

Chapter 36

1. (a) We use Eq. 36-3 to calculate the separation between the first ($m_1 = 1$) and fifth ($m_2 = 5$) minima:

$$\Delta y = D \Delta \sin \theta = D \Delta \left(\frac{m\lambda}{a} \right) = \frac{D\lambda}{a} \Delta m = \frac{D\lambda}{a} (m_2 - m_1) .$$

Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400 \text{ mm})(550 \times 10^{-6} \text{ mm})(5 - 1)}{0.35 \text{ mm}} = 2.5 \text{ mm} .$$

- (b) For $m = 1$,

$$\sin \theta = \frac{m\lambda}{a} = \frac{(1)(550 \times 10^{-6} \text{ mm})}{2.5 \text{ mm}} = 2.2 \times 10^{-4} .$$

The angle is $\theta = \sin^{-1} (2.2 \times 10^{-4}) = 2.2 \times 10^{-4} \text{ rad}$.

2. From Eq. 36-3,

$$\frac{a}{\lambda} = \frac{m}{\sin \theta} = \frac{1}{\sin 45.0^\circ} = 1.41 .$$

3. (a) A plane wave is incident on the lens so it is brought to focus in the focal plane of the lens, a distance of 70 cm from the lens.

(b) Waves leaving the lens at an angle θ to the forward direction interfere to produce an intensity minimum if $a \sin \theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The distance on the screen from the center of the pattern to the minimum is given by $y = D \tan \theta$, where D is the distance from the lens to the screen. For the conditions of this problem,

$$\sin \theta = \frac{m\lambda}{a} = \frac{(1)(590 \times 10^{-9} \text{ m})}{0.40 \times 10^{-3} \text{ m}} = 1.475 \times 10^{-3} .$$

This means $\theta = 1.475 \times 10^{-3} \text{ rad}$ and

$$y = (0.70 \text{ m}) \tan(1.475 \times 10^{-3} \text{ rad}) = 1.0 \times 10^{-3} \text{ m} .$$

4. (a) Equations 36-3 and 36-12 imply smaller angles for diffraction for smaller wavelengths. This suggests that diffraction effects in general would decrease.

(b) Using Eq. 36-3 with $m = 1$ and solving for 2θ (the angular width of the central diffraction maximum), we find

$$2\theta = 2 \sin^{-1} \left(\frac{\lambda}{a} \right) = 2 \sin^{-1} \left(\frac{0.50 \text{ m}}{5.0 \text{ m}} \right) = 11^\circ.$$

(c) A similar calculation yields 0.23° for $\lambda = 0.010 \text{ m}$.

5. (a) The condition for a minimum in a single-slit diffraction pattern is given by

$$a \sin \theta = m\lambda,$$

where a is the slit width, λ is the wavelength, and m is an integer. For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus,

$$\lambda_a = 2\lambda_b = 2(350 \text{ nm}) = 700 \text{ nm}.$$

(b) Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a , and let m_b be the integer associated with a minimum in the pattern produced by light with wavelength λ_b . A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means $m_a\lambda_a = m_b\lambda_b$. Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$. Consequently, every other minimum of the λ_b pattern coincides with a minimum of the λ_a pattern. With $m_a = 2$, we have $m_b = 4$.

(c) With $m_a = 3$, we have $m_b = 6$.

6. (a) $\theta = \sin^{-1} (1.50 \text{ cm}/2.00 \text{ m}) = 0.430^\circ$.

(b) For the m th diffraction minimum, $a \sin \theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin \theta} = \frac{2(441 \text{ nm})}{\sin 0.430^\circ} = 0.118 \text{ mm}.$$

7. The condition for a minimum of a single-slit diffraction pattern is

$$a \sin \theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 0.60° for $m = 1$. Thus,

$$a = \frac{m\lambda}{\sin \theta} = \frac{633 \times 10^{-9} \text{ m}}{\sin 0.60^\circ} = 6.04 \times 10^{-5} \text{ m}.$$

8. Let the first minimum be a distance y from the central axis that is perpendicular to the speaker. Then

$$\sin \theta = y / (D^2 + y^2)^{1/2} = m\lambda / a = \lambda / a \quad (\text{for } m = 1).$$

Therefore,

$$y = \frac{D}{\sqrt{(a/\lambda)^2 - 1}} = \frac{D}{\sqrt{(af/v_s)^2 - 1}} = \frac{100 \text{ m}}{\sqrt{[(0.300 \text{ m})(3000 \text{ Hz})/(343 \text{ m/s})]^2 - 1}} = 41.2 \text{ m}.$$

9. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a \sin \theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer.

EXPRESS To find the angular position of the first minimum to one side of the central maximum, we set $m = 1$:

$$\theta_1 = \sin^{-1} \left(\frac{\lambda}{a} \right) = \sin^{-1} \left(\frac{589 \times 10^{-9} \text{ m}}{1.00 \times 10^{-3} \text{ m}} \right) = 5.89 \times 10^{-4} \text{ rad}.$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is

$$y_1 = D \tan \theta_1 = (3.00 \text{ m}) \tan(5.89 \times 10^{-4} \text{ rad}) = 1.767 \times 10^{-3} \text{ m}.$$

To find the second minimum, we set $m = 2$:

$$\theta_2 = \sin^{-1} \left(\frac{2(589 \times 10^{-9} \text{ m})}{1.00 \times 10^{-3} \text{ m}} \right) = 1.178 \times 10^{-3} \text{ rad}.$$

ANALYZE The distance from the center of the pattern to this second minimum is

$$y_2 = D \tan \theta_2 = (3.00 \text{ m}) \tan(1.178 \times 10^{-3} \text{ rad}) = 3.534 \times 10^{-3} \text{ m}.$$

The separation of the two minima is

$$\Delta y = y_2 - y_1 = 3.534 \text{ mm} - 1.767 \text{ mm} = 1.77 \text{ mm}.$$

LEARN The angles θ_1 and θ_2 found above are quite small. In the small-angle approximation, $\sin \theta \approx \tan \theta \approx \theta$, and the separation between two adjacent diffraction minima can be approximated as

$$\Delta y = D(\tan \theta_{m+1} - \tan \theta_m) \approx D(\theta_{m+1} - \theta_m) = \frac{D\lambda}{a}.$$

10. From $y = m\lambda L/a$ we get

$$\Delta y = \Delta \left(\frac{m\lambda L}{a} \right) = \frac{\lambda L}{a} \Delta m = \frac{(632.8 \text{ nm})(2.60)}{1.37 \text{ mm}} [10 - (-10)] = 24.0 \text{ mm} .$$

11. We note that $1 \text{ nm} = 1 \times 10^{-9} \text{ m} = 1 \times 10^{-6} \text{ mm}$. From Eq. 36-4,

$$\Delta \phi = \left(\frac{2\pi}{\lambda} \right) (\Delta x \sin \theta) = \left(\frac{2\pi}{589 \times 10^{-6} \text{ mm}} \right) \left(\frac{0.10 \text{ mm}}{2} \right) \sin 30^\circ = 266.7 \text{ rad} .$$

This is equivalent to $266.7 \text{ rad} - 84\pi = 2.8 \text{ rad} = 160^\circ$.

12. (a) The slope of the plotted line is 12, and we see from Eq. 36-6 that this slope should correspond to

$$\frac{\pi a}{\lambda} = 12 \Rightarrow a = \frac{12\lambda}{\pi} = \frac{12(610 \text{ nm})}{\pi} = 2330 \text{ nm} \approx 2.33 \mu\text{m}$$

(b) Consider Eq. 36-3 with “continuously variable” m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m_{\text{max}} = \frac{a}{\lambda} (\sin \theta)_{\text{max}} = \frac{a}{\lambda} = \frac{2330 \text{ nm}}{610 \text{ nm}} \approx 3.82$$

which suggests that, on each side of the central maximum ($\theta_{\text{centr}} = 0$), there are three minima; considering both sides then implies there are six minima in the pattern.

(c) Setting $m = 1$ in Eq. 36-3 and solving for θ yields 15.2° .

(d) Setting $m = 3$ in Eq. 36-3 and solving for θ yields 51.8° .

13. (a) $\theta = \sin^{-1} (0.011 \text{ m}/3.5 \text{ m}) = 0.18^\circ$.

