

## Chapter 30

1. The flux  $\Phi_B = BA \cos\theta$  does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

2. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi rB\frac{dr}{dt} \\ &= -2\pi(0.12\text{m})(0.800\text{T})(-0.750\text{m/s}) \\ &= 0.452\text{V}.\end{aligned}$$

3. **THINK** Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil.

**EXPRESS** Using Faraday's law, the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is  $i_{\text{ind}} = |\varepsilon|/R$ , where  $R$  is the resistance of the coil.

**ANALYZE** Substituting the values given, we obtain

$$\begin{aligned}\varepsilon &= -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi \times 10^{-7}\text{T}\cdot\text{m/A})(22000/\text{m})\pi(0.016\text{m})^2\left(\frac{1.5\text{A}}{0.025\text{s}}\right) \\ &= 0.16\text{V}.\end{aligned}$$

$$\text{Ohm's law then yields } i_{\text{ind}} = \frac{|\varepsilon|}{R} = \frac{0.16\text{V}}{5.3\Omega} = 0.030\text{A}.$$

**LEARN** The direction of the induced current can be deduced from Lenz's law, which states that the direction of the induced current is such that the magnetic field which it produces opposes the change in flux that induces the current.

4. (a) We use  $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$ . For  $0 < t < 2.0\text{s}$ :

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi (0.12\text{m})^2 \left( \frac{0.5\text{T}}{2.0\text{s}} \right) = -1.1 \times 10^{-2} \text{ V}.$$

(b) For  $2.0 \text{ s} < t < 4.0 \text{ s}$ :  $\varepsilon \propto dB/dt = 0$ .

(c) For  $4.0 \text{ s} < t < 6.0 \text{ s}$ :

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi (0.12\text{m})^2 \left( \frac{-0.5\text{T}}{6.0\text{s} - 4.0\text{s}} \right) = 1.1 \times 10^{-2} \text{ V}.$$

5. The field (due to the current in the straight wire) is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

6. From the datum at  $t = 0$  in Fig. 30-37(b) we see  $0.0015 \text{ A} = V_{\text{battery}}/R$ , which implies that the resistance is

$$R = (6.00 \mu\text{V})/(0.0015 \text{ A}) = 0.0040 \Omega.$$

Now, the value of the current during  $10 \text{ s} < t < 20 \text{ s}$  leads us to equate

$$(V_{\text{battery}} + \varepsilon_{\text{induced}})/R = 0.00050 \text{ A}.$$

This shows that the induced emf is  $\varepsilon_{\text{induced}} = -4.0 \mu\text{V}$ . Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A \frac{dB}{dt} = -A a.$$

Plugging in  $\varepsilon = -4.0 \times 10^{-6} \text{ V}$  and  $A = 5.0 \times 10^{-4} \text{ m}^2$ , we obtain  $a = 0.0080 \text{ T/s}$ .

7. (a) The magnitude of the emf is

$$|\varepsilon| = \left| \frac{d\Phi_B}{dt} \right| = \frac{d}{dt} (6.0t^2 + 7.0t) = 12t + 7.0 = 12(2.0) + 7.0 = 31 \text{ mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through  $R$ .

8. The resistance of the loop is

$$R = \rho \frac{L}{A} = (1.69 \times 10^{-8} \Omega \cdot \text{m}) \frac{\pi (0.10\text{m})}{\pi (2.5 \times 10^{-3} \text{ m})^2 / 4} = 1.1 \times 10^{-3} \Omega.$$

We use  $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R) |dB/dt|$ . Thus

$$\left| \frac{dB}{dt} \right| = \frac{iR}{\pi r^2} = \frac{(10 \text{ A})(1.1 \times 10^{-3} \Omega)}{\pi (0.05 \text{ m})^2} = 1.4 \text{ T/s}.$$

9. The amplitude of the induced emf in the loop is

$$\begin{aligned} \varepsilon_m &= A\mu_0 n i_0 \omega = (6.8 \times 10^{-6} \text{ m}^2)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(85400/\text{m})(1.28 \text{ A})(212 \text{ rad/s}) \\ &= 1.98 \times 10^{-4} \text{ V}. \end{aligned}$$

10. (a) The magnetic flux  $\Phi_B$  through the loop is given by

$$\Phi_B = 2B(\pi r^2/2)(\cos 45^\circ) = \pi r^2 B / \sqrt{2}.$$

Thus,

$$\begin{aligned} \varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left( \frac{\pi r^2 B}{\sqrt{2}} \right) = -\frac{\pi r^2}{\sqrt{2}} \left( \frac{\Delta B}{\Delta t} \right) = -\frac{\pi (3.7 \times 10^{-2} \text{ m})^2}{\sqrt{2}} \left( \frac{0 - 76 \times 10^{-3} \text{ T}}{4.5 \times 10^{-3} \text{ s}} \right) \\ &= 5.1 \times 10^{-2} \text{ V}. \end{aligned}$$

(a) The direction of the induced current is clockwise when viewed along the direction of  $\vec{B}$ .

11. (a) It should be emphasized that the result, given in terms of  $\sin(2\pi ft)$ , could as easily be given in terms of  $\cos(2\pi ft)$  or even  $\cos(2\pi ft + \phi)$  where  $\phi$  is a phase constant as discussed in Chapter 15. The angular position  $\theta$  of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as  $BA \cos \theta$ ,  $BA \sin \theta$  or  $BA \cos(\theta + \phi)$ . Here our choice is such that  $\Phi_B = BA \cos \theta$ . Since the coil is rotating steadily,  $\theta$  increases linearly with time. Thus,  $\theta = \omega t$  (equivalent to  $\theta = 2\pi ft$ ) if  $\theta$  is understood to be in radians (and  $\omega$  would be the angular velocity). Since the area of the rectangular coil is  $A = ab$ , Faraday's law leads to

$$\varepsilon = -N \frac{d(BA \cos \theta)}{dt} = -NBA \frac{d \cos(2\pi ft)}{dt} = N Bab 2\pi f \sin(2\pi ft)$$

which is the desired result, shown in the problem statement. The second way this is written ( $\varepsilon_0 \sin(2\pi ft)$ ) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of  $\varepsilon_0 = 2\pi f NabB$ .

(b) We solve

$$\varepsilon_0 = 150 \text{ V} = 2\pi f NabB$$

when  $f = 60.0 \text{ rev/s}$  and  $B = 0.500 \text{ T}$ . The three unknowns are  $N$ ,  $a$ , and  $b$  which occur in a product; thus, we obtain  $Nab = 0.796 \text{ m}^2$ .

12. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For  $\vec{B} = (4.00 \times 10^{-2} \text{ T/m}) y \hat{k}$ ,  $dB/dt = 0$  and hence  $\varepsilon = 0$ .

(b) None.

(c) For  $\vec{B} = (6.00 \times 10^{-2} \text{ T/s}) t \hat{k}$ ,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A \frac{dB}{dt} = -(0.400 \text{ m} \times 0.250 \text{ m})(0.0600 \text{ T/s}) = -6.00 \text{ mV},$$

or  $|\varepsilon| = 6.00 \text{ mV}$ .

(d) Clockwise.

(e) For  $\vec{B} = (8.00 \times 10^{-2} \text{ T/m} \cdot \text{s}) y t \hat{k}$ ,  $\Phi_B = (0.400)(0.0800 t) \int y dy = 1.00 \times 10^{-3} t$ ,

in SI units. The induced emf is  $\varepsilon = -d\Phi_B/dt = -1.00 \text{ mV}$ , or  $|\varepsilon| = 1.00 \text{ mV}$ .

(f) Clockwise.

(g)  $\Phi_B = 0 \Rightarrow \varepsilon = 0$ .

(h) None.

(i)  $\Phi_B = 0 \Rightarrow \varepsilon = 0$ .

(j) None.

13. The amount of charge is

$$\begin{aligned} q(t) &= \frac{1}{R} [\Phi_B(0) - \Phi_B(t)] = \frac{A}{R} [B(0) - B(t)] = \frac{1.20 \times 10^{-3} \text{ m}^2}{13.0 \, \Omega} [1.60 \text{ T} - (-1.60 \text{ T})] \\ &= 2.95 \times 10^{-2} \text{ C}. \end{aligned}$$

14. Figure 30-42(b) demonstrates that  $dB/dt$  (the slope of that line) is  $0.003 \text{ T/s}$ . Thus, in absolute value, Faraday's law becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A \frac{dB}{dt}$$

where  $A = 8 \times 10^{-4} \text{ m}^2$ . We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-42(c) to be  $i = dq/dt = 0.002 \text{ A}$  (the slope of that line). Therefore, the resistance of the loop is

$$R = \frac{|\varepsilon|}{i} = \frac{A |dB/dt|}{i} = \frac{(8.0 \times 10^{-4} \text{ m}^2)(0.0030 \text{ T/s})}{0.0020 \text{ A}} = 0.0012 \Omega.$$

15. (a) Let  $L$  be the length of a side of the square circuit. Then the magnetic flux through the circuit is  $\Phi_B = L^2 B/2$ , and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2} \frac{dB}{dt}.$$

Now  $B = 0.042 - 0.870t$  and  $dB/dt = -0.870 \text{ T/s}$ . Thus,

$$\varepsilon_i = \frac{(2.00 \text{ m})^2}{2} (0.870 \text{ T/s}) = 1.74 \text{ V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0 \text{ V} + 1.74 \text{ V} = 21.7 \text{ V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since the flux arises from a dot product of vectors, the result of one sign for  $B_1$  and  $B_2$  and of the opposite sign for  $B_3$  (we choose the minus sign for the flux from  $B_1$  and  $B_2$ , and therefore a plus sign for the flux from  $B_3$ ). The induced emf is

$$\begin{aligned} \varepsilon &= -\Sigma \frac{d\Phi_B}{dt} = A \left( \frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt} \right) \\ &= (0.10 \text{ m})(0.20 \text{ m})(2.0 \times 10^{-6} \text{ T/s} + 1.0 \times 10^{-6} \text{ T/s} - 5.0 \times 10^{-6} \text{ T/s}) \\ &= -4.0 \times 10^{-8} \text{ V}. \end{aligned}$$

The minus sign means that the effect is dominated by the changes in  $B_3$ . Its magnitude (using Ohm's law) is  $|\varepsilon|/R = 8.0 \mu\text{A}$ .

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

17. Equation 29-10 gives the field at the center of the large loop with  $R = 1.00 \text{ m}$  and current  $i(t)$ . This is approximately the field throughout the area ( $A = 2.00 \times 10^{-4} \text{ m}^2$ ) enclosed by the small loop. Thus, with  $B = \mu_0 i/2R$  and  $i(t) = i_0 + kt$ , where  $i_0 = 200 \text{ A}$  and

$$k = (-200 \text{ A} - 200 \text{ A})/1.00 \text{ s} = -400 \text{ A/s},$$

we find

$$(a) \ B(t=0) = \frac{\mu_0 i_0}{2R} = \frac{(4\pi \times 10^{-7} \text{ H/m})(200 \text{ A})}{2(1.00 \text{ m})} = 1.26 \times 10^{-4} \text{ T},$$

$$(b) \ B(t=0.500 \text{ s}) = \frac{(4\pi \times 10^{-7} \text{ H/m})[200 \text{ A} - (400 \text{ A/s})(0.500 \text{ s})]}{2(1.00 \text{ m})} = 0, \text{ and}$$

$$(c) \ B(t=1.00 \text{ s}) = \frac{(4\pi \times 10^{-7} \text{ H/m})[200 \text{ A} - (400 \text{ A/s})(1.00 \text{ s})]}{2(1.00 \text{ m})} = -1.26 \times 10^{-4} \text{ T},$$

$$\text{or } |B(t=1.00 \text{ s})| = 1.26 \times 10^{-4} \text{ T}.$$

(d) Yes, as indicated by the flip of sign of  $B(t)$  in (c).

