINK** Light is refracted at the air–water interface. To calculate the length of the shadow of the pole, we first calculate the angle of refraction using the Snell's law.

EXPRESS Consider a ray that grazes the top of the pole, as shown in the diagram below.



Here θ_1 = 90° – θ = 90° –55° = 35°, $\ell_1 = 0.50$ m, and $\ell_2 = 1.50$ m. The length of the shadow is $d = x + L$.

ANALYZE The distance x is given by

$$x = \ell_1 \tan\theta_1 = (0.50 \text{ m}) \tan 35° = 0.35 \text{ m}.$$

According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 33-1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55°.$$

L is given by

$$L = \ell_2 \tan\theta_2 = (1.50 \text{ m}) \tan 25.55° = 0.72 \text{ m}.$$

Thus, the length of the shadow is d = 0.35 m + 0.72 m = 1.07 m.

LEARN If the pole were empty with no water, then $\theta_1 = \theta_2$ and the length of the shadow would be

$$d' = \ell_1 \tan\theta_1 + \ell_2 \tan\theta_1 = (\ell_1 + \ell_2) \tan\theta_1$$

by simple geometric consideration.

56. (a) We use subscripts *b* and *r* for the blue and red light rays. Snell's law gives

$$\theta_{2b} = \sin^{-1}\left(\frac{1}{1.343}\sin(70°)\right) = 44.403°$$
$$\theta_{2r} = \sin^{-1}\left(\frac{1}{1.331}\sin(70°)\right) = 44.911°$$

for the refraction angles at the first surface (where the normal axis is vertical). These rays strike the second surface (where *A* is) at complementary angles to those just calculated (since the normal axis is horizontal for the second surface). Taking this into consideration, we again use Snell's law to calculate the second refractions (with which the light re-enters the air):

$$\theta_{3b} = \sin^{-1}[1.343\sin(90° - \theta_{2b})] = 73.636°$$
$$\theta_{3r} = \sin^{-1}[1.331\sin(90° - \theta_{2r})] = 70.497°$$

which differ by 3.1° (thus giving a rainbow of angular width 3.1°).

(b) Both of the refracted rays emerge from the bottom side with the same angle (70°) with which they were incident on the topside (the occurrence of an intermediate reflection [from side 2] does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no rainbow in this case.

57. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. Thus, the diameter *D* of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182\text{ cm}.$$

58. The critical angle is $\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34°.$

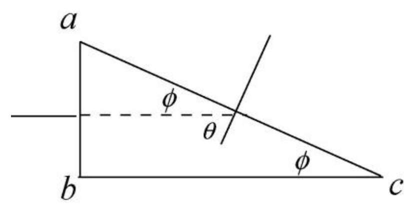
59. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

In our case, the incident light ray is perpendicular to the face *ab*. Thus, no refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $\theta = 90° - \phi$, as shown in the figure below.



ANALYZE (a) For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

LEARN Total internal reflection cannot occur if the incident light is in the medium with lower index of refraction. With $\theta_c = \sin^{-1}(n_2/n_1)$, we see that the larger the ratio n_2/n_1, the larger the value of θ_c.

60. (a) The condition (in Eq. 33-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Yes. Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

(c) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 51.1°$.

(d) No. Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

61. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 26.8°$.

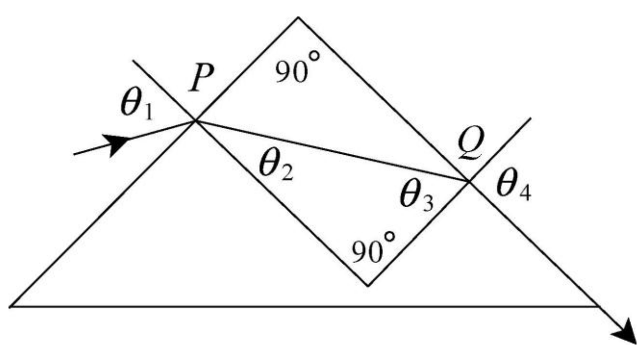
(b) Increasing θ leads to a decrease of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle; therefore, there will be some transmission of light into material 3.

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan\theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin\theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00\text{ m})(\tan 48.75°) = 4.56\text{ m}.$$

(b) The diameter *d* of the circle will increase if the fish descends (increasing *h*).

63. (a) A ray diagram is shown below.



Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is θ_4 = 90°. The law of refraction, applied to the second surface, yields $n \sin \theta_3 = \sin \theta_4 = 1$. As shown in the diagram, the normals to the surfaces at *P* and *Q* are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and

$$\sin\theta_3 = \sin(90°-\theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}.$$

According to the law of refraction, applied at *Q*, $n\sqrt{1-\sin^2\theta_2} = 1.$ The law of refraction, applied to point *P*, yields $\sin \theta_1 = n \sin \theta_2$, so $\sin \theta_2 = (\sin \theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1.$$

Squaring both sides and solving for *n*, we get

$$n = \sqrt{1+\sin^2\theta_1}.$$

(b) The greatest possible value of $\sin^2 \theta_1$ is 1, so the greatest possible value of *n* is $n_{max} = \sqrt{2} = 1.41.$

(c) For a given value of *n*, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 (= 90° – θ_2). That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at *Q*.