(b) We use Eq. 36-6:

$$\alpha = \left(\frac{\pi a}{\lambda} \right) \sin \theta = \frac{\pi (0.025 \text{ mm}) \sin 0.18^\circ}{538 \times 10^{-6} \text{ mm}} = 0.46 \text{ rad} .$$

(c) Making sure our calculator is in radian mode, Eq. 36-5 yields

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin \alpha}{\alpha} \right)^2 = 0.93 .$$

14. We will make use of arctangents and sines in our solution, even though they can be “shortcut” somewhat since the angles are small enough to justify the use of the small angle approximation.

(a) Given $y/D = 15/300$ (both expressed here in centimeters), then $\theta = \tan^{-1}(y/D) = 2.86^\circ$. Use of Eq. 36-6 (with $a = 6000$ nm and $\lambda = 500$ nm) leads to

$$\alpha = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi (6000 \text{ nm}) \sin 2.86^\circ}{500 \text{ nm}} = 1.883 \text{ rad}.$$

Thus,

$$\frac{I_p}{I_m} = \left(\frac{\sin \alpha}{\alpha} \right)^2 = 0.256.$$

(b) Consider Eq. 36-3 with “continuously variable” m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a \sin \theta}{\lambda} = \frac{(6000 \text{ nm}) \sin 2.86^\circ}{500 \text{ nm}} \approx 0.60,$$

which suggests that the angle takes us to a point between the central maximum ($\theta_{\text{centr}} = 0$) and the first minimum (which corresponds to $m = 1$ in Eq. 36-3).

15. **THINK** The relative intensity in a single-slit diffraction depends on the ratio a/λ , where a is the slit width and λ is the wavelength.

EXPRESS The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2 \alpha}{\alpha^2}$$

where I_m is the maximum intensity and $\alpha = (\pi a/\lambda) \sin \theta$. The angle θ is measured from the forward direction.

ANALYZE (a) We require $I = I_m/2$, so

$$\sin^2 \alpha = \frac{1}{2} \alpha^2.$$

(b) We evaluate $\sin^2 \alpha$ and $\alpha^2/2$ for $\alpha = 1.39$ rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda) \sin \theta$,

$$\theta = \sin^{-1} \left(\frac{\alpha \lambda}{\pi a} \right).$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1} \left(\frac{0.442\lambda}{a} \right).$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2 \sin^{-1} \left(\frac{0.442\lambda}{a} \right).$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2 \sin^{-1} (0.442/1.0) = 0.916 \text{ rad} = 52.5^\circ.$$

(e) For $a/\lambda = 5.0$,

$$\Delta\theta = 2 \sin^{-1} (0.442/5.0) = 0.177 \text{ rad} = 10.1^\circ.$$

(f) For $a/\lambda = 10$,

$$\Delta\theta = 2 \sin^{-1} (0.442/10) = 0.0884 \text{ rad} = 5.06^\circ.$$

LEARN As shown in Fig. 36-8, the wider the slit is (relative to the wavelength), the narrower is the central diffraction maximum.

16. Consider Huygens' explanation of diffraction phenomena. When A is in place only the Huygens' wavelets that pass through the hole get to point P . Suppose they produce a resultant electric field E_A . When B is in place, the light that was blocked by A gets to P and the light that passed through the hole in A is blocked. Suppose the electric field at P is now \vec{E}_B . The sum $\vec{E}_A + \vec{E}_B$ is the resultant of all waves that get to P when neither A nor B are present. Since P is in the geometric shadow, this is zero. Thus $\vec{E}_A = -\vec{E}_B$, and since the intensity is proportional to the square of the electric field, the intensity at P is the same when A is present as when B is present.

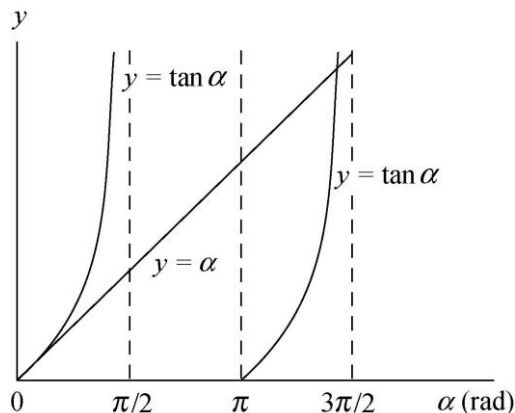
17. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2 \alpha}{\alpha^2}$$

where α is described in the text (see Eq. 36-6). To locate the extrema, we set the derivative of I with respect to α equal to zero and solve for α . The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin \alpha}{\alpha^3} (\alpha \cos \alpha - \sin \alpha).$$

The derivative vanishes if $\alpha \neq 0$ but $\sin \alpha = 0$. This yields $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: $I = 0$ for $\alpha = m\pi$. The derivative also vanishes for $\alpha \cos \alpha - \sin \alpha = 0$. This condition can be written $\tan \alpha = \alpha$. These implicitly locate the maxima.



(b) The values of α that satisfy $\tan \alpha = \alpha$ can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $(m + \frac{1}{2})\pi$ rad, so we start with these values. They can also be found graphically. As in the diagram that follows, we plot $y = \tan \alpha$ and $y = \alpha$ on the same graph. The intersections of the line with the $\tan \alpha$ curves are the solutions. The smallest α is $\alpha = 0$.

(c) We write $\alpha = (m + \frac{1}{2})\pi$ for the maxima. For the central maximum, $\alpha = 0$ and $m = -1/2 = -0.500$.

(d) The next one can be found to be $\alpha = 4.493$ rad.

(e) For $\alpha = 4.4934$, $m = 0.930$.

(f) The next one can be found to be $\alpha = 7.725$ rad.

(g) For $\alpha = 7.7252$, $m = 1.96$.

18. Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” the maximum distance is

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-3} \text{ m})(4.0 \times 10^{-3} \text{ m})}{1.22(550 \times 10^{-9} \text{ m})} = 30 \text{ m}.$$

19. (a) Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,”

$$L = \frac{D}{1.22\lambda/d} = \frac{2(50 \times 10^{-6} \text{ m})(1.5 \times 10^{-3} \text{ m})}{1.22(650 \times 10^{-9} \text{ m})} = 0.19 \text{ m}.$$

(b) The wavelength of the blue light is shorter so $L_{\max} \propto \lambda^{-1}$ will be larger.

20. Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = (6.2 \times 10^3 \text{ m}) \frac{(1.22)(1.6 \times 10^{-2} \text{ m})}{2.3 \text{ m}} = 53 \text{ m}.$$

21. **THINK** We apply the Rayleigh criterion to estimate the linear separation between the two objects.

EXPRESS If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians.

ANALYZE (a) With small angle approximation, $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. Thus,

$$D = \frac{1.22 L \lambda}{d} = \frac{1.22(8.0 \times 10^{10} \text{ m})(550 \times 10^{-9} \text{ m})}{5.0 \times 10^{-3} \text{ m}} = 1.1 \times 10^7 \text{ m} = 1.1 \times 10^4 \text{ km}.$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

$$(b) \text{ Now } d = 5.1 \text{ m and } D = \frac{1.22(8.0 \times 10^{10} \text{ m})(550 \times 10^{-9} \text{ m})}{5.1 \text{ m}} = 1.1 \times 10^4 \text{ m} = 11 \text{ km}.$$

LEARN By the Rayleigh criterion for resolvability, two objects can be resolved only if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$.

22. (a) Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{(400 \times 10^3 \text{ m})(1.22)(550 \times 10^{-9} \text{ m})}{(0.005 \text{ m})} \approx 50 \text{ m}.$$

(b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

(c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

23. **THINK** We apply the Rayleigh criterion to determine the conditions that allow the headlights to be resolved.

EXPRESS By the Rayleigh criteria, two point sources can be resolved if the central diffraction maximum of one source is centered on the first minimum of the diffraction pattern of the other. Thus, the angular separation (in radians) of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture.

ANALYZE (a) For the headlights of this problem,

$$\theta_R = \frac{1.22(550 \times 10^{-9} \text{ m})}{5.0 \times 10^{-3} \text{ m}} = 1.34 \times 10^{-4} \text{ rad},$$

or $1.3 \times 10^{-4} \text{ rad}$, in two significant figures.

(b) If L is the distance from the headlights to the eye when the headlights are just resolvable and D is the separation of the headlights, then $D = L\theta_R$, where the small angle approximation is made. This is valid for θ_R in radians. Thus,

$$L = \frac{D}{\theta_R} = \frac{1.4 \text{ m}}{1.34 \times 10^{-4} \text{ rad}} = 1.0 \times 10^4 \text{ m} = 10 \text{ km}.$$

LEARN A distance of 10 km far exceeds what human eyes can resolve. In reality, our visual resolvability depends on other factors such as the relative brightness of the source and their surroundings, turbulence in the air between the lights and the eyes, the health of one's vision.