(e) Let the area of the small loop be  $a$ . Then  $\Phi_B = Ba$ , and Faraday's law yields

$$\begin{aligned} \varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a \frac{dB}{dt} = -a \left( \frac{\Delta B}{\Delta t} \right) \\ &= -(2.00 \times 10^{-4} \text{ m}^2) \left( \frac{-1.26 \times 10^{-4} \text{ T} - 1.26 \times 10^{-4} \text{ T}}{1.00 \text{ s}} \right) \\ &= 5.04 \times 10^{-8} \text{ V}. \end{aligned}$$

18. (a) The “height” of the triangular area enclosed by the rails and bar is the same as the distance traveled in time  $v$ :  $d = vt$ , where  $v = 5.20 \text{ m/s}$ . We also note that the “base” of that triangle (the distance between the intersection points of the bar with the rails) is  $2d$ . Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2 t^2.$$

Since the field is a uniform  $B = 0.350 \text{ T}$ , then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.350)(5.20)^2 t^2 = 9.46 t^2.$$

At  $t = 3.00 \text{ s}$ , we obtain  $\Phi_B = 85.2 \text{ Wb}$ .

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 9.46 \frac{dt^2}{dt} = 18.9t$$

in SI units. At  $t = 3.00$  s, this yields  $\varepsilon = 56.8$  V.

(c) Our calculation in part (b) shows that  $n = 1$ .

19. First we write  $\Phi_B = BA \cos \theta$ . We note that the angular position  $\theta$  of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as  $BA \cos \theta$  (as opposed to, say,  $BA \sin \theta$ ). Since the coil is rotating steadily,  $\theta$  increases linearly with time. Thus,  $\theta = \omega t$  if  $\theta$  is understood to be in radians (here,  $\omega = 2\pi f$  is the angular velocity of the coil in radians per second, and  $f = 1000$  rev/min  $\approx 16.7$  rev/s is the frequency). Since the area of the rectangular coil is  $A = (0.500 \text{ m}) \times (0.300 \text{ m}) = 0.150 \text{ m}^2$ , Faraday's law leads to

$$\varepsilon = -N \frac{d(BA \cos \theta)}{dt} = -NBA \frac{d \cos(2\pi f t)}{dt} = NBA 2\pi f \sin(2\pi f t)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi f NAB = 2\pi (16.7 \text{ rev/s})(100 \text{ turns})(0.15 \text{ m}^2)(3.5 \text{ T}) = 5.50 \times 10^3 \text{ V}.$$

20. We note that  $1 \text{ gauss} = 10^{-4} \text{ T}$ . The amount of charge is

$$\begin{aligned} q(t) &= \frac{N}{R} [BA \cos 20^\circ - (-BA \cos 20^\circ)] = \frac{2NBA \cos 20^\circ}{R} \\ &= \frac{2(1000)(0.590 \times 10^{-4} \text{ T})\pi(0.100 \text{ m})^2(\cos 20^\circ)}{85.0 \Omega + 140 \Omega} = 1.55 \times 10^{-5} \text{ C}. \end{aligned}$$

Note that the axis of the coil is at  $20^\circ$ , not  $70^\circ$ , from the magnetic field of the Earth.

21. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40 \text{ rev/s})(2\pi \text{ rad/rev})}{2\pi} = 40 \text{ Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-46, such that the semicircular wire is in the  $\theta = 0$  position and a quarter of a period (of revolution) later it will be in the  $\theta = \pi/2$  position (where its midpoint will reach a distance of  $a$  above the plane of the figure). At the moment it is in the  $\theta = \pi/2$  position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area  $A_0$ , which is the area it will again appear to enclose when the wire is in the  $\theta = 3\pi/2$  position).

Since the area of the semicircle is  $\pi a^2/2$ , then the area (as it appears to us) enclosed by the circuit, as a function of our angle  $\theta$ , is

$$A = A_0 + \frac{\pi a^2}{2} \cos \theta$$

where (since  $\theta$  is increasing at a steady rate) the angle depends linearly on time, which we can write either as  $\theta = \omega t$  or  $\theta = 2\pi f t$  if we take  $t = 0$  to be a moment when the arc is in the  $\theta = 0$  position. Since  $\vec{B}$  is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B \frac{dA}{dt} = -B \frac{d(A_0 + (\pi a^2/2) \cos \theta)}{dt} = -B \frac{\pi a^2}{2} \frac{d \cos(2\pi f t)}{dt}$$

which yields  $\varepsilon = B\pi^2 a^2 f \sin(2\pi f t)$ . This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020 \text{ T})\pi^2 (0.020 \text{ m})^2 (40/\text{s}) = 3.2 \times 10^{-3} \text{ V}.$$

22. Since  $\frac{d \cos \phi}{dt} = -\sin \phi \frac{d\phi}{dt}$ , Faraday's law (with  $N = 1$ ) becomes

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(BA \cos \phi)}{dt} = BA \sin \phi \frac{d\phi}{dt}.$$

Substituting the values given yields  $|\varepsilon| = 0.018 \text{ V}$ .

23. **THINK** Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.

**EXPRESS** The magnetic flux through a surface is given by  $\Phi_B = \int \vec{B} \cdot d\vec{A}$ , where  $\vec{B}$  is the magnetic field and  $d\vec{A}$  is a vector of magnitude  $dA$  that is normal to a differential area  $dA$ . In the case where  $\vec{B}$  is uniform and perpendicular to the plane of the loop,  $\Phi_B = BA$ .

In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis.

Equation 29-27, with  $z = x$  (taken to be much greater than  $R$ ), gives  $\vec{B} = \frac{\mu_0 i R^2}{2x^3} \hat{i}$ , where the  $+x$  direction is upward in Fig. 30-47. The area of the smaller loop is  $A = \pi r^2$ .

**ANALYZE** (a) The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area of the smaller loop:



$$\Phi_B = BA = \frac{\pi\mu_0 ir^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 ir^2 R^2}{2}\right) \frac{d}{dt} \left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 ir^2 R^2}{2}\right) \left(-\frac{3}{x^4} \frac{dx}{dt}\right) = \frac{3\pi\mu_0 ir^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

**LEARN** The situation in this problem is like that shown in Fig. 30-5(d). The induced magnetic field is in the same direction as the initial magnetic field.

24. (a) Since  $\vec{B} = B\hat{i}$  uniformly, then only the area “projected” onto the  $yz$  plane will contribute to the flux (due to the scalar [dot] product). This “projected” area corresponds to one-fourth of a circle. Thus, the magnetic flux  $\Phi_B$  through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \frac{1}{4} \pi r^2 B.$$

Thus,

$$|\varepsilon| = \left| \frac{d\Phi_B}{dt} \right| = \left| \frac{d}{dt} \left( \frac{1}{4} \pi r^2 B \right) \right| = \frac{\pi r^2}{4} \left| \frac{dB}{dt} \right| = \frac{1}{4} \pi (0.10 \text{ m})^2 (3.0 \times 10^{-3} \text{ T/s}) = 2.4 \times 10^{-5} \text{ V}.$$

(b) We have a situation analogous to that shown in Fig. 30-5(a). Thus, the current in segment  $bc$  flows from  $c$  to  $b$  (following Lenz's law).

25. (a) We refer to the (very large) wire length as  $L$  and seek to compute the flux per meter:  $\Phi_B/L$ . Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of anti-parallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call  $x = \ell/2$ , where  $\ell = 20 \text{ mm} = 0.020 \text{ m}$ ); the net field at any point  $0 < x < \ell/2$  is the same at its “mirror image” point  $\ell - x$ . The central axis of one of the wires passes through the origin, and that of the other passes through  $x = \ell$ . We make use of the symmetry by integrating over  $0 < x < \ell/2$  and then multiplying by 2:

$$\Phi_B = 2 \int_0^{\ell/2} B dA = 2 \int_0^{\ell/2} B (L dx) + 2 \int_{\ell/2}^{\ell} B (L dx)$$

where  $d = 0.0025$  m is the diameter of each wire. We will use  $R = d/2$ , and  $r$  instead of  $x$  in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2 \int_0^R \left( \frac{\mu_0 i}{2\pi R^2} r + \frac{\mu_0 i}{2\pi(\ell - r)} \right) dr + 2 \int_R^{\ell/2} \left( \frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)} \right) dr \\ &= \frac{\mu_0 i}{2\pi} \left( 1 - 2 \ln \left( \frac{\ell - R}{\ell} \right) \right) + \frac{\mu_0 i}{\pi} \ln \left( \frac{\ell - R}{R} \right) \\ &= 0.23 \times 10^{-5} \text{ T} \cdot \text{m} + 1.08 \times 10^{-5} \text{ T} \cdot \text{m}\end{aligned}$$

which yields  $\Phi_B/L = 1.3 \times 10^{-5} \text{ T} \cdot \text{m}$  or  $1.3 \times 10^{-5} \text{ Wb/m}$ .

(b) The flux (per meter) existing within the regions of space occupied by one or the other wire was computed above to be  $0.23 \times 10^{-5} \text{ T} \cdot \text{m}$ . Thus,

$$\frac{0.23 \times 10^{-5} \text{ T} \cdot \text{m}}{1.3 \times 10^{-5} \text{ T} \cdot \text{m}} = 0.17 = 17\% .$$

(c) What was described in part (a) as a symmetry plane at  $x = \ell/2$  is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the  $0 < x < \ell/2$  region is now of opposite sign of the flux in the  $\ell/2 < x < \ell$  region, which causes the total flux (or, in this case, flux per meter) to be zero.

26. (a) First, we observe that a large portion of the figure contributes flux that “cancels out.” The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is  $b - a$ , then a strip below the wire (where the strip borders the long wire, and extends a distance  $b - a$  away from it) has exactly the equal but opposite flux that cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B dA = \int_{b-a}^a \left( \frac{\mu_0 i}{2\pi r} \right) (b dr) = \frac{\mu_0 i b}{2\pi} \ln \left( \frac{a}{b-a} \right) .$$

Faraday’s law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \left[ \frac{\mu_0 i b}{2\pi} \ln \left( \frac{a}{b-a} \right) \right] = -\frac{\mu_0 b}{2\pi} \ln \left( \frac{a}{b-a} \right) \frac{di}{dt} \\ &= -\frac{\mu_0 b}{2\pi} \ln \left( \frac{a}{b-a} \right) \frac{d}{dt} \left( \frac{9}{2} t^2 - 10t \right) \\ &= \frac{-\mu_0 b (9t - 10)}{2\pi} \ln \left( \frac{a}{b-a} \right) .\end{aligned}$$

With  $a = 0.120$  m and  $b = 0.160$  m, then, at  $t = 3.00$  s, the magnitude of the emf induced in the rectangular loop is

$$|\mathcal{E}| = \frac{(4\pi \times 10^{-7})(0.16)(9(3) - 10)}{2\pi} \ln\left(\frac{0.12}{0.16 - 0.12}\right) = 5.98 \times 10^{-7} \text{ V}.$$

(b) We note that  $di/dt > 0$  at  $t = 3$  s. The situation is roughly analogous to that shown in Fig. 30-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

27. (a) Consider a (thin) strip of area of height  $dy$  and width  $\ell = 0.020$  m. The strip is located at some  $0 < y < \ell$ . The element of flux through the strip is

$$d\Phi_B = BdA = (4t^2 y)(\ell dy)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^\ell (4t^2 y \ell) dy = 2t^2 \ell^3.$$

Thus, Faraday's law yields

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| = 4t\ell^3.$$

At  $t = 2.5$  s, the magnitude of the induced emf is  $8.0 \times 10^{-5}$  V.