64. (a) We refer to the entry point for the original incident ray as point *A* (which we take to be on the left side of the prism, as in Fig. 33-53), the prism vertex as point *B*, and the point where the interior ray strikes the right surface of the prism as point *C*. The angle between line *AB* and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line *BC* and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point *C*, yields $n \sin \theta_3 = 1$, so

$$\sin\theta_3 = 1/n = 1/1.60 = 0.625 \Rightarrow \theta_3 = 38.68°.$$

The interior angles of the triangle *ABC* must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point *A*, yields

$$\sin\theta_1 = n\sin\theta_2 = 1.60\sin 21.32° = 0.5817.$$

Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point *C*. Since the angle of refraction there is the same as the angle of incidence at *A*, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin\left(60° - \theta_2\right) \Rightarrow \sin\theta_1 = n\sin 60°\cos\theta_2 - n\cos 60°\sin\theta_2$$

where the trigonometric identity

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

is used. Next, we apply the law of refraction to point *A*:

$$\sin\theta_1 = n\sin\theta_2 \Rightarrow \sin\theta_2 = \left(1/n\right)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(1/n^2\right)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60°\sqrt{1-\left(1/n\right)^2\sin^2\theta_1} - \cos 60°\sin\theta_1$$

or

$$\left(1+\cos 60°\right)\sin\theta_1 = \sin 60°\sqrt{n^2 - \sin^2\theta_1}.$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{\left(1+\cos 60°\right)^2 + \sin^2 60°}} = 0.80$$

and $\theta_1 = 53.1°$.

65. When examining Fig. 33-61, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta$$

assuming $n_{\text{air}} = 1$. The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp}, and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1-\sin^2\theta'}.$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1-\sin^2\theta'} = \sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$. With n_1 = 1.58 and n_2 = 1.53, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2°.$$

66. (a) We note that the upper-right corner is at an angle (measured from the point where the light enters, and measured relative to a normal axis established at that point the normal at that point would be horizontal in Fig. 33-62) is at $\tan^{-1}(2/3) = 33.7°$. The angle of refraction is given by

$$n_{\text{air}} \sin 40° = 1.56 \sin \theta_2$$

which yields θ_2 = 24.33° if we use the common approximation n_{air} = 1.0, and yields θ_2 = 24.34° if we use the more accurate value for n_{air} found in Table 33-1. The value is less than 33.7°, which means that the light goes to side 3.

(b) The ray strikes a point on side 3, which is 0.643 cm below that upper-right corner, and then (using the fact that the angle is symmetrical upon reflection) strikes the top surface (side 2) at a point 1.42 cm to the left of that corner. Since 1.42 cm is certainly less than 3 cm we have a self-consistency check to the effect that the ray does indeed strike side 2 as its second reflection (if we had gotten 3.42 cm instead of 1.42 cm, then the situation would be quite different).

(c) The normal axes for sides 1 and 3 are both horizontal, so the angle of incidence (in the plastic) at side 3 is the same as the angle of refraction was at side 1. Thus,

$$1.56 \sin 24.3° = n_{\text{air}} \sin \theta_{\text{air}} \quad \Rightarrow \quad \theta_{\text{air}} = 40° .$$

(d) It strikes the top surface (side 2) at an angle (measured from the normal axis there, which in this case would be a vertical axis) of $90° - \theta_2 = 66°$, which is much greater than the critical angle for total internal reflection ($\sin^{-1}(n_{air}/1.56) = 39.9°$). Therefore, no refraction occurs when the light strikes side 2.

(e) In this case, we have

$$n_{air}\sin 70° = 1.56 \sin\theta_2$$

which yields $\theta_2 = 37.04°$ if we use the common approximation $n_{air} = 1.0$, and yields $\theta_2 = 37.05°$ if we use the more accurate value for n_{air} found in Table 33-1. This is greater than the 33.7° mentioned above (regarding the upper-right corner), so the ray strikes side 2 instead of side 3.

(f) After bouncing from side 2 (at a point fairly close to that corner) it goes to side 3.

(g) When it bounced from side 2, its angle of incidence (because the normal axis for side 2 is orthogonal to that for side 1) is $90° - \theta_2 = 53°$, which is much greater than the critical angle for total internal reflection (which, again, is $\sin^{-1}(n_{air}/1.56) = 39.9°$). Therefore, no refraction occurs when the light strikes side 2.

(h) For the same reasons implicit in the calculation of part (c), the refracted ray emerges from side 3 with the same angle (70°) that it entered side 1. We see that the occurrence of an intermediate reflection (from side 2) does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side.

67. (a) In the notation of this problem, Eq. 33-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60°$.

(b) Applying Eq. 33-44 to the interface between material 1 and material 2, we have

$$n_2 \sin 30° = n_1 \sin\theta$$

which yields $\theta = 28.1°$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

68. (a) We use Eq. 33-49: $\theta_B = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.

69. **THINK** A reflected wave will be fully polarized if it strikes the boundary at the Brewster angle.