24. We use Eq. 36-12 with $\theta = 2.5^\circ/2 = 1.25^\circ$. Thus,

$$d = \frac{1.22\lambda}{\sin \theta} = \frac{1.22(550 \text{ nm})}{\sin 1.25^\circ} = 31 \mu\text{m}.$$

25. Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” the minimum separation is

$$D = L\theta_R = L \left(1.22 \frac{\lambda}{d} \right) = (3.82 \times 10^8 \text{ m}) \frac{(1.22)(550 \times 10^{-9} \text{ m})}{5.1 \text{ m}} = 50 \text{ m}.$$

26. Using the same notation found in Sample Problem — “Pointillistic paintings use the diffraction of your eye,”

$$\frac{D}{L} = \theta_R = 1.22 \frac{\lambda}{d}$$

where we will assume a “typical” wavelength for visible light: $\lambda \approx 550 \times 10^{-9}$ m.

(a) With $L = 400 \times 10^3$ m and $D = 0.85$ m, the above relation leads to $d = 0.32$ m.

(b) Now with $D = 0.10$ m, the above relation leads to $d = 2.7$ m.

(c) The military satellites do not use Hubble Telescope-sized apertures. A great deal of very sophisticated optical filtering and digital signal processing techniques go into the final product, for which there is not space for us to describe here.

27. Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,”

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-2} \text{ m})(4.0 \times 10^{-3} \text{ m})}{1.22(0.10 \times 10^{-9} \text{ m})} = 1.6 \times 10^6 \text{ m} = 1.6 \times 10^3 \text{ km} .$$

28. Eq. 36-14 gives $\theta_R = 1.22\lambda/d$, where in our case $\theta_R \approx D/L$, with $D = 60 \mu\text{m}$ being the size of the object your eyes must resolve, and L being the maximum viewing distance in question. If $d = 3.00 \text{ mm} = 3000 \mu\text{m}$ is the diameter of your pupil, then

$$L = \frac{Dd}{1.22\lambda} = \frac{(60 \mu\text{m})(3000 \mu\text{m})}{1.22(0.55 \mu\text{m})} = 2.7 \times 10^5 \mu\text{m} = 27 \text{ cm} .$$

29. (a) Using Eq. 36-14, the angular separation is

$$\theta_R = \frac{1.22\lambda}{d} = \frac{(1.22)(550 \times 10^{-9} \text{ m})}{0.76 \text{ m}} = 8.8 \times 10^{-7} \text{ rad} .$$

(b) Using the notation of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” the distance between the stars is

$$D = L\theta_R = \frac{(10 \text{ ly})(9.46 \times 10^{12} \text{ km/ly})(0.18)\pi}{(3600)(180)} = 8.4 \times 10^7 \text{ km} .$$

(c) The diameter of the first dark ring is

$$d = 2\theta_R L = \frac{2(0.18)(\pi)(14 \text{ m})}{(3600)(180)} = 2.5 \times 10^{-5} \text{ m} = 0.025 \text{ mm} .$$

30. From Fig. 36-42(a), we find the diameter D' on the retina to be

$$D' = D \frac{L'}{L} = (2.00 \text{ mm}) \frac{2.00 \text{ cm}}{45.0 \text{ cm}} = 0.0889 \text{ mm} .$$

Next, using Fig. 36-42(b), the angle from the axis is

$$\theta = \tan^{-1}\left(\frac{D'/2}{x}\right) = \tan^{-1}\left(\frac{0.0889 \text{ mm}/2}{6.00 \text{ mm}}\right) = 0.424^\circ.$$

Since the angle corresponds to the first minimum in the diffraction pattern, we have $\sin \theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the defect. With $\lambda = 550 \text{ nm}$, we obtain

$$d = \frac{1.22\lambda}{\sin \theta} = \frac{1.22(550 \text{ nm})}{\sin(0.424^\circ)} = 9.06 \times 10^{-5} \text{ m} \approx 91 \mu\text{m}.$$

31. THINK We apply the Rayleigh criterion to calculate the angular width of the central maxima.

EXPRESS The first minimum in the diffraction pattern is at an angular position θ , measured from the center of the pattern, such that $\sin \theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{220 \times 10^9 \text{ Hz}} = 1.36 \times 10^{-3} \text{ m}.$$

ANALYZE (a) Thus, we have

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(1.36 \times 10^{-3} \text{ m})}{55.0 \times 10^{-2} \text{ m}}\right) = 3.02 \times 10^{-3} \text{ rad}.$$

The angular width of the central maximum is twice this, or $6.04 \times 10^{-3} \text{ rad}$ (0.346°).

(b) Now $\lambda = 1.6 \text{ cm}$ and $d = 2.3 \text{ m}$, so

$$\theta = \sin^{-1}\left(\frac{1.22(1.6 \times 10^{-2} \text{ m})}{2.3 \text{ m}}\right) = 8.5 \times 10^{-3} \text{ rad}.$$

The angular width of the central maximum is $1.7 \times 10^{-2} \text{ rad}$ (or 0.97°).

LEARN Using small angle approximation, we can write the angular width as

$$2\theta \approx 2\left(\frac{1.22\lambda}{d}\right) = \frac{2.44\lambda}{d}.$$

32. (a) We use Eq. 36-12:

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left[\frac{1.22(v_s/f)}{d}\right] = \sin^{-1}\left[\frac{(1.22)(1450\text{ m/s})}{(25 \times 10^3 \text{ Hz})(0.60\text{ m})}\right] = 6.8^\circ.$$

(b) Now $f = 1.0 \times 10^3 \text{ Hz}$ so

$$\frac{1.22\lambda}{d} = \frac{(1.22)(1450\text{ m/s})}{(1.0 \times 10^3 \text{ Hz})(0.60\text{ m})} = 2.9 > 1.$$

Since $\sin \theta$ cannot exceed 1 there is no minimum.

33. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — “Pointillistic paintings use the diffraction of your eye.”

(a) We are asked to solve for D and are given $\lambda = 1.40 \times 10^{-9} \text{ m}$, $d = 0.200 \times 10^{-3} \text{ m}$, and $L = 2000 \times 10^3 \text{ m}$. Consequently, we obtain $D = 17.1 \text{ m}$.

(b) Intensity is power over area (with the area assumed spherical in this case, which means it is proportional to radius-squared), so the ratio of intensities is given by the square of a ratio of distances: $(d/D)^2 = 1.37 \times 10^{-10}$.

34. (a) Since $\theta = 1.22\lambda/d$, the larger the wavelength the larger the radius of the first minimum (and second maximum, etc). Therefore, the white pattern is outlined by red lights (with longer wavelength than blue lights).

(b) The diameter of a water drop is

$$d = \frac{1.22\lambda}{\theta} \approx \frac{1.22(7 \times 10^{-7} \text{ m})}{1.5(0.50^\circ)(\pi/180^\circ)/2} = 1.3 \times 10^{-4} \text{ m}.$$

35. Bright interference fringes occur at angles θ given by $d \sin \theta = m\lambda$, where m is an integer. For the slits of this problem, we have $d = 11a/2$, so

$$a \sin \theta = 2m\lambda/11.$$

The first minimum of the diffraction pattern occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, and the second occurs at the angle θ_2 given by $a \sin \theta_2 = 2\lambda$, where a is the slit width. We

should count the values of m for which $\theta_1 < \theta < \theta_2$, or, equivalently, the values of m for which $\sin \theta_1 < \sin \theta < \sin \theta_2$. This means $1 < (2m/11) < 2$. The values are $m = 6, 7, 8, 9$, and 10. There are five bright fringes in all.

36. Following the method of Sample Problem — “Double-slit experiment with diffraction of each slit included,” we find

$$\frac{d}{a} = \frac{0.30 \times 10^{-3} \text{ m}}{46 \times 10^{-6} \text{ m}} = 6.52$$

which we interpret to mean that the first diffraction minimum occurs slightly farther “out” than the $m = 6$ interference maximum. This implies that the central diffraction envelope includes the central ($m = 0$) interference maximum as well as six interference maxima on each side of it. Therefore, there are $6 + 1 + 6 = 13$ bright fringes (interference maxima) in the central diffraction envelope.

37. In a manner similar to that discussed in Sample Problem — “Double-slit experiment with diffraction of each slit included,” we find the number is $2(d/a) - 1 = 2(2a/a) - 1 = 3$.