(b) Its "direction" (or "sense") is clockwise, by Lenz's law.

28. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left( \frac{\mu_0 i}{2\pi r} \right) (a dr) = \frac{\mu_0 i a}{2\pi} \ln \left( \frac{r+b/2}{r-b/2} \right).$$

When  $r = 1.5b$ , we have

$$|\Phi_B| = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.7 \text{ A})(0.022 \text{ m})}{2\pi} \ln(2.0) = 1.4 \times 10^{-8} \text{ Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that  $dr/dt = v$ . The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$\begin{aligned} i_{\text{loop}} &= \left| \frac{\varepsilon}{R} \right| = -\frac{\mu_0 i a}{2\pi R} \left| \frac{d}{dt} \ln \left( \frac{r+b/2}{r-b/2} \right) \right| = \frac{\mu_0 i a b v}{2\pi R [r^2 - (b/2)^2]} \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.7 \text{ A})(0.022 \text{ m})(0.0080 \text{ m})(3.2 \times 10^{-3} \text{ m/s})}{2\pi(4.0 \times 10^{-4} \Omega)[2(0.0080 \text{ m})^2]} \\ &= 1.0 \times 10^{-5} \text{ A}. \end{aligned}$$

29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350 \text{ T})(0.250 \text{ m})(0.55 \text{ m/s}) = 0.0481 \text{ V}.$$

(b) By Ohm's law, the induced current is

$$i = 0.0481 \text{ V} / 18.0 \Omega = 0.00267 \text{ A}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to  $P = i^2 R = 0.000129 \text{ W}$ .

30. Equation 26-28 gives  $\varepsilon^2/R$  as the rate of energy transfer into thermal forms ( $dE_{\text{th}}/dt$ , which, from Fig. 30-53(c), is roughly 40 nJ/s). Interpreting  $\varepsilon$  as the induced emf (in absolute value) in the single-turn loop ( $N = 1$ ) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A \frac{dB}{dt}.$$

Equation 29-23 gives  $B = \mu_0 ni$  for the solenoid (and note that the field is zero outside of the solenoid, which implies that  $A = A_{\text{coil}}$ ), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = A \frac{dB}{dt} = A_{\text{coil}} \frac{d}{dt}(\mu_0 ni_{\text{coil}}) = \mu_0 n A_{\text{coil}} \frac{di_{\text{coil}}}{dt}.$$

where Fig. 30-53(b) suggests that  $di_{\text{coil}}/dt = 0.5 \text{ A/s}$ . With  $n = 8000$  (in SI units) and  $A_{\text{coil}} = \pi(0.02)^2$  (note that the loop radius does not come into the computations of this problem, just the coil's), we find  $V = 6.3 \mu\text{V}$ . Returning to our earlier observations, we can now solve for the resistance:

$$R = \varepsilon^2 / (dE_{\text{th}}/dt) = 1.0 \text{ m}\Omega.$$

31. **THINK** Thermal energy is generated at the rate given by  $P = \varepsilon^2/R$  (see Eq. 27-23), where  $\varepsilon$  is the emf in the wire and  $R$  is the resistance of the wire.

**EXPRESS** Using Eq. 27-16, the resistance is given by  $R = \rho L/A$ , where the resistivity is  $1.69 \times 10^{-8} \Omega \cdot \text{m}$  (by Table 27-1) and  $A = \pi d^2/4$  is the cross-sectional area of the wire ( $d = 0.00100 \text{ m}$  is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi \left( \frac{L}{2\pi} \right)^2$$

since the length of the wire ( $L = 0.500 \text{ m}$ ) is the circumference of the loop. This enclosed area is used in Faraday's law to give the induced emf:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A_{\text{loop}} \frac{dB}{dt} = -\frac{L^2}{4\pi} \frac{dB}{dt}.$$

**ANALYZE** The rate of change of the field is  $dB/dt = 0.0100 \text{ T/s}$ . Thus, we obtain

$$P = \frac{|\mathcal{E}|^2}{R} = \frac{(L^2/4\pi)^2 (dB/dt)^2}{\rho L/(\pi d^2/4)} = \frac{d^2 L^3}{64\pi\rho} \left( \frac{dB}{dt} \right)^2 = \frac{(1.00 \times 10^{-3} \text{ m})^2 (0.500 \text{ m})^3}{64\pi (1.69 \times 10^{-8} \Omega \cdot \text{m})} (0.0100 \text{ T/s})^2$$

$$= 3.68 \times 10^{-6} \text{ W}.$$

**LEARN** The rate of thermal energy generated is proportional to  $(dB/dt)^2$ .

32. Noting that  $|\Delta B| = B$ , we find the thermal energy is

$$P_{\text{thermal}} \Delta t = \frac{\mathcal{E}^2 \Delta t}{R} = \frac{1}{R} \left( -\frac{d\Phi_B}{dt} \right)^2 \Delta t = \frac{1}{R} \left( -A \frac{\Delta B}{\Delta t} \right)^2 \Delta t = \frac{A^2 B^2}{R \Delta t}$$

$$= \frac{(2.00 \times 10^{-4} \text{ m}^2)^2 (17.0 \times 10^{-6} \text{ T})^2}{(5.21 \times 10^{-6} \Omega)(2.96 \times 10^{-3} \text{ s})} = 7.50 \times 10^{-10} \text{ J}.$$

33. (a) Letting  $x$  be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is  $B = \mu_0 i / 2\pi r$ , where  $r$  is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length  $x$  and width  $dr$ , parallel to the wire and a distance  $r$  from it; it has area  $A = x dr$  and the flux is

$$d\Phi_B = B dA = \frac{\mu_0 i}{2\pi r} x dr.$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 i x}{2\pi} \int_a^{a+L} \frac{dr}{r} = \frac{\mu_0 i x}{2\pi} \ln \left( \frac{a+L}{a} \right).$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi} \frac{dx}{dt} \ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 i v}{2\pi} \ln\left(\frac{a+L}{a}\right) \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100 \text{ A})(5.00 \text{ m/s})}{2\pi} \ln\left(\frac{1.00 \text{ cm} + 10.0 \text{ cm}}{1.00 \text{ cm}}\right) = 2.40 \times 10^{-4} \text{ V}.\end{aligned}$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon / R = (2.40 \times 10^{-4} \text{ V}) / (0.400 \Omega) = 6.00 \times 10^{-4} \text{ A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = (6.00 \times 10^{-4} \text{ A})^2 (0.400 \Omega) = 1.44 \times 10^{-7} \text{ W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length  $dr$  at a distance  $r$  from the long straight wire, is

$$dF_B = i_\ell B dr = (\mu_0 i_\ell i / 2\pi r) dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned}F_B &= \frac{\mu_0 i_\ell i}{2\pi} \int_a^{a+L} \frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi} \ln\left(\frac{a+L}{a}\right) \\ &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(6.00 \times 10^{-4} \text{ A})(100 \text{ A})}{2\pi} \ln\left(\frac{1.00 \text{ cm} + 10.0 \text{ cm}}{1.00 \text{ cm}}\right) \\ &= 2.87 \times 10^{-8} \text{ N}.\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of  $2.87 \times 10^{-8} \text{ N}$ , to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8} \text{ N})(5.00 \text{ m/s}) = 1.44 \times 10^{-7} \text{ W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

34. Noting that  $F_{\text{net}} = BiL - mg = 0$ , we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\mathcal{E}|}{R} = \frac{1}{R} \left| \frac{d\Phi_B}{dt} \right| = \frac{B}{R} \left| \frac{dA}{dt} \right| = \frac{Bv_t L}{R},$$

which yields  $v_t = mgR/B^2L^2$ .

35. (a) Equation 30-8 leads to

$$\mathcal{E} = BLv = (1.2 \text{ T})(0.10 \text{ m})(5.0 \text{ m/s}) = 0.60 \text{ V}.$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is  $i = 0.60 \text{ V}/0.40 \Omega = 1.5 \text{ A}$ .

(d) The direction is clockwise.

(e) Equation 26-28 leads to  $P = i^2R = 0.90 \text{ W}$ .

(f) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5 \text{ A})(0.10 \text{ m})(1.2 \text{ T}) = 0.18 \text{ N}.$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(g) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent:

$$P = Fv = (0.18 \text{ N})(5.0 \text{ m/s}) = 0.90 \text{ W},$$

which is the same as our result from part (e).

36. (a) For path 1, we have

$$\begin{aligned} \oint_1 \vec{E} \cdot d\vec{s} &= -\frac{d\Phi_{B1}}{dt} = \frac{d}{dt}(B_1 A_1) = A_1 \frac{dB_1}{dt} = \pi r_1^2 \frac{dB_1}{dt} = \pi (0.200 \text{ m})^2 (-8.50 \times 10^{-3} \text{ T/s}) \\ &= -1.07 \times 10^{-3} \text{ V}. \end{aligned}$$

(b) For path 2, the result is

$$\oint_2 \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{B2}}{dt} = \pi r_2^2 \frac{dB_2}{dt} = \pi (0.300\text{ m})^2 (-8.50 \times 10^{-3} \text{ T/s}) = -2.40 \times 10^{-3} \text{ V}.$$

(c) For path 3, we have

$$\oint_3 \vec{E} \cdot d\vec{s} = \oint_1 \vec{E} \cdot d\vec{s} - \oint_2 \vec{E} \cdot d\vec{s} = -1.07 \times 10^{-3} \text{ V} - (-2.4 \times 10^{-3} \text{ V}) = 1.33 \times 10^{-3} \text{ V}.$$

37. **THINK** Changing magnetic field induces an electric field.

**EXPRESS** The induced electric field is given by Eq. 30-20:  $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ .

**ANALYZE** (a) The point at which we are evaluating the field is inside the solenoid, so

$$E(2\pi r) = -(\pi r^2) \frac{dB}{dt} \Rightarrow E = -\frac{1}{2} \frac{dB}{dt} r.$$

The magnitude of the induced electric field is

$$|E| = \frac{1}{2} \frac{dB}{dt} r = \frac{1}{2} (6.5 \times 10^{-3} \text{ T/s}) (0.0220 \text{ m}) = 7.15 \times 10^{-5} \text{ V/m}.$$

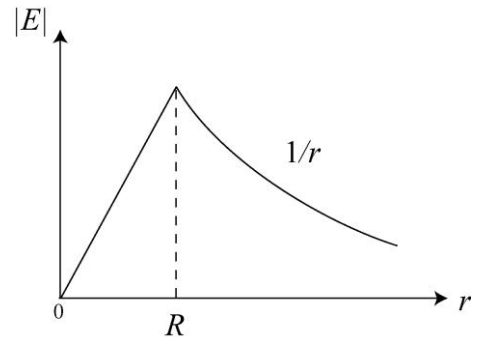
(b) Now the point at which we are evaluating the field is outside the solenoid, so

$$E(2\pi r) = -(\pi R^2) \frac{dB}{dt} \Rightarrow E = -\frac{1}{2} \frac{dB}{dt} \frac{R^2}{r}.$$

The magnitude of the induced field is

$$|E| = \frac{1}{2} \frac{dB}{dt} \frac{R^2}{r} = \frac{1}{2} (6.5 \times 10^{-3} \text{ T/s}) \frac{(0.0600 \text{ m})^2}{0.0820 \text{ m}} = 1.43 \times 10^{-4} \text{ V/m}.$$

**LEARN** The magnitude of the induced electric field as a function of  $r$  is shown to the right. Inside the solenoid,  $r < R$ , the field  $|E|$  is linear in  $r$ . However, outside the solenoid,  $r > R$ ,  $|E| \sim 1/r$ .