EXPRESS The angle of incidence for which reflected light is fully polarized is given by Eq. 33-48:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium. The angle θ_B is called the Brewster angle.

ANALYZE With $n_1 = 1.33$ and $n_2 = 1.53$, we obtain

$$\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 49.0°.$$

LEARN In general, reflected light is partially polarized, having components both parallel and perpendicular to the plane of incidence. However, it can be completely polarized when incident at the Brewster angle.

70. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 33-64). We recall that as part of the derivation of Eq. 33-49 (Brewster's angle), the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_c = 90° - \theta_1.$$

We apply Eq. 33-49 to both refractions, setting up a product:

$$\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) = (\tan\theta_{B1\to2})(\tan\theta_{B2\to3}) \quad \Rightarrow \quad \frac{n_3}{n_1} = (\tan\theta_1)(\tan\theta_2).$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_c = \frac{1}{\tan\theta_1}.$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

71. **THINK** All electromagnetic waves, including visible light, travel at the same speed c in vacuum.

EXPRESS The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

ANALYZE (a) We take d to be 150 km = 150 × 10^3 m. Then,

$$t = \frac{d}{c} = \frac{150\times10^3 \text{ m}}{3.00\times10^8 \text{ m/s}} = 5.00\times10^{-4} \text{ s.}$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is

$$d = (1.5 \times 10^8 \text{ km}) + 2\,(3.8 \times 10^5 \text{ km}) = 1.51 \times 10^8 \text{ km} = 1.51 \times 10^{11} \text{ m.}$$

The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51\times10^{11} \text{ m}}{3.00\times10^8 \text{ m/s}} = 500 \text{ s} = 8.4 \text{ min.}$$

(c) We take d to be 2(1.3 × 10^9 km) = 2.6 × 10^{12} m. Then,

$$t = \frac{d}{c} = \frac{2.6\times10^{12} \text{ m}}{3.00\times10^8 \text{ m/s}} = 8.7\times10^3 \text{ s} = 2.4 \text{ h.}$$

(d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500 \text{ ly}}{1.00 \text{ ly/y}} = 6500 \text{ y.}$$

The explosion took place in the year 1054 – 6500 = –5446 or 5446 B.C.

LEARN Since the speed c is constant, the travel time is proportional to the distance. The radio signals at 150 km away reach you almost instantly.

72. (a) The expression $E_y = E_m \sin(kx - \omega t)$ fits the requirement "at point P … [it] is decreasing with time" if we imagine P is just to the right ($x > 0$) of the coordinate origin (but at a value of x less than $\pi/2k = \lambda/4$ which is where there would be a maximum, at t = 0). It is important to bear in mind, in this description, that the wave is moving to the right. Specifically, $x_P = (1/k)\sin^{-1}(1/4)$ so that E_y = (1/4) E_m at t = 0, there. Also, E_y = 0 with our choice of expression for E_y . Therefore, part (a) is answered simply by solving for x_P. Since $k = 2\pi f/c$ we find

$$x_P = \frac{c}{2\pi f}\sin^{-1}\left(\frac{1}{4}\right) = 30.1 \text{ nm}.$$

(b) If we proceed to the right on the x axis (still studying this "snapshot" of the wave at t = 0) we find another point where E_y = 0 at a distance of one-half wavelength from the previous point where $E_y = 0$. Thus (since $\lambda = c/f$) the next point is at $x = \frac{1}{2}\lambda = \frac{1}{2}c/f$ and is consequently a distance $c/2f - x_P = 345$ nm to the right of P.

73. **THINK** The electric and magnetic components of the electromagnetic waves are always in phase, perpendicular to each other, and perpendicular to the direction of propagation of the wave.

EXPRESS The electric and magnetic fields can be written as sinusoidal functions of position and time as:

$$E = E_m \sin(kx + \omega t), \quad B = B_m \sin(kx + \omega t)$$

where E_m and B_m are the amplitudes of the fields, and ω and k, are the angular frequency and angular wave number of the wave, respectively. The two amplitudes are related by Eq. 33-4: $E_m / B_m = c,$ where c is the speed of the wave.

ANALYZE (a) From $kc = \omega$ where $k = 1.00 \times 10^6$ m^{-1}, we obtain $\omega = 3.00 \times 10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 33-5,

$$B_m = E_m/c = (5.00 \text{ V/m})/c = 1.67 \times 10^{-8} \text{ T}.$$

From the argument of the sinusoidal fucntion for E, we see that the direction of propagation is in the –z direction. Since $\vec{E} = E_y\hat{\text{j}}$, and that $\vec{B}$ is perpendicular to $\vec{E}$ and $\vec{E}\times\vec{B},$, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have

$$B_x = (1.67\times10^{-8} \text{ T})\sin[(1.00\times10^6 / \text{m})z + (3.00\times10^{14} / \text{s})t].$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09 \times 10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\,\text{V/m}}{\sqrt{2}}\right)^2 = 0.0332\,\text{W/m}^2\,.$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

LEARN Electromagnetic wave is a transverse wave. Knowing the functional form of the electric field allows us to determine the corresponding magnetic field, and vice versa.

74. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}.$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}.$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{Pr^2}{4R^2 c}.$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}.$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}$ kg and $P = 3.90 \times 10^{26}$ W. Thus,

$$r = \frac{3(3.90\times 10^{26}\ \text{W})}{16\pi(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times 10^{30}\ \text{kg})(1.0\times 10^3\ \text{kg/m}^3)(3.00\times 10^8\ \text{m/s})}$$
$$= 5.8\times 10^{-7}\ \text{m}.$$

75. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

Referring to Fig. 33-65, let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is

$$n \sin \theta_3 \geq 1.$$

We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields

$$n \sin \theta_2 = \sin \theta_1.$$

Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes

$$1 \leq n \sin(90° - \theta_2) = n \cos \theta_2.$$

Squaring this equation and using $\sin^2 \theta_2 + \cos^2 \theta_2 = 1$, we obtain $1 \leq n^2 (1 - \sin^2 \theta_2)$. Substituting $\sin \theta_2 = (1/n) \sin \theta_1$ now leads to

$$1 \leq n^2 \left(1 - \frac{\sin^2 \theta_1}{n^2}\right) = n^2 - \sin^2 \theta_1.$$

The smallest value of n for which this equation is true is given by $1 = n^2 - \sin^2 \theta_1$. We solve for n:

$$n = \sqrt{1 + \sin^2 \theta_1} = \sqrt{1 + \sin^2 45°} = 1.22.$$

LEARN With $n = 1.22$, we have $\theta_2 = \sin^{-1}[(1/1.22)\sin 45°] = 35°$, which gives $\theta_3 = 90° - 35° = 55°$ as the angle of incidence at the second surface. We can readily verify that $n \sin \theta_3 = (1.22) \sin 55° = 1$, meeting the threshold condition for total internal reflection.

76. Since some of the angles in Fig. 33-66 are measured from vertical axes and some are measured from horizontal axes, we must be very careful in taking differences. For instance, the angle difference between the first polarizer struck by the light and the second is 110º (or 70º depending on how we measure it; it does not matter in the final result whether we put $\Delta\theta_1 = 70°$ or put $\Delta\theta_1 = 110°$). Similarly, the angle difference between the second and the third is $\Delta\theta_2 = 40°$, and between the third and the fourth is $\Delta\theta_3$

= 40º, also. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is the incident intensity multiplied by

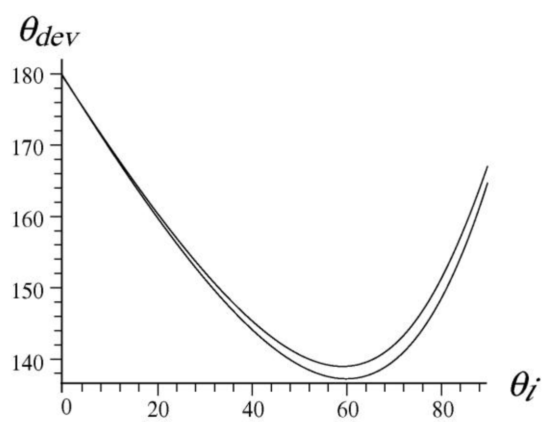
$$\frac{1}{2}\cos^2(\Delta\theta_1)\cos^2(\Delta\theta_2)\cos^2(\Delta\theta_3)\,.$$

Thus, the light that emerges from the system has intensity equal to 0.50 W/m^2.

77. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180° - 2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180° + 2\theta_i - 4\theta_r.$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the θ_{dev} minimum for red light is $137.63° \approx 137.6°$, and this occurs at θ_i = 59.52°.

(d) For blue light, we find that the θ_{dev} minimum is $139.35° \approx 139.4°$, and this occurs at θ_i = 59.52°.

(e) The difference in θ_{dev} in the previous two parts is 1.72°.

78. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s).

Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2(\theta_i - \theta_r) + k(180° - 2\theta_r) = k(180°) + 2\theta_i - 2(k+1)\theta_r.$$

(b) For k = 2 and n = 1.331 (given in Problem 33-77), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is $230.37° \approx 230.4°$, and this occurs at θ_i = 71.90°.

(c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which n = 1.343) is $233.48° \approx 233.5°$, and this occurs at θ_i = 71.52°.

(d) The difference in θ_{dev} in the previous two parts is approximately 3.1°.

(e) Setting k = 3, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.5°, and this occurs at θ_i = 76.88°.

(f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.9°, and this occurs at θ_i = 76.62°.

(g) The difference in θ_{dev} in the previous two parts is 4.4°.

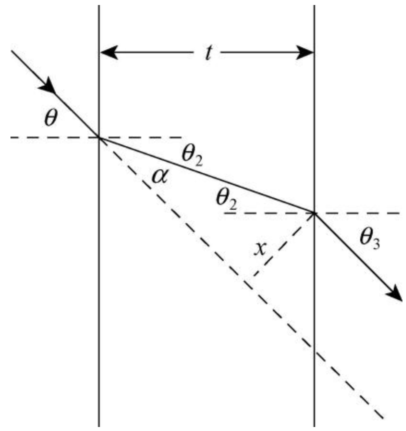
79. **THINK** We apply law of refraction to both interfaces to calculate the sideway displacement.