38. We note that the central diffraction envelope contains the central bright interference fringe (corresponding to $m = 0$ in Eq. 36-25) plus ten on either side of it. Since the eleventh order bright interference fringe is not seen in the central envelope, then we conclude the first diffraction minimum (satisfying $\sin \theta = \lambda/a$) coincides with the $m = 11$ instantiation of Eq. 36-25:

$$d = \frac{m\lambda}{\sin \theta} = \frac{11 \lambda}{\lambda/a} = 11 a .$$

Thus, the ratio d/a is equal to 11.

39. (a) The first minimum of the diffraction pattern is at 5.00° , so

$$a = \frac{\lambda}{\sin \theta} = \frac{0.440 \mu\text{m}}{\sin 5.00^\circ} = 5.05 \mu\text{m} .$$

(b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05 \mu\text{m}) = 20.2 \mu\text{m}$.

(c) For the $m = 1$ bright fringe,

$$\alpha = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi (5.05 \mu\text{m}) \sin 1.25^\circ}{0.440 \mu\text{m}} = 0.787 \text{ rad} .$$

Consequently, the intensity of the $m = 1$ fringe is

$$I = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2 = (7.0 \text{ mW/cm}^2) \left(\frac{\sin 0.787 \text{ rad}}{0.787} \right)^2 = 5.7 \text{ mW/cm}^2 ,$$

which agrees with Fig. 36-45. Similarly for $m = 2$, the intensity is $I = 2.9 \text{ mW/cm}^2$, also in agreement with Fig. 36-45.

40. (a) We note that the slope of the graph is 80, and that Eq. 36-20 implies that the slope should correspond to

$$\frac{\pi d}{\lambda} = 80 \Rightarrow d = \frac{80\lambda}{\pi} = \frac{80(435 \text{ nm})}{\pi} = 11077 \text{ nm} \approx 11.1 \mu\text{m}.$$

(b) Consider Eq. 36-25 with “continuously variable” m (of course, m should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m_{\max} = \frac{d}{\lambda} (\sin \theta)_{\max} = \frac{d}{\lambda} = \frac{11077 \text{ nm}}{435 \text{ nm}} \approx 25.5$$

which indicates (on one side of the interference pattern) there are 25 bright fringes. Thus on the other side there are also 25 bright fringes. Including the one in the middle, then, means there are a total of 51 maxima in the interference pattern (assuming, as the problem remarks, that none of the interference maxima have been eliminated by diffraction minima).

(c) Clearly, the maximum closest to the axis is the middle fringe at $\theta = 0^\circ$.

(d) If we set $m = 25$ in Eq. 36-25, we find

$$m\lambda = d \sin \theta \Rightarrow \theta = \sin^{-1} \left(\frac{m\lambda}{d} \right) = \sin^{-1} \left(\frac{(25)(435 \text{ nm})}{11077 \text{ nm}} \right) = 79.0^\circ$$

41. We will make use of arctangents and sines in our solution, even though they can be “shortcut” somewhat since the angles are [almost] small enough to justify the use of the small angle approximation.

(a) Given $y/D = (0.700 \text{ m})/(4.00 \text{ m})$, then

$$\theta = \tan^{-1} \left(\frac{y}{D} \right) = \tan^{-1} \left(\frac{0.700 \text{ m}}{4.00 \text{ m}} \right) = 9.93^\circ = 0.173 \text{ rad}.$$

Equation 36-20 then gives

$$\beta = \frac{\pi d \sin \theta}{\lambda} = \frac{\pi (24.0 \mu\text{m}) \sin 9.93^\circ}{0.600 \mu\text{m}} = 21.66 \text{ rad}.$$

Thus, use of Eq. 36-21 (with $a = 12 \mu\text{m}$ and $\lambda = 0.60 \mu\text{m}$) leads to

$$\alpha = \frac{\pi a \sin \theta}{\lambda} = \frac{\pi (12.0 \mu\text{m}) \sin 9.93^\circ}{0.600 \mu\text{m}} = 10.83 \text{ rad} .$$

Thus,

$$\frac{I}{I_m} = \left(\frac{\sin \alpha}{\alpha} \right)^2 (\cos \beta)^2 = \left(\frac{\sin 10.83 \text{ rad}}{10.83} \right)^2 (\cos 21.66 \text{ rad})^2 = 0.00743 .$$

(b) Consider Eq. 36-25 with “continuously variable” m (of course, m should be an integer for interference maxima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{d \sin \theta}{\lambda} = \frac{(24.0 \mu\text{m}) \sin 9.93^\circ}{0.600 \mu\text{m}} \approx 6.9$$

which suggests that the angle takes us to a point between the sixth minimum (which would have $m = 6.5$) and the seventh maximum (which corresponds to $m = 7$).

(c) Similarly, consider Eq. 36-3 with “continuously variable” m (of course, m should be an integer for diffraction minima, but for the moment we will solve for it as if it could be any real number):

$$m = \frac{a \sin \theta}{\lambda} = \frac{(12.0 \mu\text{m}) \sin 9.93^\circ}{0.600 \mu\text{m}} \approx 3.4$$

which suggests that the angle takes us to a point between the third diffraction minimum ($m = 3$) and the fourth one ($m = 4$). The maxima (in the smaller peaks of the diffraction pattern) are not exactly midway between the minima; their location would make use of mathematics not covered in the prerequisites of the usual sophomore-level physics course.

42. (a) In a manner similar to that discussed in Sample Problem — “Double-slit experiment with diffraction of each slit included,” we find the ratio should be $d/a = 4$. Our reasoning is, briefly, as follows: we let the location of the fourth bright fringe coincide with the first minimum of diffraction pattern, and then set $\sin \theta = 4\lambda/d = \lambda/a$ (so $d = 4a$).

(b) Any bright fringe that happens to be at the same location with a diffraction minimum will vanish. Thus, if we let

$$\sin \theta = \frac{m_1 \lambda}{d} = \frac{m_2 \lambda}{a} = \frac{m_1 \lambda}{4a} ,$$

or $m_1 = 4m_2$ where $m_2 = 1, 2, 3, \dots$. The fringes missing are the 4th, 8th, 12th, and so on. Hence, every fourth fringe is missing.

43. **THINK** For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity; instead, the intensities are modified by diffraction of light passing through each slit.

EXPRESS The angular positions θ of the bright interference fringes are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which

$$-\sin \theta_1 < \sin \theta < +\sin \theta_1.$$

The intensity at the screen is given by

$$I = I_m (\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2$$

where $\alpha = (\pi a/\lambda) \sin \theta$, $\beta = (\pi d/\lambda) \sin \theta$, and I_m is the intensity at the center of the pattern.

ANALYZE (a) The condition above means $-1/a < m/d < 1/a$, or $-d/a < m < +d/a$. Now

$$d/a = (0.150 \times 10^{-3} \text{ m}) / (30.0 \times 10^{-6} \text{ m}) = 5.00,$$

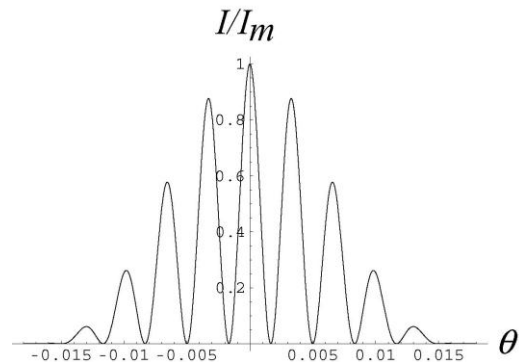
so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are 9 fringes.

(b) For the third bright interference fringe, $d \sin \theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2 \beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin \alpha}{\alpha} \right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi} \right)^2 = 0.255.$$

The intensity ratio is $I/I_m = 0.255$.

LEARN The expression for intensity contains two factors: (1) the interference factor $\cos^2 \beta$ due to the interference between two slits with separation d , and (2) the diffraction factor $[(\sin \alpha)/\alpha]^2$ which arises due to diffraction by a single slit of width a . In the limit $a \rightarrow 0$, $(\sin \alpha)/\alpha \rightarrow 1$, and we recover Eq. 35-22 for the interference between two slits of vanishingly narrow slits separated by d . Similarly, setting $d = 0$ or equivalently, $\beta = 0$, we recover Eq. 36-5 for the diffraction of a single slit of width a . A plot of the relative intensity is shown to the right.