38. From the “kink” in the graph of Fig. 30-57, we conclude that the radius of the circular region is 2.0 cm. For values of  $r$  less than that, we have (from the absolute value of Eq. 30-20)



$$E(2\pi r) = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A \frac{dB}{dt} = \pi r^2 a$$

which means that  $E/r = a/2$ . This corresponds to the slope of that graph (the linear portion for small values of  $r$ ) which we estimate to be 0.015 (in SI units). Thus,  $a = 0.030$  T/s.

39. The magnetic field  $B$  can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where  $B_0 = (30.0 \text{ T} + 29.6 \text{ T})/2 = 29.8 \text{ T}$  and  $B_1 = (30.0 \text{ T} - 29.6 \text{ T})/2 = 0.200 \text{ T}$ . Then from Eq. 30-25

$$E = \frac{1}{2} \left( \frac{dB}{dt} \right) r = \frac{r}{2} \frac{dB}{dt} B_0 + B_1 \sin(\omega t + \phi_0) = \frac{1}{2} B_1 \omega r \cos(\omega t + \phi_0).$$

We note that  $\omega = 2\pi f$  and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2} B_1 (2\pi f) r = \frac{1}{2} (0.200 \text{ T}) (2\pi) (15 \text{ Hz}) (1.6 \times 10^{-2} \text{ m}) = 0.15 \text{ V/m}.$$

40. Since  $N\Phi_B = Li$ , we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0 \times 10^{-3} \text{ H})(5.0 \times 10^{-3} \text{ A})}{400} = 1.0 \times 10^{-7} \text{ Wb}.$$

41. (a) We interpret the question as asking for  $N$  multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60 \times 10^{-3} \text{ T})(\pi)(0.100 \text{ m})^2 = 2.45 \times 10^{-3} \text{ Wb}.$$

(b) Equation 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45 \times 10^{-3} \text{ Wb}}{3.80 \text{ A}} = 6.45 \times 10^{-4} \text{ H}.$$

42. (a) We imagine dividing the one-turn solenoid into  $N$  small circular loops placed along the width  $W$  of the copper strip. Each loop carries a current  $\Delta i = i/N$ . Then the magnetic field inside the solenoid is

$$B = \mu_0 n \Delta i = \mu_0 \left( \frac{N}{W} \right) \left( \frac{i}{N} \right) = \frac{\mu_0 i}{W} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.035 \text{ A})}{0.16 \text{ m}} = 2.7 \times 10^{-7} \text{ T}.$$

(b) Equation 30-33 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2 (\mu_0 i / W)}{i} = \frac{\pi \mu_0 R^2}{W} = \frac{\pi (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (0.018 \text{ m})^2}{0.16 \text{ m}} = 8.0 \times 10^{-9} \text{ H}.$$

43. We refer to the (very large) wire length as  $\ell$  and seek to compute the flux per meter:  $\Phi_B / \ell$ . Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at  $x = d/2$ ); the net field at any point  $0 < x < d/2$  is the same at its “mirror image” point  $d - x$ . The central axis of one of the wires passes through the origin, and that of the other passes through  $x = d$ . We make use of the symmetry by integrating over  $0 < x < d/2$  and then multiplying by 2:

$$\Phi_B = 2 \int_0^{d/2} B \, dA = 2 \int_0^a B(\ell \, dx) + 2 \int_a^{d/2} B(\ell \, dx)$$

where  $d = 0.0025 \text{ m}$  is the diameter of each wire. We will use  $r$  instead of  $x$  in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned} \frac{\Phi_B}{\ell} &= 2 \int_0^a \left( \frac{\mu_0 i}{2\pi a^2} r + \frac{\mu_0 i}{2\pi (d-r)} \right) dr + 2 \int_a^{d/2} \left( \frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi (d-r)} \right) dr \\ &= \frac{\mu_0 i}{2\pi} \left( 1 - 2 \ln \left( \frac{d-a}{d} \right) \right) + \frac{\mu_0 i}{\pi} \ln \left( \frac{d-a}{a} \right) \end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately  $\Phi_B \approx \mu_0 i \ell / \pi \ln((d-a)/a)$ . Now, we use Eq. 30-33 (with  $N = 1$ ) to obtain the inductance per unit length:

$$\frac{L}{\ell} = \frac{\Phi_B}{\ell i} = \frac{\mu_0}{\pi} \ln \left( \frac{d-a}{a} \right) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})}{\pi} \ln \left( \frac{142 - 1.53}{1.53} \right) = 1.81 \times 10^{-6} \text{ H/m}.$$

44. Since  $\mathcal{E} = -L(di/dt)$ , we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\mathcal{E}}{L} = -\frac{60 \text{ V}}{12 \text{ H}} = -5.0 \text{ A/s},$$

or  $|di/dt| = 5.0 \text{ A/s}$ . We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then  $i$  must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left| \frac{\varepsilon}{di/dt} \right| = \frac{17 \text{ V}}{2.5 \text{ kA/s}} = 6.8 \times 10^{-4} \text{ H}.$$

46. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes  $|\varepsilon| = L |\Delta i / \Delta t|$ . For simplicity, we omit the absolute value signs in the following.

(a) For  $0 < t < 2 \text{ ms}$ ,

$$\varepsilon = L \frac{\Delta i}{\Delta t} = \frac{(4.6 \text{ H})(7.0 \text{ A} - 0)}{2.0 \times 10^{-3} \text{ s}} = 1.6 \times 10^4 \text{ V}.$$

(b) For  $2 \text{ ms} < t < 5 \text{ ms}$ ,

$$\varepsilon = L \frac{\Delta i}{\Delta t} = \frac{(4.6 \text{ H})(5.0 \text{ A} - 7.0 \text{ A})}{(5.0 - 2.0)10^{-3} \text{ s}} = 3.1 \times 10^3 \text{ V}.$$

(c) For  $5 \text{ ms} < t < 6 \text{ ms}$ ,

$$\varepsilon = L \frac{\Delta i}{\Delta t} = \frac{(4.6 \text{ H})(0 - 5.0 \text{ A})}{(6.0 - 5.0)10^{-3} \text{ s}} = 2.3 \times 10^4 \text{ V}.$$

47. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ( $V_1 + V_2$ ), then inductances in series must add,  $L_{\text{eq}} = L_1 + L_2$ , just as was the case for resistances.

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors,  $L_{\text{eq}} = \sum_{n=1}^N L_n$ .

48. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ( $V_1 = V_2$ ), and the currents (which are generally functions of time) add ( $i_1(t) + i_2(t) = i(t)$ ). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also apply to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that the field of one inductor not to have significant influence (or “coupling”) in the next.

(b) Just as with resistors, 
$$\frac{1}{L_{\text{eq}}} = \sum_{n=1}^N \frac{1}{L_n}.$$

49. Using the results from Problems 30-47 and 30-48, the equivalent resistance is

$$\begin{aligned} L_{\text{eq}} &= L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3} = 30.0 \text{ mH} + 15.0 \text{ mH} + \frac{(50.0 \text{ mH})(20.0 \text{ mH})}{50.0 \text{ mH} + 20.0 \text{ mH}} \\ &= 59.3 \text{ mH}. \end{aligned}$$

50. The steady state value of the current is also its maximum value,  $\mathcal{E}/R$ , which we denote as  $i_m$ . We are told that  $i = i_m/3$  at  $t_0 = 5.00$  s. Equation 30-41 becomes  $i = i_m(1 - e^{-t_0/\tau_L})$ , which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00 \text{ s}}{\ln(1 - 1/3)} = 12.3 \text{ s}.$$

51. The current in the circuit is given by  $i = i_0 e^{-t/\tau_L}$ , where  $i_0$  is the current at time  $t = 0$  and  $\tau_L$  is the inductive time constant ( $L/R$ ). We solve for  $\tau_L$ . Dividing by  $i_0$  and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0 \text{ s}}{\ln((10 \times 10^{-3} \text{ A})/(1.0 \text{ A}))} = 0.217 \text{ s}.$$

Therefore,  $R = L/\tau_L = 10 \text{ H}/0.217 \text{ s} = 46 \Omega$ .

52. (a) Immediately after the switch is closed,  $\mathcal{E} - \mathcal{E}_L = iR$ . But  $i = 0$  at this instant, so  $\mathcal{E}_L = \mathcal{E}$ , or  $\mathcal{E}_L/\mathcal{E} = 1.00$ .

(b)  $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon$ , or  $\varepsilon_L/\varepsilon = 0.135$ .

(c) From  $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$  we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \Rightarrow t = \tau_L \ln 2 = 0.693\tau_L \Rightarrow t/\tau_L = 0.693.$$

53. **THINK** The inductor in the  $RL$  circuit initially acts to oppose changes in current through it.

**EXPRESS** If the battery is switched into the circuit at  $t = 0$ , then the current at a later time  $t$  is given by

$$i = \frac{\varepsilon}{R} (1 - e^{-t/\tau_L}),$$

where  $\tau_L = L/R$ .

(a) We want to find the time at which  $i = 0.800\varepsilon/R$ . This means

$$0.800 = 1 - e^{-t/\tau_L} \Rightarrow e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30 \times 10^{-6} \text{ H})}{1.20 \times 10^3 \Omega} = 8.45 \times 10^{-9} \text{ s}.$$

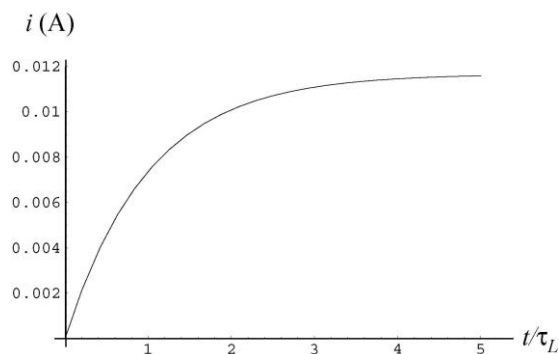
(b) At  $t = 1.0\tau_L$  the current in the circuit is

$$i = \frac{\varepsilon}{R} (1 - e^{-1.0}) = \left( \frac{14.0 \text{ V}}{1.20 \times 10^3 \Omega} \right) (1 - e^{-1.0}) = 7.37 \times 10^{-3} \text{ A}.$$

**LEARN** At  $t = 0$ , the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

$$i_0 = \frac{\varepsilon}{R} = \frac{14.0 \text{ V}}{1.20 \times 10^3 \Omega} = 0.0117 \text{ A}.$$

The current as a function of  $t/\tau_L$  is plotted to the right.



54. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100 \text{ V}}{10.0 \Omega + 20.0 \Omega} = 3.33 \text{ A}.$$

(b)  $i_2 = i_1 = 3.33 \text{ A}$ .

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in  $R_3$  is  $i_1 - i_2$ . Kirchhoff's loop rule gives

$$\begin{aligned}\varepsilon - i_1 R_1 - i_2 R_2 &= 0 \\ \varepsilon - i_1 R_1 - (i_1 - i_2) R_3 &= 0.\end{aligned}$$

We solve these simultaneously for  $i_1$  and  $i_2$ , and find

$$\begin{aligned}i_1 &= \frac{\varepsilon(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100 \text{ V})(20.0 \Omega + 30.0 \Omega)}{(10.0 \Omega)(20.0 \Omega) + (10.0 \Omega)(30.0 \Omega) + (20.0 \Omega)(30.0 \Omega)} \\ &= 4.55 \text{ A},\end{aligned}$$

(d) and

$$\begin{aligned}i_2 &= \frac{\varepsilon R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100 \text{ V})(30.0 \Omega)}{(10.0 \Omega)(20.0 \Omega) + (10.0 \Omega)(30.0 \Omega) + (20.0 \Omega)(30.0 \Omega)} \\ &= 2.73 \text{ A}.\end{aligned}$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is,  $i_1 = 0$ ).

(f) The current in  $R_3$  changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is  $4.55 \text{ A} - 2.73 \text{ A} = 1.82 \text{ A}$ . The current in  $R_2$  is the same but in the opposite direction as that in  $R_3$ , that is,  $i_2 = -1.82 \text{ A}$ .

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g)  $i_1 = 0$ , and

(h)  $i_2 = 0$ .

55. **THINK** The inductor in the  $RL$  circuit initially acts to oppose changes in current through it.

**EXPRESS** Starting with zero current at  $t = 0$  (the moment the switch is closed) the current in the circuit increases according to

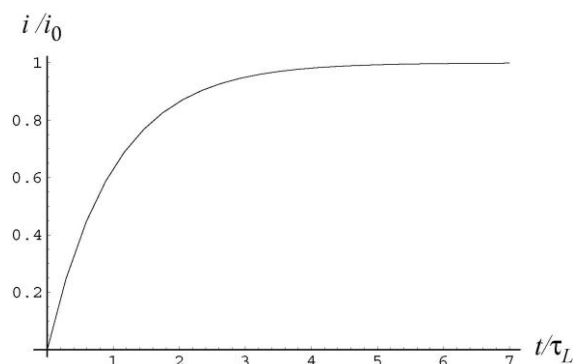
$$i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}),$$

where  $\tau_L = L/R$  is the inductive time constant and  $\mathcal{E}$  is the battery emf.

**ANALYZE** To calculate the time at which  $i = 0.9990\mathcal{E}/R$ , we solve for  $t$ :

$$0.9990 \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \Rightarrow \ln(0.0010) = -\frac{t}{\tau_L} \Rightarrow \frac{t}{\tau_L} = 6.91.$$

**LEARN** At  $t = 0$ , the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is  $i_0 = \mathcal{E}/R$ . The current (in terms of  $i/i_0$ ) as a function of  $t/\tau_L$  is plotted below.



56. From the graph we get  $\Phi/i = 2 \times 10^{-4}$  in SI units. Therefore, with  $N = 25$ , we find the self-inductance is  $L = N\Phi/i = 5 \times 10^{-3}$  H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol  $V$  to stand for the battery emf)

$$\frac{di}{dt} = \frac{V}{R} \frac{R}{L} e^{-t/\tau_L} = \frac{V}{L} e^{-t/\tau_L} = 7.1 \times 10^2 \text{ A/s}.$$

57. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop:  $\mathcal{E} - L di/dt = 0$ . So  $i = \mathcal{E}t/L$ . As the fuse blows at  $t = t_0$ ,  $i = i_0 = 3.0$  A. Thus,

$$t_0 = \frac{i_0 L}{\mathcal{E}} = \frac{(3.0 \text{ A})(5.0 \text{ H})}{10 \text{ V}} = 1.5 \text{ s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

58. Applying the loop theorem,

$$\varepsilon - L \left( \frac{di}{dt} \right) = iR ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\begin{aligned} \varepsilon &= L \frac{di}{dt} + iR = L \frac{d}{dt} (3.0 + 5.0t) + (3.0 + 5.0t)R = (6.0)(5.0) + (3.0 + 5.0t)(4.0) \\ &= (42 + 20t). \end{aligned}$$

59. **THINK** The inductor in the  $RL$  circuit initially acts to oppose changes in current through it. We are interested in the currents in the resistor and the current in the inductor as a function of time.

**EXPRESS** We assume  $i$  to be from left to right through the closed switch. We let  $i_1$  be the current in the resistor and take it to be downward. Let  $i_2$  be the current in the inductor, also assumed downward. The junction rule gives  $i = i_1 + i_2$  and the loop rule gives  $i_1 R - L(di_2/dt) = 0$ . According to the junction rule,  $(di_1/dt) = -(di_2/dt)$ . We substitute into the loop equation to obtain

$$L \frac{di_1}{dt} + i_1 R = 0.$$

This equation is similar to Eq. 30-46, and its solution is the function given as Eq. 30-47:  $i_1 = i_0 e^{-Rt/L}$ , where  $i_0$  is the current through the resistor at  $t = 0$ , just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment  $i_2 = 0$  and  $i_1 = i$ . Thus  $i_0 = i$ .

**ANALYZE** (a) The currents in the resistor and the inductor as a function of time are:

$$i_1 = i e^{-Rt/L}, \quad i_2 = i - i_1 = i(1 - e^{-Rt/L}).$$

(b) When  $i_2 = i_1$ , we have

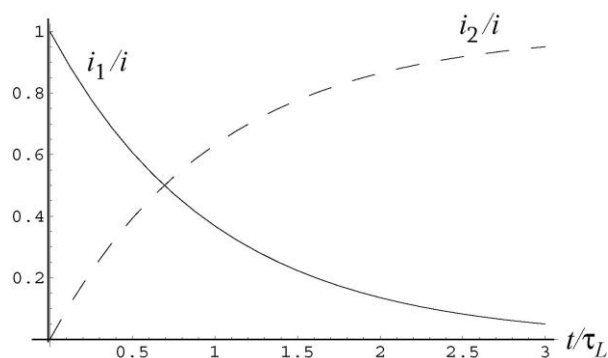
$$e^{-Rt/L} = 1 - e^{-Rt/L} \Rightarrow e^{-Rt/L} = \frac{1}{2}.$$

Taking the natural logarithm of both sides and using  $\ln(1/2) = -\ln 2$ , we obtain

$$\left( \frac{Rt}{L} \right) = \ln 2 \Rightarrow t = \frac{L}{R} \ln 2.$$

**LEARN** A plot of  $i_1/i$  (solid line, for resistor) and  $i_2/i$  (dashed line, for inductor) as a function of  $t/\tau_L$  is shown next.





60. (a) Our notation is as follows:  $h$  is the height of the toroid,  $a$  its inner radius, and  $b$  its outer radius. Since it has a square cross section,  $h = b - a = 0.12 \text{ m} - 0.10 \text{ m} = 0.02 \text{ m}$ . We derive the flux using Eq. 29-24 and the self-inductance using Eq. 30-33:

$$\Phi_B = \int_a^b B dA = \int_a^b \left( \frac{\mu_0 Ni}{2\pi r} \right) h dr = \frac{\mu_0 Nih}{2\pi} \ln\left(\frac{b}{a}\right)$$

and

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right).$$

Now, since the inner circumference of the toroid is  $l = 2\pi a = 2\pi(10 \text{ cm}) \approx 62.8 \text{ cm}$ , the number of turns of the toroid is roughly  $N \approx 62.8 \text{ cm}/1.0 \text{ mm} = 628$ . Thus

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \approx \frac{(4\pi \times 10^{-7} \text{ H/m}) (628)^2 (0.02 \text{ m})}{2\pi} \ln\left(\frac{12}{10}\right) = 2.9 \times 10^{-4} \text{ H}.$$

(b) Noting that the perimeter of a square is four times its sides, the total length  $\ell$  of the wire is  $\ell = (628)4(2.0 \text{ cm}) = 50 \text{ m}$ , and the resistance of the wire is

$$R = (50 \text{ m})(0.02 \Omega/\text{m}) = 1.0 \Omega.$$

Thus,

$$\tau_L = \frac{L}{R} = \frac{2.9 \times 10^{-4} \text{ H}}{1.0 \Omega} = 2.9 \times 10^{-4} \text{ s}.$$

61. **THINK** Inductance  $L$  is related to the inductive time constant of an  $RL$  circuit by  $L = \tau_L R$ , where  $R$  is the resistance in the circuit. The energy stored by an inductor carrying current  $i$  is given by  $U_B = Li^2/2$ .

**EXPRESS** If the battery is applied at time  $t = 0$  the current is given by

$$i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}),$$

where  $\mathcal{E}$  is the emf of the battery,  $R$  is the resistance, and  $\tau_L$  is the inductive time constant ( $L/R$ ). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\mathcal{E}} \Rightarrow -\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\mathcal{E}}\right).$$

Since

$$\ln\left(1 - \frac{iR}{\mathcal{E}}\right) = \ln\left[1 - \frac{(2.00 \times 10^{-3} \text{ A})(10.0 \times 10^3 \Omega)}{50.0 \text{ V}}\right] = -0.5108,$$

the inductive time constant is  $\tau_L = t/0.5108 = (5.00 \times 10^{-3} \text{ s})/0.5108 = 9.79 \times 10^{-3} \text{ s}$ .

**ANALYZE** (a) The inductance is

$$L = \tau_L R = (9.79 \times 10^{-3} \text{ s})(10.0 \times 10^3 \Omega) = 97.9 \text{ H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2} Li^2 = \frac{1}{2} (97.9 \text{ H})(2.00 \times 10^{-3} \text{ A})^2 = 1.96 \times 10^{-4} \text{ J}.$$

**LEARN** Note the similarity between  $U_B = \frac{1}{2} Li^2$  and  $U_C = \frac{q^2}{2C}$ , the electric energy stored in a capacitor.

62. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2} Li^2\right)}{dt} = Li \frac{di}{dt} = L \left( \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right) \left( \frac{\mathcal{E}}{R} \frac{1}{\tau_L} e^{-t/\tau_L} \right) = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0 \text{ H}/10 \Omega = 0.20 \text{ s}$$

and  $\mathcal{E} = 100 \text{ V}$ , so the above expression yields  $dU_B/dt = 2.4 \times 10^2 \text{ W}$  when  $t = 0.10 \text{ s}$ .

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2} (1 - e^{-t/\tau_L})^2 R = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2.$$

At  $t = 0.10 \text{ s}$ , this yields  $P_{\text{thermal}} = 1.5 \times 10^2 \text{ W}$ .