EXPRESS Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields

$$\sin\theta = n\sin\theta_2.$$

The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then

$$n\sin\theta_2 = \sin\theta_3.$$



ANALYZE (a) Combining the two expressions gives sin θ_3 = sin θ, which implies that $\theta_3 = \theta$. Thus, the emerging ray is parallel to the incident ray.

(b) We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D \cos \theta_2 = t$ and $D = t/\cos \theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and

$$x = D \sin \alpha = D \sin (\theta - \theta_2).$$

Thus,

$$x = \frac{t \sin(\theta - \theta_2)}{\cos \theta_2}.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then sin $\theta \approx \theta$, sin $\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos \theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}.$$

LEARN The thicker the glass, the greater the displacement x. Note in the limit $n = 1$ (no glass), $x = 0$, as expected.

80. (a) The magnitude of the magnetic field is

$$B = \frac{E}{c} = \frac{100\ \text{V/m}}{3.0 \times 10^8\ \text{m/s}} = 3.3 \times 10^{-7}\ \text{T}.$$

(b) With $\vec{E} \times \vec{B} = \mu_0 \vec{S}$, where $\vec{E} = E\hat{\text{k}}$ and $\vec{S} = S(-\hat{\text{j}})$, one can verify easily that since $\hat{\text{k}} \times (-\hat{\text{i}}) = -\hat{\text{j}}$, $\vec{B}$ has to be in the $-x$ direction.

81. (a) The polarization direction is defined by the electric field (which is perpendicular to the magnetic field in the wave, and also perpendicular to the direction of wave travel). The given function indicates the magnetic field is along the x axis (by the subscript on B)

and the wave motion is along $-y$ axis (see the argument of the sine function). Thus, the electric field direction must be parallel to the z axis.

(b) Since k is given as 1.57×10^7/m, then $\lambda = 2\pi/k = 4.0 \times 10^{-7}$ m, which means $f = c/\lambda = 7.5 \times 10^{14}$ Hz.

(c) The magnetic field amplitude is given as $B_m = 4.0 \times 10^{-6}$ T. The electric field amplitude E_m is equal to B_m divided by the speed of light c. The rms value of the electric field is then E_m divided by $\sqrt{2}$. Equation 33-26 then gives $I = 1.9$ kW/m^2.

82. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta_1' \cos^2 \theta_2'$$

where $\theta_1' = 90° - \theta_1 = 60°$ and $\theta_2' = 90° - \theta_2 = 60°$. This yields $I/I_0 = 0.031$.

83. **THINK** The index of refraction encountered by light generally depends on the wavelength of the light.

EXPRESS The critical angle for total internal reflection is given by $\sin \theta_c = 1/n$. With an index of refraction $n = 1.456$ at the red end, the critical angle is $\theta_c = 43.38°$ for red. Similarly, with $n = 1.470$ at the blue end, the critical angle is $\theta_c = 42.86°$ for blue.

ANALYZE (a) An angle of incidence of $\theta_1 = 42.00°$ is less than the critical angles for both red and blue light, so the refracted light is white.

(b) An angle of incidence of $\theta_1 = 43.10°$ is slightly less than the critical angle for red light but greater than the critical angle for blue light, so the refracted light is dominated by red end.

(c) An angle of incidence of $\theta_1 = 44.00°$ is greater than the critical angles for both red and blue light, so there is no refracted light.

LEARN The dependence of the index of refraction of fused quartz on wavelength is shown in Fig. 33-18. From the figure, we see that the index of refraction is greater for a shorter wavelength. Such dependence results in the spreading of light as it enters or leaves quartz, a phenomenon called "chromatic dispersion."

84. Using Eqs. 33-40 and 33-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{(I_0/2)(\cos^2 45°)(\cos^2 45°)}{I_0} = \frac{1}{8} = 0.125.$$

85. We write $m = \rho\mathcal{V}$ where $\mathcal{V} = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 33-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho\frac{4\pi R^3}{3}a = \frac{I\pi R^2}{c}.$$

This simplifies to

$$a = \frac{3I}{4\rho cR}$$

which yields $a = 1.5 \times 10^{-9}$ m/s^2.

86. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is

$$\tfrac{1}{2}(\cos^2(30^\circ))^3 = 0.21.$$

87. **THINK** Since the radar beam is emitted uniformly over a hemisphere, the source power is also the same everywhere within the hemisphere.

EXPRESS The intensity of the beam is given by

$$I = \frac{P}{A} = \frac{P}{2\pi r^2}$$

where $A = 2\pi r^2$ is the area of a hemisphere. The power of the aircraft's reflection is equal to the product of the intensity at the aircraft's location and its cross-sectional area: $P_r = IA_r$. The intensity is related to the amplitude of the electric field by Eq. 33-26: $I = E_{\text{rms}}^2/c\mu_0 = E_m^2/2c\mu_0$.