44. We use Eq. 36-25 for diffraction maxima: $d \sin \theta = m\lambda$. In our case, since the angle between the $m = 1$ and $m = -1$ maxima is 26° , the angle θ corresponding to $m = 1$ is $\theta = 26^\circ/2 = 13^\circ$. We solve for the grating spacing:

$$d = \frac{m\lambda}{\sin \theta} = \frac{(1)(550\text{nm})}{\sin 13^\circ} = 2.4\mu\text{m} \approx 2\mu\text{m}.$$

45. The distance between adjacent rulings is

$$d = 20.0\text{ mm}/6000 = 0.00333\text{ mm} = 3.33\mu\text{m}.$$

(a) Let $d \sin \theta = m\lambda$ ($m = 0, \pm 1, \pm 2, \dots$). Since $|m|\lambda/d > 1$ for $|m| \geq 6$, the largest value of θ corresponds to $|m| = 5$, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{5(0.589\mu\text{m})}{3.33\mu\text{m}}\right) = 62.1^\circ.$$

(b) The second largest value of θ corresponds to $|m| = 4$, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{4(0.589\mu\text{m})}{3.33\mu\text{m}}\right) = 45.0^\circ.$$

(c) The third largest value of θ corresponds to $|m| = 3$, which yields

$$\theta = \sin^{-1}(|m|\lambda/d) = \sin^{-1}\left(\frac{3(0.589\mu\text{m})}{3.33\mu\text{m}}\right) = 32.0^\circ.$$

46. The angular location of the m th order diffraction maximum is given by $m\lambda = d \sin \theta$. To be able to observe the fifth-order maximum, we must let $\sin \theta_{m=5} = 5\lambda/d < 1$, or

$$\lambda < \frac{d}{5} = \frac{1.00\text{ nm}/315}{5} = 635\text{ nm}.$$

Therefore, the longest wavelength that can be used is $\lambda = 635\text{ nm}$.

47. **THINK** Diffraction lines occur at angles θ such that $d \sin \theta = m\lambda$, where d is the grating spacing, λ is the wavelength and m is an integer.

EXPRESS The ruling separation is

$$d = 1/(400\text{ mm}^{-1}) = 2.5 \times 10^{-3}\text{ mm}.$$

Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. We take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90° . That is, find the greatest integer value of m for which $m\lambda < d$.

ANALYZE Since

$$\frac{d}{\lambda} = \frac{2.5 \times 10^{-6} \text{ m}}{700 \times 10^{-9} \text{ m}} \approx 3.57,$$

that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

LEARN From $\theta = \sin^{-1}(m\lambda/d)$, the condition for maxima or lines, we see that for a given diffraction grating, the angle from the central axis to any line depends on the wavelength of the light being used.

48. (a) For the maximum with the greatest value of $m = M$ we have $M\lambda = a \sin \theta < d$, so $M < d/\lambda = 900 \text{ nm}/600 \text{ nm} = 1.5$, or $M = 1$. Thus three maxima can be seen, with $m = 0, \pm 1$.

(b) From Eq. 36-28, we obtain

$$\begin{aligned} \Delta\theta_{\text{hw}} &= \frac{\lambda}{N d \cos \theta} = \frac{d \sin \theta}{N d \cos \theta} = \frac{\tan \theta}{N} = \frac{1}{N} \tan \left[\sin^{-1} \left(\frac{\lambda}{d} \right) \right] \\ &= \frac{1}{1000} \tan \left[\sin^{-1} \left(\frac{600 \text{ nm}}{900 \text{ nm}} \right) \right] = 0.051^\circ. \end{aligned}$$

49. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS If two lines are adjacent, then their order numbers differ by unity. Let m be the order number for the line with $\sin \theta = 0.2$ and $m + 1$ be the order number for the line with $\sin \theta = 0.3$. Then,

$$0.2d = m\lambda, \quad 0.3d = (m + 1)\lambda.$$

ANALYZE (a) We subtract the first equation from the second to obtain $0.1d = \lambda$, or

$$d = \lambda/0.1 = (600 \times 10^{-9} \text{ m})/0.1 = 6.0 \times 10^{-6} \text{ m}.$$

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a \sin \theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must

fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a \sin \theta = \lambda$. It is also given by $d \sin \theta = 4\lambda$, so

$$a = d/4 = (6.0 \times 10^{-6} \text{ m})/4 = 1.5 \times 10^{-6} \text{ m}.$$

(c) First, we set $\theta = 90^\circ$ and find the largest value of m for which $m\lambda < d \sin \theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since

$$d/\lambda = (6.0 \times 10^{-6} \text{ m})/(600 \times 10^{-9} \text{ m}) = 10.0,$$

the highest order seen is the $m = 9$ order. The fourth and eighth orders are missing, so the observable orders are $m = 0, 1, 2, 3, 5, 6, 7$, and 9. Thus, the largest value of the order number is $m = 9$.

(d) Using the result obtained in (c), the second largest value of the order number is $m = 7$.

(e) Similarly, the third largest value of the order number is $m = 6$.

LEARN Interference maxima occur when $d \sin \theta = m\lambda$, while the condition for diffraction minima is $a \sin \theta = m'\lambda$. Thus, a particular interference maximum with order m may coincide with the diffraction minimum of order m' . The value of m is given by

$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{m'\lambda} \Rightarrow m = \left(\frac{d}{a}\right)m'.$$

Since $m = 4$ when $m' = 1$, we conclude that $d/a = 4$. Thus, $m = 8$ would correspond to the second diffraction minimum ($m' = 2$).

50. We use Eq. 36-25. For $m = \pm 1$

$$\lambda = \frac{d \sin \theta}{m} = \frac{(1.73 \mu\text{m}) \sin(\pm 17.6^\circ)}{\pm 1} = 523 \text{ nm},$$

and for $m = \pm 2$,

$$\lambda = \frac{(1.73 \mu\text{m}) \sin(\pm 37.3^\circ)}{\pm 2} = 524 \text{ nm}.$$

Similarly, we may compute the values of λ corresponding to the angles for $m = \pm 3$. The average value of these λ 's is 523 nm.

51. (a) Since $d = (1.00 \text{ mm})/180 = 0.0056 \text{ mm}$, we write Eq. 36-25 as

$$\theta = \sin^{-1} \left(\frac{m\lambda}{d} \right) = \sin^{-1} (180)(2)\lambda$$

where $\lambda_1 = 4 \times 10^{-4}$ mm and $\lambda_2 = 5 \times 10^{-4}$ mm. Thus, $\Delta\theta = \theta_2 - \theta_1 = 2.1^\circ$.

(b) Use of Eq. 36-25 for each wavelength leads to the condition

$$m_1\lambda_1 = m_2\lambda_2$$

for which the smallest possible choices are $m_1 = 5$ and $m_2 = 4$. Returning to Eq. 36-25, then, we find

$$\theta = \sin^{-1}\left(\frac{m_1\lambda_1}{d}\right) = \sin^{-1}\left(\frac{5(4.0 \times 10^{-4} \text{ mm})}{0.0056 \text{ mm}}\right) = \sin^{-1}(0.36) = 21^\circ.$$

(c) There are no refraction angles greater than 90° , so we can solve for “ m_{\max} ” (realizing it might not be an integer):

$$m_{\max} = \frac{d \sin 90^\circ}{\lambda_2} = \frac{d}{\lambda_2} = \frac{0.0056 \text{ mm}}{5.0 \times 10^{-4} \text{ mm}} \approx 11$$

where we have rounded down. There are no values of m (for light of wavelength λ_2) greater than $m = 11$.

52. We are given the “number of lines per millimeter” (which is a common way to express $1/d$ for diffraction gratings); thus,

$$\frac{1}{d} = 160 \text{ lines/mm} \Rightarrow d = 6.25 \times 10^{-6} \text{ m}.$$

(a) We solve Eq. 36-25 for θ with various values of m and λ . We show here the $m = 2$ and $\lambda = 460$ nm calculation:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{2(460 \times 10^{-9} \text{ m})}{6.25 \times 10^{-6} \text{ m}}\right) = \sin^{-1}(0.1472) = 8.46^\circ.$$

Similarly, we get 11.81° for $m = 2$ and $\lambda = 640$ nm, 12.75° for $m = 3$ and $\lambda = 460$ nm, and 17.89° for $m = 3$ and $\lambda = 640$ nm. The first indication of overlap occurs when we compute the angle for $m = 4$ and $\lambda = 460$ nm; the result is 17.12° which clearly shows overlap with the large-wavelength portion of the $m = 3$ spectrum.