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

63. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d(Li^2/2)}{dt} = Li \frac{di}{dt} = L \left( \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right) \left( \frac{\mathcal{E}}{R} \frac{1}{\tau_L} e^{-t/\tau_L} \right) = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L}$$

where  $\tau_L = L/R$  has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2} (1 - e^{-t/\tau_L})^2 R = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2.$$

We equate this to  $dU_B/dt$ , and solve for the time:

$$\frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L})^2 = \frac{\mathcal{E}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L} \Rightarrow t = \tau_L \ln 2 = (37.0 \text{ ms}) \ln 2 = 25.6 \text{ ms}.$$

64. Let  $U_B(t) = \frac{1}{2} Li^2(t)$ . We require the energy at time  $t$  to be half of its final value:  $U(t) = \frac{1}{2} U_B(t \rightarrow \infty) = \frac{1}{4} Li_f^2$ . This gives  $i(t) = i_f / \sqrt{2}$ . But  $i(t) = i_f (1 - e^{-t/\tau_L})$ , so

$$1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \Rightarrow \frac{t}{\tau_L} = -\ln \left( 1 - \frac{1}{\sqrt{2}} \right) = 1.23.$$

65. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\begin{aligned} \int_0^t P_{\text{battery}} dt &= \int_0^t \frac{\mathcal{E}^2}{R} (1 - e^{-Rt/L}) dt = \frac{\mathcal{E}^2}{R} \left[ t + \frac{L}{R} (e^{-Rt/L} - 1) \right] \\ &= \frac{(10.0 \text{ V})^2}{6.70 \, \Omega} \left[ 2.00 \text{ s} + \frac{(5.50 \text{ H}) (e^{-(6.70 \, \Omega)(2.00 \text{ s})/5.50 \text{ H}} - 1)}{6.70 \, \Omega} \right] \\ &= 18.7 \text{ J}. \end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B = \frac{1}{2} Li^2(t) = \frac{1}{2} L \left( \frac{\mathcal{E}}{R} \right)^2 (1 - e^{-Rt/L})^2 = \frac{1}{2} (5.50 \text{ H}) \left( \frac{10.0 \text{ V}}{6.70 \Omega} \right)^2 \left[ 1 - e^{-(6.70 \Omega)(2.00 \text{ s})/5.50 \text{ H}} \right]^2$$

$$= 5.10 \text{ J} .$$

(c) The difference of the previous two results gives the amount “lost” in the resistor:  
 $18.7 \text{ J} - 5.10 \text{ J} = 13.6 \text{ J} .$

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi \times 10^{-7} \text{ H/m})(100 \text{ A})}{2(50 \times 10^{-3} \text{ m})} = 1.3 \times 10^{-3} \text{ T} .$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3 \times 10^{-3} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ H/m})} = 0.63 \text{ J/m}^3 .$$

67. **THINK** The magnetic energy density is given by  $u_B = B^2/2\mu_0$ , where  $B$  is the magnitude of the magnetic field at that point.

**EXPRESS** Inside a solenoid, the magnitude of the magnetic field is  $B = \mu_0 ni$ , where

$$n = (950 \text{ turns})/(0.850 \text{ m}) = 1.118 \times 10^3 \text{ m}^{-1} .$$

Thus, the energy density is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(\mu_0 ni)^2}{2\mu_0} = \frac{1}{2} \mu_0 n^2 i^2 .$$

Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is  $U_B = u_B \mathcal{V}$ , where  $\mathcal{V}$  is the volume of the solenoid.

**ANALYZE** (a) Substituting the values given, we find the magnetic energy density to be

$$u_B = \frac{1}{2} \mu_0 n^2 i^2 = \frac{1}{2} (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (1.118 \times 10^3 \text{ m}^{-1})^2 (6.60 \text{ A})^2 = 34.2 \text{ J/m}^3 .$$

(b) The volume  $\mathcal{V}$  is calculated as the product of the cross-sectional area and the length. Thus,

$$U_B = (34.2 \text{ J/m}^3) (17.0 \times 10^{-4} \text{ m}^2) (0.850 \text{ m}) = 4.94 \times 10^{-2} \text{ J} .$$

**LEARN** Note the similarity between  $u_B = \frac{B^2}{2\mu_0}$ , the energy density at a point in a magnetic field, and  $u_E = \frac{1}{2}\epsilon_0 E^2$ , the energy density at a point in an electric field. Both quantities are proportional to the square of the fields.

68. The magnetic energy stored in the toroid is given by  $U_B = \frac{1}{2} Li^2$ , where  $L$  is its inductance and  $i$  is the current. By Eq. 30-54, the energy is also given by  $U_B = u_B \mathcal{V}$ , where  $u_B$  is the average energy density and  $\mathcal{V}$  is the volume. Thus

$$i = \sqrt{\frac{2u_B \mathcal{V}}{L}} = \sqrt{\frac{2(70.0 \text{ J/m}^3)(0.0200 \text{ m}^3)}{90.0 \times 10^{-3} \text{ H}}} = 5.58 \text{ A}.$$

69. We set  $u_E = \frac{1}{2}\epsilon_0 E^2 = u_B = \frac{1}{2} B^2 / \mu_0$  and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\epsilon_0 \mu_0}} = \frac{0.50 \text{ T}}{\sqrt{(8.85 \times 10^{-12} \text{ F/m})(4\pi \times 10^{-7} \text{ H/m})}} = 1.5 \times 10^8 \text{ V/m}.$$

70. It is important to note that the  $x$  that is used in the graph of Fig. 30-67(b) is not the  $x$  at which the energy density is being evaluated. The  $x$  in Fig. 30-67(b) is the location of wire 2. The energy density (Eq. 30-54) is being evaluated at the coordinate origin throughout this problem. We note the curve in Fig. 30-67(b) has a zero; this implies that the magnetic fields (caused by the individual currents) are in opposite directions (at the origin), which further implies that the currents have the same direction. Since the magnitudes of the fields must be equal (for them to cancel) when the  $x$  of Fig. 30-67(b) is equal to 0.20 m, then we have (using Eq. 29-4)  $B_1 = B_2$ , or

$$\frac{\mu_0 i_1}{2\pi d} = \frac{\mu_0 i_2}{2\pi(0.20 \text{ m})}$$

which leads to  $d = (0.20 \text{ m})/3$  once we substitute  $i_1 = i_2/3$  and simplify. We can also use the given fact that when the energy density is completely caused by  $B_1$  (this occurs when  $x$  becomes infinitely large because then  $B_2 = 0$ ) its value is  $u_B = 1.96 \times 10^{-9}$  (in SI units) in order to solve for  $B_1$ :

$$B_1 = \sqrt{2\mu_0 u_B}.$$

(a) This combined with  $B_1 = \mu_0 i_1 / 2\pi d$  allows us to find wire 1's current:  $i_1 \approx 23 \text{ mA}$ .

(b) Since  $i_2 = 3i_1$  then  $i_2 = 70 \text{ mA}$  (approximately).

71. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} \left( \frac{\mu_0 i}{2\pi R} \right)^2 = \frac{\mu_0 i^2}{8\pi^2 R^2} = \frac{(4\pi \times 10^{-7} \text{ H/m})(10 \text{ A})^2}{8\pi^2 (2.5 \times 10^{-3} \text{ m/2})^2} = 1.0 \text{ J/m}^3.$$

(b) The electric energy density is

$$\begin{aligned} u_E &= \frac{1}{2} \epsilon_0 E^2 = \frac{\epsilon_0}{2} (\rho J)^2 = \frac{\epsilon_0}{2} \left( \frac{iR}{\ell} \right)^2 = \frac{1}{2} (8.85 \times 10^{-12} \text{ F/m}) [(10 \text{ A})(3.3 \Omega / 10^3 \text{ m})]^2 \\ &= 4.8 \times 10^{-15} \text{ J/m}^3. \end{aligned}$$

Here we used  $J = i/A$  and  $R = \rho \ell / A$  to obtain  $\rho J = iR/\ell$ .

72. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25 \text{ mH})(6.0 \text{ mA})}{100} = 1.5 \mu \text{ Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1 \frac{di_1}{dt} = (25 \text{ mH})(4.0 \text{ A/s}) = 1.0 \times 10^2 \text{ mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{M i_1}{N_2} = \frac{(3.0 \text{ mH})(6.0 \text{ mA})}{200} = 90 \text{ nWb}.$$

(d) The mutually induced emf is

$$\epsilon_{21} = M \frac{di_1}{dt} = (3.0 \text{ mH})(4.0 \text{ A/s}) = 12 \text{ mV}.$$

73. **THINK** If two coils are near each other, mutual induction can take place whereby a changing current in one coil can induce an emf in the other.

**EXPRESS** The mutual inductance is given by

$$\epsilon_1 = -M \frac{di_2}{dt}$$

where  $\epsilon_1$  is the induced emf in coil 1 due to the changing current in coil 2. The flux linkage in coil 2 is  $N_2 \Phi_{21} = M i_1$ .

**ANALYZE** (a) From the equation above, we find the mutual inductance to be

$$M = \frac{|\varepsilon_1|}{di_2/dt} = \frac{25.0 \text{ mV}}{15.0 \text{ A/s}} = 1.67 \text{ mH}.$$

(b) Similarly, the flux linkage in coil 2 is

$$N_2 \Phi_{21} = M i_1 = (1.67 \text{ mH})(3.60 \text{ A}) = 6.00 \text{ mWb}.$$

**LEARN** The emf induced in one coil is proportional to the rate at which current in the other coil is changing:

$$\varepsilon_1 = -M_{12} \frac{di_2}{dt}, \quad \varepsilon_2 = -M_{21} \frac{di_1}{dt}.$$

The proportionality constants,  $M_{12}$  and  $M_{21}$ , are the same,  $M_{12} = M_{21} = M$ , so we simply write

$$\varepsilon_1 = -M \frac{di_2}{dt}, \quad \varepsilon_2 = -M \frac{di_1}{dt}.$$

74. We use  $\varepsilon_2 = -M di_1/dt \approx M|\Delta i/\Delta t|$  to find  $M$ :

$$M = \left| \frac{\varepsilon}{\Delta i_1/\Delta t} \right| = \frac{30 \times 10^3 \text{ V}}{6.0 \text{ A}/(2.5 \times 10^{-3} \text{ s})} = 13 \text{ H}.$$

75. The flux over the loop cross section due to the current  $i$  in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} \ln \left( 1 + \frac{b}{a} \right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln \left( 1 + \frac{b}{a} \right).$$

From the formula for  $M$  obtained above, we have

$$M = \frac{(100)(4\pi \times 10^{-7} \text{ H/m})(0.30 \text{ m})}{2\pi} \ln \left( 1 + \frac{8.0}{1.0} \right) = 1.3 \times 10^{-5} \text{ H}.$$

76. (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N(\mu_0 i_s n \pi R^2)}{i_s} = \mu_0 \pi R^2 n N .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross section of the coil we have  $\Phi_{sc} = B_s A_s = B_s \pi R^2$ , regardless of the shape, size, or possible lack of close-packing of the coil.

**77. THINK** To find the equivalent inductance, we calculate the total emf across both coils.

**EXPRESS** We assume the current to be changing at (nonzero) a rate  $di/dt$ . The induced emf's can take on the following form:

$$\varepsilon_1 = -(L_1 \pm M) \frac{di}{dt}, \quad \varepsilon_2 = -(L_2 \pm M) \frac{di}{dt}$$

The relative sign between  $L$  and  $M$  depends on how the coils are connected, as we shall see below.