ANALYZE (a) Substituting the values given we get

$$I = \frac{P}{2\pi r^2} = \frac{180\times10^3 \text{ W}}{2\pi(90\times10^3 \text{ m})^2} = 3.5\times10^{-6} \text{ W/m}^2.$$

(b) The power of the aircraft's reflection is

$$P_r = IA_r = (3.5\times10^{-6} \text{ W/m}^2)(0.22 \text{ m}^2) = 7.8\times10^{-7} \text{ W}.$$

(c) Back at the radar site, the intensity is

$$I_r = \frac{P_r}{2\pi r^2} = \frac{7.8\times10^{-7} \text{ W}}{2\pi(90\times10^3 \text{ m})^2} = 1.5\times10^{-17} \text{ W/m}^2.$$

(d) From $I_r = E_m^2 / 2c\mu_0$, we find the amplitude of the electric field to be

$$E_m = \sqrt{2c\mu_0 I_r} = \sqrt{2(3.0\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.5\times10^{-17}\ \text{W/m}^2)}$$
$$= 1.1\times10^{-7}\ \text{V/m}.$$

(e) The rms value of the magnetic field is

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{E_m}{\sqrt{2}c} = \frac{1.1\times10^{-7}\ \text{V/m}}{\sqrt{2}(3.0\times10^8\ \text{m/s})} = 2.5\times10^{-16}\text{T}.$$

LEARN The intensity due to a power source decreases with the square of the distance. Also, as emphasized in Sample Problem — "Light wave: rms values of the electric and magnetic fields," one cannot compare the values of the two fields because they are measured in different units. Both components are on the same basis from the perspective of wave propagation, and they have the same average energy.

88. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20\times10^{-4}\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.07\times10^{-12}\ \text{T}.$$

89. From Fig. 33-19 we find $n_{\max} = 1.470$ for $\lambda = 400$ nm and $n_{\min} = 1.456$ for $\lambda = 700$ nm.
(a) The corresponding Brewster's angles are

$$\theta_{\text{B,max}} = \tan^{-1} n_{\max} = \tan^{-1}(1.470) = 55.8°,$$

(b) and $\theta_{\text{B,min}} = \tan^{-1}(1.456) = 55.5°$.

90. (a) Suppose there are a total of N transparent layers ($N = 5$ in our case). We label these layers from left to right with indices 1, 2, …, N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the $(j + 1)$-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1},$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2},$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f},$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right).$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray *a*,

(b) and $\theta_f = 20°$ for ray *b*.

(c) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. That is, we still have $\theta_f = 0$ for ray *a*,

(d) and $\theta_f = 20°$ for ray *b*.

Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

91. (a) At $r = 40$ m, the intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} = \frac{4(3.0\times10^{-3}\,\text{W})}{\pi\left[\left(0.17\times10^{-3}\,\text{rad}\right)\left(40\,\text{m}\right)\right]^2} = 83\,\text{W/m}^2.$$

(b) $P' = 4\pi r^2 I = 4\pi(40\text{m})^2(83\,\text{W/m}^2) = 1.7\times10^6\,\text{W}.$

92. The law of refraction requires that

$$\sin\theta_1/\sin\theta_2 = n_{\text{water}} = \text{const.}$$

We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example, sin 10° / sin 8° = 1.3, and sin 20° / sin 15°30′ = 1.3, etc. Therefore, the index of refraction of water is $n_{\text{water}} = 1.3$.

93. We remind ourselves that when the unpolarized light passes through the first sheet, its intensity is reduced by a factor of 2. Thus, to end up with an overall reduction of one-third, the second sheet must cause a further decrease by a factor of two-thirds (since (1/2)(2/3) = 1/3). Thus, $\cos^2\theta = 2/3 \;\Rightarrow\; \theta = 35°$.

94. (a) The magnitude of the electric field at point *P* is

$$E = \frac{V}{l} = \frac{iR}{l} = (25.0\text{ A})\left(\frac{1.00\,\Omega}{300\text{ m}}\right) = 0.0833\text{ V/m}.$$

The direction of $\vec{E}$ at point *P* is in the +*x* direction, same as the current.

(b) We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and *i* is the net current through the loop. The magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{2\pi r} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(25.0\text{ A}\right)}{2\pi\left(1.25\times10^{-3}\text{m}\right)} = 4.00\times10^{-3}\text{ T}.$$

The direction of $\vec{B}$ at point *P* is in the +*z* direction (out of the page).

(c) From $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, we find the magnitude of the Poynting vector to be

$$S = \frac{EB}{\mu_0} = \frac{(0.0833\text{ V/m})(4.0\times10^{-3}\text{ T})}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)} = 265\text{ W/m}^2.$$

(d) Since $\vec{S}$ points in the direction of $\vec{E}\times\vec{B}$, using the right-hand-rule, the direction of $\vec{S}$ at point *P* is in the –*y* direction.

95. (a) For the cylindrical resistor shown in Figure 33-74, the magnetic field is in the $-\hat{\theta}$, or clockwise direction. On the other hand, the electric field is in the same direction as the current, $-\hat{z}$. Since $\vec{S} = \vec{E}\times\vec{B}/\mu_0$, $\vec{S}$ is in the direction of $(-\hat{z})\times(-\hat{\theta}) = -\hat{r}$, or radially inward.