(b) We solve Eq. 36-25 for m with $\theta = 90^\circ$ and $\lambda = 640$ nm. In this case, we obtain $m = 9.8$ which means that the largest order in which the full range (which must include that largest wavelength) is seen is ninth order.

(c) Now with $m = 9$, Eq. 36-25 gives $\theta = 41.5^\circ$ for $\lambda = 460$ nm.

(d) It similarly gives $\theta = 67.2^\circ$ for $\lambda = 640$ nm.

(e) We solve Eq. 36-25 for m with $\theta = 90^\circ$ and $\lambda = 460$ nm. In this case, we obtain $m = 13.6$ which means that the largest order in which the wavelength is seen is the thirteenth order. Now with $m = 13$, Eq. 36-25 gives $\theta = 73.1^\circ$ for $\lambda = 460$ nm.

53. At the point on the screen where we find the inner edge of the hole, we have $\tan \theta = 5.0 \text{ cm}/30 \text{ cm}$, which gives $\theta = 9.46^\circ$. We note that d for the grating is equal to $1.0 \text{ mm}/350 = 1.0 \times 10^6 \text{ nm}/350$.

(a) From $m\lambda = d \sin \theta$, we find

$$m = \frac{d \sin \theta}{\lambda} = \frac{(1.0 \times 10^6 \text{ nm}/350)(0.1644)}{\lambda} = \frac{470 \text{ nm}}{\lambda}.$$

Since for white light $\lambda > 400$ nm, the only integer m allowed here is $m = 1$. Thus, at one edge of the hole, $\lambda = 470$ nm. This is the shortest wavelength of the light that passes through the hole.

(b) At the other edge, we have $\tan \theta' = 6.0 \text{ cm}/30 \text{ cm}$, which gives $\theta' = 11.31^\circ$. This leads to

$$\lambda' = d \sin \theta' = \left(\frac{1.0 \times 10^6 \text{ nm}}{350} \right) \sin(11.31^\circ) = 560 \text{ nm}.$$

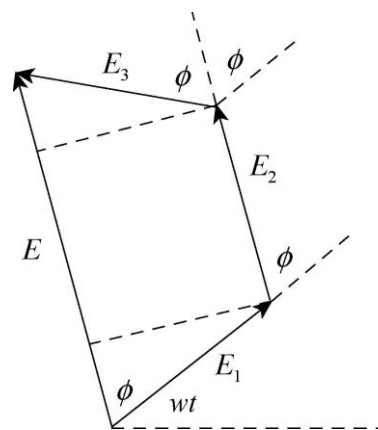
This corresponds to the longest wavelength of the light that passes through the hole.

54. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written as

$$\begin{aligned} E_1 &= E_0 \sin(\omega t) \\ E_2 &= E_0 \sin(\omega t + \phi) \\ E_3 &= E_0 \sin(\omega t + 2\phi) \end{aligned}$$

where $\phi = (2\pi d/\lambda) \sin \theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown on the right. It yields

$$E = E_0 \cos \phi + E_0 \cos \phi = E_0(1 + 2 \cos \phi).$$



for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1 + 2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}(1 + 2\cos\phi)^2 = \frac{I_m}{9}(1 + 4\cos\phi + 4\cos^2\phi).$$

55. THINK If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{\text{avg}}/\Delta\lambda$.

EXPRESS As shown in Eq. 36-32, the resolving power can also be written as Nm , where N is the number of rulings in the grating and m is the order of the lines.

ANALYZE Thus $\lambda_{\text{avg}}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{\text{avg}}}{m\Delta\lambda} = \frac{656.3 \text{ nm}}{(1)(0.18 \text{ nm})} = 3.65 \times 10^3 \text{ rulings.}$$

LEARN A large N (more rulings) means greater resolving power.

56. (a) From $R = \lambda/\Delta\lambda = Nm$ we find

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(415.496 \text{ nm} + 415.487 \text{ nm})/2}{2(415.96 \text{ nm} - 415.487 \text{ nm})} = 23100.$$

(b) We note that $d = (4.0 \times 10^7 \text{ nm})/23100 = 1732 \text{ nm}$. The maxima are found at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(2)(415.5 \text{ nm})}{1732 \text{ nm}}\right] = 28.7^\circ.$$

57. (a) We note that $d = (76 \times 10^6 \text{ nm})/40000 = 1900 \text{ nm}$. For the first order maxima $\lambda = d \sin \theta$, which leads to

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589 \text{ nm}}{1900 \text{ nm}}\right) = 18^\circ.$$

Now, substituting $m = d \sin \theta/\lambda$ into Eq. 36-30 leads to

$$D = \tan \theta/\lambda = \tan 18^\circ/589 \text{ nm} = 5.5 \times 10^{-4} \text{ rad/nm} = 0.032^\circ/\text{nm}.$$

(b) For $m = 1$, the resolving power is $R = Nm = 40000$ $m = 40000 = 4.0 \times 10^4$.

(c) For $m = 2$ we have $\theta = 38^\circ$, and the corresponding value of dispersion is $0.076^\circ/\text{nm}$.

(d) For $m = 2$, the resolving power is $R = Nm = 40000$ $m = (40000)2 = 8.0 \times 10^4$.

(e) Similarly for $m = 3$, we have $\theta = 68^\circ$, and the corresponding value of dispersion is $0.24^\circ/\text{nm}$.

(f) For $m = 3$, the resolving power is $R = Nm = 40000$ $m = (40000)3 = 1.2 \times 10^5$.

58. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500 \text{ nm}}{(600/\text{mm})(5.0 \text{ mm})(3)} = 0.056 \text{ nm} = 56 \text{ pm}.$$

(b) Since $\sin \theta = m_{\text{max}}\lambda/d < 1$,

$$m_{\text{max}} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500 \times 10^{-6} \text{ mm})} = 3.3.$$

Therefore, $m_{\text{max}} = 3$. No higher orders of maxima can be seen.

59. Assuming all $N = 2000$ lines are uniformly illuminated, we have

$$\frac{\lambda_{\text{av}}}{\Delta\lambda} = Nm$$

from Eq. 36-31 and Eq. 36-32. With $\lambda_{\text{av}} = 600 \text{ nm}$ and $m = 2$, we find $\Delta\lambda = 0.15 \text{ nm}$.

60. Letting $R = \lambda/\Delta\lambda = Nm$, we solve for N :

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(589.6 \text{ nm} + 589.0 \text{ nm})/2}{2(589.6 \text{ nm} - 589.0 \text{ nm})} = 491.$$

61. (a) From $d \sin \theta = m\lambda$ we find

$$d = \frac{m\lambda_{\text{avg}}}{\sin \theta} = \frac{3(589.3 \text{ nm})}{\sin 10^\circ} = 1.0 \times 10^4 \text{ nm} = 10 \mu\text{m}.$$

(b) The total width of the ruling is

$$L = Nd = \left(\frac{R}{m}\right)d = \frac{\lambda_{\text{avg}}d}{m\Delta\lambda} = \frac{(589.3 \text{ nm})(10 \mu\text{m})}{3(589.59 \text{ nm} - 589.00 \text{ nm})} = 3.3 \times 10^3 \mu\text{m} = 3.3 \text{ mm}.$$

62. (a) From the expression for the half-width $\Delta\theta_{\text{hw}}$ (given by Eq. 36-28) and that for the resolving power R (given by Eq. 36-32), we find the product of $\Delta\theta_{\text{hw}}$ and R to be

$$\Delta\theta_{\text{hw}} R = \left(\frac{\lambda}{N d \cos \theta} \right) Nm = \frac{m\lambda}{d \cos \theta} = \frac{d \sin \theta}{d \cos \theta} = \tan \theta,$$

where we used $m\lambda = d \sin \theta$ (see Eq. 36-25).