**ANALYZE** (a) The connection is shown in Fig. 30-70. First consider coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emfs in the same direction. Thus, the induced emfs are

$$\varepsilon_1 = -(L_1 + M) \frac{di}{dt}, \quad \varepsilon_2 = -(L_2 + M) \frac{di}{dt} .$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -(L_1 + L_2 + 2M) \frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance  $L_{eq} = L_1 + L_2 + 2M$ .

(b) We imagine reversing the leads of coil 2 so the current enters at the back of the coil rather than the front (as pictured in Fig. 30-70). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -(L_1 - M) \frac{di}{dt} .$$

Similarly, the emf across coil 2 is



$$\mathcal{E}_2 = -(L_2 - M) \frac{di}{dt}.$$

The total emf across both coils is

$$\mathcal{E} = -(L_1 + L_2 - 2M) \frac{di}{dt}.$$

This is the same as the emf that would be produced by a single coil with inductance

$$L_{\text{eq}} = L_1 + L_2 - 2M.$$

**LEARN** The sign of the mutual inductance term is determined by the senses of the coil winding. The induced emfs can either reinforce one another ( $L + M$ ), or oppose one another ( $L - M$ ).

78. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt} = \frac{d}{dt} \left[ \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right] = \frac{\mathcal{E}}{R\tau_L} e^{-t/\tau_L} = \frac{\mathcal{E}}{L} e^{-t/\tau_L}.$$

With  $\tau_L = L/R$  (Eq. 30-42),  $L = 0.023 \text{ H}$  and  $\mathcal{E} = 12 \text{ V}$ ,  $t = 0.00015 \text{ s}$ , and  $di/dt = 280 \text{ A/s}$ , we obtain  $e^{-t/\tau_L} = 0.537$ . Taking the natural log and rearranging leads to  $R = 95.4 \Omega$ .

79. **THINK** The inductor in the  $RL$  circuit initially acts to oppose changes in current through it.

**EXPRESS** When the switch  $S$  is just closed,  $V_1 = \mathcal{E}$  and no current flows through the inductor. A long time later, the currents have reached their equilibrium values and the inductor acts as an ordinary connecting wire; we can solve the multi-loop circuit problem by applying Kirchhoff's junction and loop rules.

**ANALYZE** (a) Applying the loop rule to the left loop gives  $\mathcal{E} - i_1 R_1 = 0$ , so

$$i_1 = \mathcal{E}/R_1 = 10 \text{ V}/5.0 \Omega = 2.0 \text{ A}.$$

(b) Since now  $\mathcal{E}_L = \mathcal{E}$ , we have  $i_2 = 0$ .

(c) The junction rule gives  $i_s = i_1 + i_2 = 2.0 \text{ A} + 0 = 2.0 \text{ A}$ .

(d) Since  $V_L = \mathcal{E}$ , the potential difference across resistor 2 is  $V_2 = \mathcal{E} - \mathcal{E}_L = 0$ .

(e) The potential difference across the inductor is  $V_L = \mathcal{E} = 10 \text{ V}$ .

(f) The rate of change of current is  $\frac{di_2}{dt} = \frac{V_L}{L} = \frac{\mathcal{E}}{L} = \frac{10 \text{ V}}{5.0 \text{ H}} = 2.0 \text{ A/s}$ .

- (g) After a long time, we still have  $V_1 = \mathcal{E}$ , so  $i_1 = 2.0$  A.
- (h) Since now  $V_L = 0$ ,  $i_2 = \mathcal{E}/R_2 = 10 \text{ V}/10 \Omega = 1.0$  A.
- (i) The current through the switch is now  $i_s = i_1 + i_2 = 2.0 \text{ A} + 1.0 \text{ A} = 3.0$  A.
- (j) Since  $V_L = 0$ ,  $V_2 = \mathcal{E} - V_L = \mathcal{E} = 10$  V.
- (k) With the inductor acting as an ordinary connecting wire, we have  $V_L = 0$ .
- (l) The rate of change of current in resistor 2 is  $\frac{di_2}{dt} = \frac{V_L}{L} = 0$ .

**LEARN** In analyzing an  $RL$  circuit immediately after closing the switch and a very long time after that, there is no need to solve any differential equation.

80. Using Eq. 30-41:  $i = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L})$ , where  $\tau_L = 2.0$  ns, we find

$$t = \tau_L \ln \left( \frac{1}{1 - iR/\mathcal{E}} \right) \approx 1.0 \text{ ns}.$$

81. Using Ohm's law, we relate the induced current to the emf and (the absolute value of) Faraday's law:

$$i = \frac{|\mathcal{E}|}{R} = \frac{1}{R} \left| \frac{d\Phi}{dt} \right|.$$

As the loop is crossing the boundary between regions 1 and 2 (so that “ $x$ ” amount of its length is in region 2 while “ $D - x$ ” amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}H(B_2 - B_1) = vH(B_2 - B_1) \Rightarrow i = vH(B_2 - B_1)/R.$$

Similar considerations hold (replacing “ $B_1$ ” with 0 and “ $B_2$ ” with  $B_1$ ) for the loop crossing initially from the zero-field region (to the left of Fig. 30-72(a)) into region 1.

(a) In this latter case, appeal to Fig. 30-72(b) leads to

$$3.0 \times 10^{-6} \text{ A} = (0.40 \text{ m/s})(0.015 \text{ m}) B_1 / (0.020 \Omega)$$

which yields  $B_1 = 10 \mu\text{T}$ .

(b) Lenz's law considerations lead us to conclude that the direction of the region 1 field is *out of the page*.

(c) Similarly,  $i = \nu H(B_2 - B_1)/R$  leads to  $B_2 = 3.3 \mu\text{T}$ .

(d) The direction of  $\vec{B}_2$  is out of the page.

82. Faraday's law (for a single turn, with  $B$  changing in time) gives

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}.$$

In this problem, we find  $\frac{dB}{dt} = -\frac{B_0}{\tau} e^{-t/\tau}$ . Thus,  $\mathcal{E} = \pi r^2 \frac{B_0}{\tau} e^{-t/\tau}$ .

83. Equation 30-41 applies, and the problem requires

$$iR = L \frac{di}{dt} = \mathcal{E} - iR$$

at some time  $t$  (where Eq. 30-39 has been used in that last step). Thus, we have  $2iR = \mathcal{E}$ , or

$$\mathcal{E} = 2iR = 2 \left[ \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right] R = 2\mathcal{E} (1 - e^{-t/\tau_L})$$

where Eq. 30-42 gives the inductive time constant as  $\tau_L = L/R$ . We note that the emf  $\mathcal{E}$  cancels out of that final equation, and we are able to rearrange (and take the natural log) and solve. We obtain  $t = 0.520$  ms.

84. In absolute value, Faraday's law (for a single turn, with  $B$  changing in time) gives

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A \frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

for the magnitude of the induced emf. Dividing it by  $R^2$  then allows us to relate this to the slope of the graph in Fig. 30-73(b) [particularly the first part of the graph], which we estimate to be  $80 \mu\text{V}/\text{m}^2$ .

(a) Thus,  $\frac{dB_1}{dt} = (80 \mu\text{V}/\text{m}^2)/\pi \approx 25 \mu\text{T}/\text{s}$ .

(b) Similar reasoning for region 2 (corresponding to the slope of the second part of the graph in Fig. 30-73(b)) leads to an emf equal to

$$\pi r_1^2 \left( \frac{dB_1}{dt} - \frac{dB_2}{dt} \right) + \pi R^2 \frac{dB_2}{dt}$$

which means the second slope (which we estimate to be  $40 \mu\text{V}/\text{m}^2$ ) is equal to  $\pi \frac{dB_2}{dt}$ .

Therefore,  $\frac{dB_2}{dt} = (40 \mu\text{V}/\text{m}^2)/\pi \approx 13 \mu\text{T}/\text{s}$ .

(c) Considerations of Lenz's law leads to the conclusion that  $B_2$  is increasing.

85. **THINK** Changing magnetic field induces an electric field.

**EXPRESS** The induced electric field is given by Eq. 30-20:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}.$$

The electric field lines are circles that are concentric with the cylindrical region. Thus,

$$E(2\pi r) = -(\pi r^2) \frac{dB}{dt} \Rightarrow E = -\frac{1}{2} \frac{dB}{dt} r.$$

The force on the electron is  $\vec{F} = -e\vec{E}$ , so by Newton's second law, the acceleration is  $\vec{a} = -e\vec{E}/m$ .

**ANALYZE** (a) At point  $a$ ,

$$E = -\frac{r}{2} \left( \frac{dB}{dt} \right) = -\frac{1}{2} (5.0 \times 10^{-2} \text{ m}) (-10 \times 10^{-3} \text{ T/s}) = 2.5 \times 10^{-4} \text{ V/m}.$$

With the normal taken to be into the page, in the direction of the magnetic field, the positive direction for  $\vec{E}$  is clockwise. Thus, the direction of the electric field at point  $a$  is to the left, that is  $\vec{E} = -(2.5 \times 10^{-4} \text{ V/m})\hat{i}$ . The resulting acceleration is

$$\vec{a}_a = \frac{-e\vec{E}}{m} = \frac{(-1.60 \times 10^{-19} \text{ C})(-2.5 \times 10^{-4} \text{ V/m})}{9.11 \times 10^{-31} \text{ kg}} \hat{i} = (4.4 \times 10^7 \text{ m/s}^2)\hat{i}.$$

The acceleration is to the right.

(b) At point  $b$  we have  $r_b = 0$ , so the acceleration is zero.

(c) The electric field at point  $c$  has the same magnitude as the field in  $a$ , but with its direction reversed. Thus, the acceleration of the electron released at point  $c$  is

$$\vec{a}_c = -\vec{a}_a = -(4.4 \times 10^7 \text{ m/s}^2) \hat{i}.$$

**LEARN** Inside the cylindrical region, the induced electric field increases with  $r$ . Therefore, the greater the value of  $r$ , the greater the magnitude of acceleration.

86. Because of the decay of current (Eq. 30-45) that occurs after the switches are closed on  $B$ , the flux will decay according to

$$\Phi_1 = \Phi_{10} e^{-t/\tau_{L1}}, \quad \Phi_2 = \Phi_{20} e^{-t/\tau_{L2}}$$

where each time constant is given by Eq. 30-42. Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{20}/\Phi_{10})}{(R_2/L_2) - (R_1/L_1)} = \frac{\ln(1.50)}{(30.0 \, \Omega/0.0030 \text{ H}) - (25 \, \Omega/0.0050 \text{ H})} = 81.1 \, \mu\text{s}.$$

87. **THINK** Changing the area of the loop changes the flux through it. An induced emf is produced to oppose this change.

**EXPRESS** The magnetic flux through the loop is  $\Phi_B = BA$ , where  $B$  is the magnitude of the magnetic field and  $A$  is the area of the loop. According to Faraday's law, the magnitude of the average induced emf is

$$\mathcal{E}_{\text{avg}} = \left| \frac{-d\Phi_B}{dt} \right| = \left| \frac{\Delta\Phi_B}{\Delta t} \right| = \frac{B|\Delta A|}{\Delta t}.$$

**ANALYZE** (a) substituting the values given, we obtain

$$\mathcal{E}_{\text{avg}} = \frac{B|\Delta A|}{\Delta t} = \frac{(2.0 \text{ T})(0.20 \text{ m})^2}{0.20 \text{ s}} = 0.40 \text{ V}.$$

(b) The average induced current is  $i_{\text{avg}} = \frac{\mathcal{E}_{\text{avg}}}{R} = \frac{0.40 \text{ V}}{20 \times 10^{-3} \, \Omega} = 20 \text{ A}.$

**LEARN** By Lenz's law, the more rapidly the area is changing, the greater the induced current in

88. (a) From Eq. 30-28, we have

$$L = \frac{N\Phi}{i} = \frac{(150)(50 \times 10^{-9} \text{ T} \cdot \text{m}^2)}{2.00 \times 10^{-3} \text{ A}} = 3.75 \text{ mH}.$$

(b) The answer for  $L$  (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is  $2(50) = 100 \text{ nWb}$ .