(b) The magnitudes of the electric and magnetic fields are $E = V/l = iR/l$ and $B = \mu_0 i/2\pi a$, respectively. Thus,

$$S = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\left(\frac{iR}{l}\right)\left(\frac{\mu_0 i}{2\pi a}\right) = \frac{i^2 R}{2\pi a l}.$$

Noting that the magnitude of the Poynting vector S is constant, we have

$$\int \vec{S}\cdot d\vec{A} = SA = \left(\frac{i^2 R}{2\pi a l}\right)(2\pi a l) = i^2 R.$$

96. The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, implying that the rate of energy absorbed is $P_{\text{abs}} = IA = E_m^2 A/2\mu_0 c$. If all the energy is used to heat up the sheet (converting to its internal energy), then

$$P_{\text{abs}} = \frac{dE_{\text{int}}}{dt} = mc_s\frac{dT}{dt},$$

where c_s is the specific heat of the material. Solving for dT/dt, we find

$$mc_s\frac{dT}{dt} = \frac{E_m^2 A}{2\mu_0 c} \quad\Rightarrow\quad \frac{dT}{dt} = \frac{E_m^2 A}{2mc_s\mu_0 c}.$$

97. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is, by one-half rule, $I_1 = \frac{1}{2}I_0$. For the second sheet, we apply the cosine-squared rule:

$$I_2 = I_1\cos^2\theta = \frac{1}{2}I_0\cos^2\theta$$

where θ is the angle between the direction of polarization of the two sheets. With $I_2/I_0 = p/100$, we solve for θ and obtain

$$\frac{I_2}{I_0} = \frac{p}{100} = \frac{1}{2}\cos^2\theta \quad\Rightarrow\quad \theta = \cos^{-1}\left(\sqrt{\frac{p}{50}}\right).$$

98. The cross-sectional area of the beam on the surface is $A\cos\theta$. In a time interval Δt, the volume of the beam that's been reflected is $\Delta V = (A\cos\theta)c\Delta t$, and the momentum carried by this volume is $p = (I/c^2)(A\cos\theta)c\Delta t$. Upon being reflected, the change in momentum is

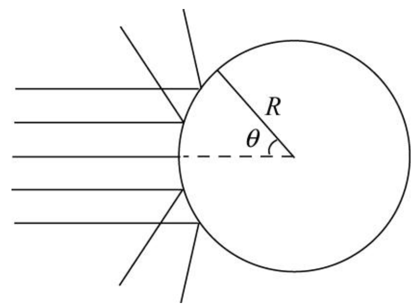
$$\Delta p = 2p\cos\theta = 2IA\cos^2\theta\Delta t/c$$

Thus, the radiation pressure is

$$p_r = \frac{F_r}{A} = \frac{\Delta p}{A\Delta t} = \frac{2I}{c}\cos^2\theta = p_{r\perp}\cos^2\theta$$

where $p_{r\perp} = 2I/c$ is the radiation pressure when $\theta = 0$.

99. Consider the figure shown to the right. The y-component of the force cancels out, and we're left with the x-component:



$$dF_x = 2dF\cos\theta = 2(p_r dA)\cos\theta .$$

Using the result from Problem 98: $p_r = (2I/c)\cos^2\theta$, and $dA = RLd\theta$, where L is the length of the cylinder, we obtain

$$\frac{F_x}{L} = \int 2(2I\cos\theta/c)\cos\theta\, Rd\theta = \frac{4IR}{c}\int_0^{\pi/2}\cos^3\theta d\theta = \frac{8IR}{3c}.$$

100. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta_1'\cos^2\theta_2'$$

where $\theta_1' = (90° - \theta_1) + \theta_2 = 110°$ is the relative angle between the first and the second polarizing sheets, and $\theta_2' = 90° - \theta_2 = 50°$ is the relative angle between the second and the third polarizing sheets. Thus, we have $I/I_0 = 0.024$.

101. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2}I_0\cos^2\theta'\cos^2\theta''.$$

With $\theta' = \theta_2 - \theta_1$ = 60° – 20° = 40° and $\theta'' = \theta_3 + (\pi/2 - \theta_2)$ = 40° + 30° = 70°, we get $I/I_0 = 0.034$.

102. We use Eq. 33-33 for the force, where A is the area of the reflecting surface (4.0 m^2). The intensity is gotten from Eq. 33-27 where $P = P_S$ is in Appendix C (see also Sample Problem 33-2) and $r = 3.0 \times 10^{11}$ m (given in the problem statement). Our result for the force is 9.2 μN.

103. Eq. 33-5 gives $B = E/c$, which relates the field values at any instant — and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is

$$B_{\text{rms}} = (0.200 \text{ V/m})/(3 \times 10^8 \text{ m/s}) = 6.67 \times 10^{-10} \text{ T},$$

which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.43×10^{-10} T.

104. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \geq y \geq h$ from the top of the wall, then it is clear from the Figure that $\tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15 \text{ m/s})\tan 30° = -8.7 \text{ m/s},$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \leq \theta < 90°$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt|$ = 45 m/s, we find

$$v_{\text{hawk}} = \left|\frac{dx}{dt}\right| = \frac{|dy/dt|}{\tan\theta}$$

so that we obtain $\theta = 72°$ if we assume v_{hawk} = 15 m/s.

105. (a) The wave is traveling in the $-y$ direction (see §16-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 33-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is B_m = 4.00 μT, then (by Eq. 33-5) E_m = 1199 V/m $\approx 1.20\times10^3$ V/m. Dividing by $\sqrt{2}$ yields E_{rms} = 848 V/m. Then, Eq. 33-26 gives

$$I = \frac{I}{c\mu_0}E_{\text{rms}}^2 = 1.91\times10^3 \text{ W/m}^2.$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k = \frac{2.00\times10^{15}}{c} = 6.67\times10^{6}\ \mathrm{m}^{-1}.$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z = (1.2\times10^{3}\ \mathrm{V/m})\sin[(6.67\times10^{6}/\mathrm{m})y + (2.00\times10^{15}/\mathrm{s})t].$$

(e) $\lambda = 2\pi/k = 942$ nm.

(f) This is an infrared light.

106. (a) The angle of incidence $\theta_{B,1}$ at B is the complement of the critical angle at A; its sine is

$$\sin\theta_{B,1} = \cos\theta_c = \sqrt{1-\left(\frac{n_3}{n_2}\right)^2}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1^\circ\,.$$

(b) From $n_1 \sin\theta = n_2 \sin\theta_c = n_2(n_3/n_2)$, we find

$$\theta = \sin^{-1}\left(\frac{n_3}{n_1}\right) = 49.9^\circ\,.$$

(c) The angle of incidence $\theta_{A,1}$ at A is the complement of the critical angle at B; its sine is

$$\sin\theta_{A,1} = \cos\theta_c = \sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\,.$$

so that the angle of refraction $\theta_{A,2}$ at A becomes

$$\theta_{A,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1-\left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1^\circ\,.$$

(d) From

$$n_1\sin\theta = n_2\sin\theta_{A,1} = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}\ ,$$

we find

$$\theta = \sin^{-1}\left(\frac{\sqrt{n_2^2 - n_3^2}}{n_1}\right) = 26.1°$$

(e) The angle of incidence $\theta_{B,1}$ at B is the complement of the Brewster angle at A; its sine is

$$\sin\theta_{B,1} = \frac{n_2}{\sqrt{n_2^2 + n_3^2}}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2^2}{n_3\sqrt{n_2^2 + n_3^2}}\right) = 60.7°\,.$$

(f) From

$$n_1 \sin\theta = n_2 \sin\theta_{\text{Brewster}} = n_2 \frac{n_3}{\sqrt{n_2^2 + n_3^2}}\ ,$$

we find

$$\theta = \sin^{-1}\left(\frac{n_2 n_3}{n_1\sqrt{n_2^2 + n_3^2}}\right) = 35.3°\ .$$

107. (a) and (b) At the Brewster angle, $\theta_{\text{incident}} + \theta_{\text{refracted}} = \theta_B + 32.0° = 90.0°$, so $\theta_B = 58.0°$ and

$$n_{\text{glass}} = \tan\theta_B = \tan 58.0° = 1.60.$$

108. We take the derivative with respect to x of both sides of Eq. 33-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t}.$$

Now we differentiate both sides of Eq. 33-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0\mu_0\frac{\partial E}{\partial t}\right) = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}.$$

Substituting $\partial^2 E/\partial x^2 = -\partial^2 B/\partial x\partial t$ from the first equation above into the second one, we get

$$\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} \qquad \Rightarrow \qquad \frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2} = c^2\frac{\partial^2 E}{\partial x^2}.$$

Similarly, we differentiate both sides of Eq. 33-11 with respect to *t*

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2},$$

and differentiate both sides of Eq. 33-18 with respect to *x*

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 - \frac{\partial^2 E}{\partial x \partial t}.$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

109. (a) From Eq. 33-1,

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2} \; E_m \sin(kx - \omega t) \; = -\omega^2 E_m \sin(kx - \omega t),$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2}{\partial x^2} \; E_m \sin(kx - \omega t) \; = -k^2 c^2 \sin(kx - \omega t) = -\omega^2 E_m \sin(kx - \omega t).$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 33-2 satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

(b) From $E = E_m f(kx \pm \omega t)$,

$$\frac{\partial^2 E}{\partial t^2} = E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = \omega^2 E_m \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = c^2 E_m k^2 \left.\frac{d^2 f}{du^2}\right|_{u = kx \pm \omega t}$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}.$$

Changing E to B and repeating the derivation above shows that $B = B_m f(kx \pm \omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}.$$

110. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of r = 20 m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\,\text{W/m}^2.$$

as illustrated in Sample Problem 33-2. Now, in Eq. 33-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\text{ m})^2 = 0.0013\text{ m}^2$. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3\times10^8} = 1.7\times10^{-13}\,\text{N}.$$