(b) For first order $m = 1$, so the corresponding angle θ_1 satisfies $d \sin \theta_1 = m\lambda = \lambda$. Thus the product in question is given by

$$\begin{aligned} \tan \theta_1 &= \frac{\sin \theta_1}{\cos \theta_1} = \frac{\sin \theta_1}{\sqrt{1 - \sin^2 \theta_1}} = \frac{1}{\sqrt{(1/\sin \theta_1)^2 - 1}} = \frac{1}{\sqrt{(d/\lambda)^2 - 1}} \\ &= \frac{1}{\sqrt{(900\text{nm}/600\text{nm})^2 - 1}} = 0.89. \end{aligned}$$

63. The angular positions of the first-order diffraction lines are given by $d \sin \theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta + \Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20^\circ$. Then,

$$\lambda_1 = d \sin \theta, \quad \lambda_2 = d \sin(\theta + \Delta\theta).$$

We write

$$\sin(\theta + \Delta\theta) \text{ as } \sin \theta \cos \Delta\theta + \cos \theta \sin \Delta\theta,$$

then use the equation for the first line to replace $\sin \theta$ with λ_1/d , and $\cos \theta$ with $\sqrt{1 - \lambda_1^2/d^2}$. After multiplying by d , we obtain

$$\lambda_1 \cos \Delta\theta + \sqrt{d^2 - \lambda_1^2} \sin \Delta\theta = \lambda_2.$$

Solving for d , we find

$$\begin{aligned} d &= \sqrt{\frac{(\lambda_2 - \lambda_1 \cos \Delta\theta)^2 + (\lambda_1 \sin \Delta\theta)^2}{\sin^2 \Delta\theta}} \\ &= \sqrt{\frac{(680 \text{ nm}) - (430 \text{ nm}) \cos 20^\circ^2 + (430 \text{ nm}) \sin 20^\circ^2}{\sin^2 20^\circ}} \\ &= 914 \text{ nm} = 9.14 \times 10^{-4} \text{ mm}. \end{aligned}$$

There are $1/d = 1/(9.14 \times 10^{-4} \text{ mm}) = 1.09 \times 10^3$ rulings per mm.

64. We use Eq. 36-34. For smallest value of θ , we let $m = 1$. Thus,

$$\theta_{\min} = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(30\text{ pm})}{2(0.30 \times 10^3 \text{ pm})}\right] = 2.9^\circ.$$

65. (a) For the first beam $2d \sin \theta_1 = \lambda_A$ and for the second one $2d \sin \theta_2 = 3\lambda_B$. The values of d and λ_A can then be determined:

$$d = \frac{3\lambda_B}{2 \sin \theta_2} = \frac{3(97 \text{ pm})}{2 \sin 60^\circ} = 1.7 \times 10^2 \text{ pm}.$$

(b) $\lambda_A = 2d \sin \theta_1 = 2(1.7 \times 10^2 \text{ pm})(\sin 23^\circ) = 1.3 \times 10^2 \text{ pm}.$

66. The x-ray wavelength is $\lambda = 2d \sin \theta = 2(39.8 \text{ pm}) \sin 30.0^\circ = 39.8 \text{ pm}.$

67. We use Eq. 36-34.

(a) From the peak on the left at angle 0.75° (estimated from Fig. 36-46), we have

$$\lambda_1 = 2d \sin \theta_1 = 2(0.94 \text{ nm}) \sin(0.75^\circ) = 0.025 \text{ nm} = 25 \text{ pm}.$$

This is the shorter wavelength of the beam. Notice that the estimation should be viewed as reliable to within $\pm 2 \text{ pm}$.

(b) We now consider the next peak:

$$\lambda_2 = 2d \sin \theta_2 = 2(0.94 \text{ nm}) \sin 1.15^\circ = 0.038 \text{ nm} = 38 \text{ pm}.$$

This is the longer wavelength of the beam. One can check that the third peak from the left is the second-order one for λ_1 .

68. For x-ray (“Bragg”) scattering, we have $2d \sin \theta_m = m \lambda$. This leads to

$$\frac{2d \sin \theta_2}{2d \sin \theta_1} = \frac{2 \lambda}{1 \lambda} \quad \Rightarrow \quad \sin \theta_2 = 2 \sin \theta_1.$$

Thus, with $\theta_1 = 3.4^\circ$, this yields $\theta_2 = 6.8^\circ$. The fact that θ_2 is very nearly twice the value of θ_1 is due to the small angles involved (when angles are small, $\sin \theta_2 / \sin \theta_1 = \theta_2 / \theta_1$).

69. Bragg’s law gives the condition for diffraction maximum:

$$2d \sin \theta = m\lambda$$

where d is the spacing of the crystal planes and λ is the wavelength. The angle θ is measured from the surfaces of the planes. For a second-order reflection $m = 2$, so

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{2(0.12 \times 10^{-9} \text{ m})}{2 \sin 28^\circ} = 2.56 \times 10^{-10} \text{ m} \approx 0.26 \text{ nm}.$$

70. The angle of incidence on the reflection planes is $\theta = 63.8^\circ - 45.0^\circ = 18.8^\circ$, and the plane-plane separation is $d = a_0 / \sqrt{2}$. Thus, using $2d \sin \theta = \lambda$, we get

$$a_0 = \sqrt{2}d = \frac{\sqrt{2}\lambda}{2 \sin \theta} = \frac{0.260 \text{ nm}}{\sqrt{2} \sin 18.8^\circ} = 0.570 \text{ nm}.$$

71. **THINK** The criterion for diffraction maxima is given by the Bragg's law.

EXPRESS We want the reflections to obey the Bragg condition: $2d \sin \theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. We solve for θ .

$$\theta = \sin^{-1} \left(\frac{m\lambda}{2d} \right) = \sin^{-1} \left(\frac{(0.125 \times 10^{-9} \text{ m})m}{2(0.252 \times 10^{-9} \text{ m})} \right) = 0.2480m.$$

ANALYZE (a) For $m = 2$ the above equation gives $\theta = 29.7^\circ$. The crystal should be turned $\phi = 45^\circ - 29.7^\circ = 15.3^\circ$ clockwise.

(b) For $m = 1$ the above equation gives $\theta = 14.4^\circ$. The crystal should be turned $\phi = 45^\circ - 14.4^\circ = 30.6^\circ$ clockwise.

(c) For $m = 3$ the above equation gives $\theta = 48.1^\circ$. The crystal should be turned $\phi = 48.1^\circ - 45^\circ = 3.1^\circ$ counterclockwise.

(d) For $m = 4$ the above equation gives $\theta = 82.8^\circ$. The crystal should be turned $\phi = 82.8^\circ - 45^\circ = 37.8^\circ$ counterclockwise.

LEARN Note that there are no intensity maxima for $m > 4$ as one can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4.

72. The wavelengths satisfy

$$m\lambda = 2d \sin \theta = 2(275 \text{ pm})(\sin 45^\circ) = 389 \text{ pm}.$$

In the range of wavelengths given, the allowed values of m are $m = 3, 4$.

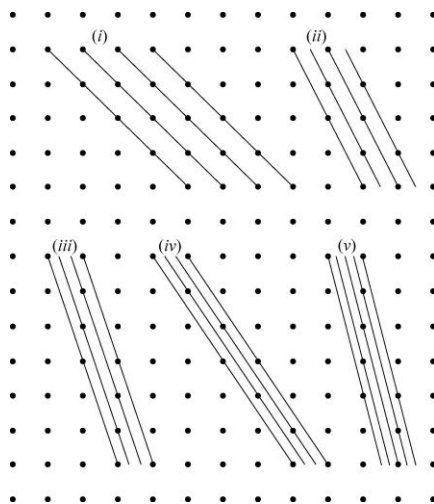
(a) The longest wavelength is $389 \text{ pm}/3 = 130 \text{ pm}$.

(b) The associated order number is $m = 3$.

(c) The shortest wavelength is $389 \text{ pm}/4 = 97.2 \text{ pm}$.

(d) The associated order number is $m = 4$.

73. The sets of planes with the next five smaller interplanar spacings (after a_0) are shown in the diagram that follows.



(a) In terms of a_0 , the second largest interplanar spacing is $a_0/\sqrt{2} = 0.7071a_0$.

(b) The third largest interplanar spacing is $a_0/\sqrt{5} = 0.4472a_0$.

(c) The fourth largest interplanar spacing is $a_0/\sqrt{10} = 0.3162a_0$.

(d) The fifth largest interplanar spacing is $a_0/\sqrt{13} = 0.2774a_0$.

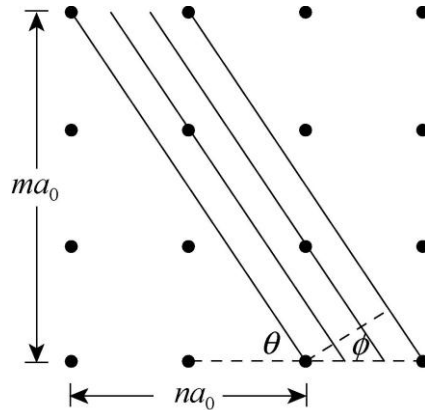
(e) The sixth largest interplanar spacing is $a_0/\sqrt{17} = 0.2425a_0$.