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L \left. \frac{di}{dt} \right|_{\text{max}} = (0.00375 \text{ H})(0.0030 \text{ A})(377 \text{ rad/s}) = 4.24 \times 10^{-3} \text{ V}.$$

89. (a)  $i_0 = \mathcal{E}/R = 100 \text{ V}/10 \Omega = 10 \text{ A}$ .

(b)  $U_B = \frac{1}{2} Li_0^2 = \frac{1}{2} (2.0 \text{ H})(10 \text{ A})^2 = 1.0 \times 10^2 \text{ J}$ .

90. We write  $i = i_0 e^{-t/\tau_L}$  and note that  $i = 10\% i_0$ . We solve for  $t$ :

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R} \ln\left(\frac{i_0}{i}\right) = \frac{2.00 \text{ H}}{3.00 \Omega} \ln\left(\frac{i_0}{0.100 i_0}\right) = 1.54 \text{ s}.$$

91. **THINK** We have an  $RL$  circuit in which the inductor is in series with the battery.

**EXPRESS** As the switch closes at  $t = 0$ , the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit.

**ANALYZE** (a) At  $t = 0$ , the current through the battery is also zero.

(b) With no current anywhere in the circuit at  $t = 0$ , the loop rule requires the emf of the inductor  $\mathcal{E}_L$  to cancel that of the battery ( $\mathcal{E} = 40 \text{ V}$ ). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{40 \text{ V}}{0.050 \text{ H}} = 8.0 \times 10^2 \text{ A/s}.$$

(c) This circuit becomes equivalent to that analyzed in Section 30-9 when we replace the parallel set of  $20000 \Omega$  resistors with  $R = 10000 \Omega$ . Now, with  $\tau_L = L/R = 5 \times 10^{-6} \text{ s}$ , we have  $t/\tau_L = 3/5$ , and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\mathcal{E}}{R} (1 - e^{-3/5}) \approx 1.8 \times 10^{-3} \text{ A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\mathcal{E} - i_{\text{bat}} R - |\mathcal{E}_L| = 0.$$

Using the values from part (c), we obtain  $|\mathcal{E}_L| \approx 22 \text{ V}$ . Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{22 \text{ V}}{0.050 \text{ H}} \approx 4.4 \times 10^2 \text{ A/s}.$$

(e) As  $t \rightarrow \infty$ , the circuit reaches a steady-state condition, so that  $di_{\text{bat}}/dt = 0$  and  $\mathcal{E}_L = 0$ . The loop rule then leads to

$$\mathcal{E} - i_{\text{bat}} R - |\mathcal{E}_L| = 0 \Rightarrow i_{\text{bat}} = \frac{40 \text{ V}}{10000 \Omega} = 4.0 \times 10^{-3} \text{ A}.$$

(f) As  $t \rightarrow \infty$ , the circuit reaches a steady-state condition,  $di_{\text{bat}}/dt = 0$ .

**LEARN** In summary, at  $t = 0$  immediately after the switch is closed, the inductor opposes any change in current, and with the inductor and the battery being connected in series, the induced emf in the inductor is equal to the emf of the battery,  $\mathcal{E}_L = \mathcal{E}$ . A long time later after all the currents have reached their steady-state values,  $\mathcal{E}_L = 0$ , and the inductor can be treated as an ordinary connecting wire. In this limit, the circuit can be analyzed as if  $L$  were not present.

92. (a)  $L = \Phi/i = 26 \times 10^{-3} \text{ Wb}/5.5 \text{ A} = 4.7 \times 10^{-3} \text{ H}.$

(b) We use Eq. 30-41 to solve for  $t$ :

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{iR}{\mathcal{E}}\right) = -\frac{L}{R} \ln\left(1 - \frac{iR}{\mathcal{E}}\right) = -\frac{4.7 \times 10^{-3} \text{ H}}{0.75 \Omega} \ln\left[1 - \frac{(2.5 \text{ A})(0.75 \Omega)}{6.0 \text{ V}}\right] \\ &= 2.4 \times 10^{-3} \text{ s}. \end{aligned}$$

93. The energy stored when the current is  $i$  is  $U_B = \frac{1}{2} Li^2$ , where  $L$  is the self-inductance.

The rate at which this is developed is

$$\frac{dU_B}{dt} = Li \frac{di}{dt}$$

where  $i$  is given by Eq. 30-41 and  $di/dt$  is obtained by taking the derivative of that equation (or by using Eq. 30-37). Thus, using the symbol  $V$  to stand for the battery voltage (12.0 volts) and  $R$  for the resistance (20.0  $\Omega$ ), we have, at  $t = 1.61\tau_L$ ,

$$\frac{dU_B}{dt} = \frac{V^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L} = \frac{(12.0 \text{ V})^2}{20.0 \Omega} (1 - e^{-1.61}) e^{-1.61} = 1.15 \text{ W}.$$

94. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = (4\pi \times 10^{-7} \text{ H/m}) (100 \text{ turns/cm})^2 (\pi) (1.6 \text{ cm})^2 = 0.10 \text{ H/m}.$$

(b) The induced emf per meter is

$$\frac{\mathcal{E}}{\ell} = \frac{L}{\ell} \frac{di}{dt} = (0.10 \text{ H/m}) (13 \text{ A/s}) = 1.3 \text{ V/m}.$$

95. (a) As the switch closes at  $t = 0$ , the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf ( $\mathcal{E}_{L1}$ ) of the  $L_1 = 0.30 \text{ H}$  inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt} = \frac{|\mathcal{E}_{L1}|}{L_1} = \frac{6.0}{0.30} = 20 \text{ A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as  $t \rightarrow \infty$ ). Applying the loop rule to the outer loop, with  $R_1 = 8.0 \Omega$ , we have

$$\mathcal{E} - iR_1 - |\mathcal{E}_{L1}| - |\mathcal{E}_{L2}| = 0 \Rightarrow i = \frac{6.0 \text{ V}}{R_1} = 0.75 \text{ A}.$$

96. Since  $A = \ell^2$ , we have  $dA/dt = 2\ell d\ell/dt$ . Thus, Faraday's law, with  $N = 1$ , becomes

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B \frac{dA}{dt} = -2\ell B \frac{d\ell}{dt}$$

which yields  $\mathcal{E} = 0.0029 \text{ V}$ .

97. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt} = \frac{d}{dt} \left[ \frac{\mathcal{E}}{R} (1 - e^{-t/\tau_L}) \right] = \frac{\mathcal{E}}{R\tau_L} e^{-t/\tau_L} = \frac{\mathcal{E}}{L} e^{-t/\tau_L}$$



With  $\tau_L = 0.28$  ms (by Eq. 30-42),  $L = 0.050$  H, and  $\mathcal{E} = 45$  V, we obtain  $di/dt = 12$  A/s when  $t = 1.2$  ms.

98. (a) From Eq. 30-35, we find  $L = (3.00 \text{ mV})/(5.00 \text{ A/s}) = 0.600 \text{ mH}$ .

(b) Since  $N\Phi = iL$  (where  $\Phi = 40.0 \mu\text{Wb}$  and  $i = 8.00$  A), we obtain  $N = 120$ .

99. We use  $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$ , and use the symbol  $\mathcal{V}$  for volume.

$$U_B = \mathcal{V}u_B = \frac{\mathcal{V}B^2}{2\mu_0} = \frac{(9.46 \times 10^{15} \text{ m})^3 (1 \times 10^{-10} \text{ T})^2}{2(4\pi \times 10^{-7} \text{ H/m})} = 3 \times 10^{36} \text{ J}.$$

100. (a) The total length of the closed loop formed by the two radii plus the arc is

$$L = 2r + r\theta = r(2 + \theta),$$

where  $r$  is the radius. The total resistance is

$$\begin{aligned} R &= \frac{\rho L}{A} = \frac{\rho r(2 + \theta)}{A} = \frac{(1.7 \times 10^{-8} \Omega \cdot \text{m})(0.24 \text{ m})(2 + \theta)}{1.20 \times 10^{-6} \text{ m}^2} \\ &= (3.4 \times 10^{-3})(2 + \theta) \Omega. \end{aligned}$$

(b) The area of the loop is  $A = \frac{1}{2}r^2\theta$ . Thus, the magnetic flux through the loop is

$$\Phi_B = BA = \frac{1}{2}Br^2\theta = \frac{1}{2}(0.150 \text{ T})(0.240 \text{ m})^2\theta = (4.32 \times 10^{-3}\theta) \text{ Wb}.$$

(c) The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Br^2\theta\right) = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega$$

which gives

$$i = \frac{|\mathcal{E}|}{R} = \frac{Br^2\omega}{2R} = \frac{Br^2\omega}{2(3.4 \times 10^{-3})(2 + \theta)} = \frac{Br^2\alpha t}{2(3.4 \times 10^{-3})(2 + \alpha t^2/2)}$$

as the magnitude of the induced current. Note that in the last step, we have substituted  $\omega = \alpha t$  and  $\theta = \frac{1}{2}\alpha t^2$ , for constant angular acceleration  $\alpha$ . Differentiating  $i$  with respect to  $t$  gives

$$\frac{di}{dt} = \frac{Br^2\alpha(4 - \alpha t^2)}{(3.4 \times 10^{-3})(4 + \alpha t^2)^2}.$$

The induced current is at a maximum when  $4 - \alpha t^2 = 0$ , or  $t = \sqrt{4/\alpha}$ . At this instant, the angle is

$$\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha \left( \frac{4}{\alpha} \right) = 2.0 \text{ rad.}$$

(d) When current is at a maximum,  $\omega = \alpha t = \alpha \sqrt{4/\alpha} = \sqrt{4\alpha}$ . Thus,

$$i_{\max} = \frac{Br^2\omega}{2R} = \frac{Br^2\sqrt{4\alpha}}{2R} = \frac{Br^2\sqrt{4\alpha}}{2(3.4 \times 10^{-3})(2 + \theta)} = \frac{(0.150 \text{ T})(0.24 \text{ m})^2 \sqrt{4(12 \text{ rad/s}^2)}}{2(3.4 \times 10^{-3})(2 + 2.0)} = 2.20 \text{ A.}$$

101. (a) We use  $U_B = \frac{1}{2} Li^2$  to solve for the self-inductance:

$$L = \frac{2U_B}{i^2} = \frac{2(25.0 \times 10^{-3} \text{ J})}{(60.0 \times 10^{-3} \text{ A})^2} = 13.9 \text{ H.}$$

(b) Since  $U_B \propto i^2$ , for  $U_B$  to increase by a factor of 4,  $i$  must increase by a factor of 2. Therefore,  $i$  should be increased to  $2(60.0 \text{ mA}) = 120 \text{ mA}$ .