(f) Since a crystal plane passes through lattice points, its slope can be written as the ratio of two integers. Consider a set of planes with slope m/n , as shown in the diagram that follows. The first and last planes shown pass through adjacent lattice points along a horizontal line and there are $m - 1$ planes between. If h is the separation of the first and last planes, then the interplanar spacing is $d = h/m$. If the planes make the angle θ with the horizontal, then the normal to the planes (shown dashed) makes the angle $\phi = 90^\circ - \theta$. The distance h is given by $h = a_0 \cos \phi$ and the interplanar spacing is $d = h/m = (a_0/m) \cos \phi$. Since $\tan \theta = m/n$, $\tan \phi = n/m$ and

$$\cos \phi = 1/\sqrt{1 + \tan^2 \phi} = m/\sqrt{n^2 + m^2}.$$

Thus,

$$d = \frac{h}{m} = \frac{a_0 \cos \phi}{m} = \frac{a_0}{\sqrt{n^2 + m^2}}.$$



74. (a) We use Eq. 36-14:

$$\theta_R = 1.22 \frac{\lambda}{d} = \frac{(1.22)(540 \times 10^{-6} \text{ mm})}{5.0 \text{ mm}} = 1.3 \times 10^{-4} \text{ rad}.$$

(b) The linear separation is $D = L\theta_R = (160 \times 10^3 \text{ m})(1.3 \times 10^{-4} \text{ rad}) = 21 \text{ m}$.

75. **THINK** Maxima of a diffraction grating pattern occur at angles θ given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer.

EXPRESS The ruling separation is given by

$$d = \frac{1}{200 \text{ mm}^{-1}} = 5.00 \times 10^{-3} \text{ mm} = 5.00 \times 10^{-6} \text{ m} = 5000 \text{ nm}.$$

Letting $d \sin \theta = m\lambda$, we solve for λ :

$$\lambda = \frac{d \sin \theta}{m} = \frac{(5000 \text{ nm})(\sin 30^\circ)}{m} = \frac{2500 \text{ nm}}{m}$$

where $m = 1, 2, 3, \dots$. In the visible light range m can assume the following values: $m_1 = 4$, $m_2 = 5$ and $m_3 = 6$.

(a) The longest wavelength corresponds to $m_1 = 4$ with $\lambda_1 = 2500 \text{ nm}/4 = 625 \text{ nm}$.

(b) The second longest wavelength corresponds to $m_2 = 5$ with $\lambda_2 = 2500 \text{ nm}/5 = 500 \text{ nm}$.

(c) The third longest wavelength corresponds to $m_3 = 6$ with $\lambda_3 = 2500 \text{ nm}/6 = 416 \text{ nm}$.

LEARN As shown above, only three values of m give wavelengths that are in the visible spectrum. Note that if the light incident on the diffraction grating is not monochromatic, a *spectrum* would be observed since the grating spreads out light into its component wavelength,

76. We combine Eq. 36-31 ($R = \lambda_{\text{avg}}/\Delta\lambda$) with Eq. 36-32 ($R = Nm$) and solve for N :

$$N = \frac{\lambda_{\text{avg}}}{m \Delta\lambda} = \frac{590.2 \text{ nm}}{2 (0.061 \text{ nm})} = 4.84 \times 10^3.$$

77. **THINK** The condition for a minimum of intensity in a single-slit diffraction pattern is given by $a \sin \theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer.

EXPRESS As a slit is narrowed, the pattern spreads outward, so the question about “minimum width” suggests that we are looking at the lowest possible values of m (the label for the minimum produced by light $\lambda = 600 \text{ nm}$) and m' (the label for the minimum produced by light $\lambda' = 500 \text{ nm}$). Since the angles are the same, then Eq. 36-3 leads to

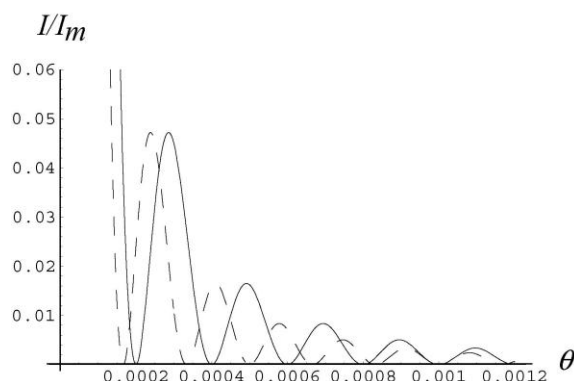
$$m\lambda = m'\lambda'$$

which leads to the choices $m = 5$ and $m' = 6$.

ANALYZE We find the slit width from Eq. 36-3:

$$a = \frac{m\lambda}{\sin \theta} = \frac{5(600 \times 10^{-9} \text{ m})}{\sin(1.00 \times 10^{-9} \text{ rad})} = 3.00 \times 10^{-3} \text{ m}.$$

LEARN The intensities of the diffraction are shown next (solid line for orange light, and dashed line for blue-green light). The angle $\theta = 0.001 \text{ rad}$ corresponds to $m = 5$ for the orange light, but $m' = 6$ for the blue-green light.



78. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$. The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \leq m \leq +5$. Thus, we find eleven values of m that satisfy this requirement.

79. **THINK** We relate the resolving power of a diffraction grating to the frequency range.

EXPRESS Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm , the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\Delta\lambda + \lambda\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f} \Delta f = -\frac{\lambda^2}{c} \Delta f$$

where $f = c/\lambda$ is used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength.

ANALYZE (a) Equating the two expressions for $\Delta\lambda$, we have

$$\frac{\lambda^2}{c} \Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are

separated by $(N - 1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N - 1)d \sin \theta$ and the time difference is

$$\Delta t = \frac{(N - 1)d \sin \theta}{c}.$$

If N is large, this may be approximated by $\Delta t = (Nd/c) \sin \theta$. The lens does not affect the travel time.

(c) Substituting the expressions we derived for Δt and Δf , we obtain

$$\Delta f \Delta t = \left(\frac{c}{Nm\lambda} \right) \left(\frac{Nd \sin \theta}{c} \right) = \frac{d \sin \theta}{m\lambda} = 1.$$

The condition $d \sin \theta = m\lambda$ for a diffraction line is used to obtain the last result.

LEARN We take Δf to be positive and interpret it as the range of frequencies that can be resolved.

80. Eq. 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — “Pointillistic paintings use the diffraction of your eye.” We are asked to solve for D and are given $\lambda = 500 \times 10^{-9} \text{ m}$, $d = 5.00 \times 10^{-3} \text{ m}$, and $L = 0.250 \text{ m}$. Consequently, $D = 3.05 \times 10^{-5} \text{ m}$.

81. Consider two of the rays shown in Fig. 36-49, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray’s paths are here referred to as points A and C . Where the bottom ray changes direction is point B . We note that angle $\angle APB$ is the same as ψ , and angle $\angle BPC$ is the same as θ (see Fig. 36-49). The difference in path lengths between the two adjacent light rays is

$$\Delta x = |AB| + |BC| = d \sin \psi + d \sin \theta.$$

The condition for bright fringes to occur is therefore

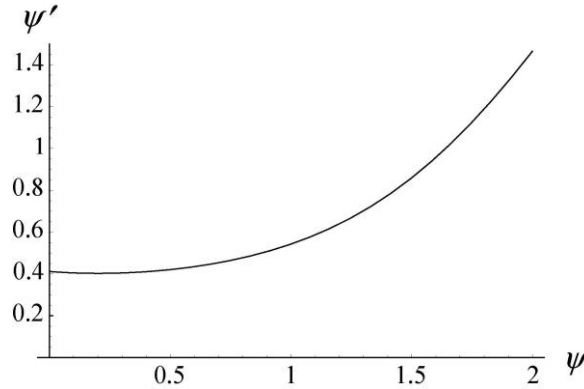
$$\Delta x = d(\sin \psi + \sin \theta) = m\lambda$$

where $m = 0, 1, 2, \dots$. If we set $\psi = 0$ then this reduces to Eq. 36-25.

82. The angular deviation of a diffracted ray (the angle between the forward extrapolation of the incident ray and its diffracted ray) is $\psi' = \psi + \theta$. For $m = 1$, this becomes

$$\psi' = \psi + \theta = \psi + \sin^{-1} \left(\frac{\lambda}{d} - \sin \psi \right)$$

where the ratio $\lambda/d = 0.40$ using the values given in the problem statement. The graph of this is shown next (with radians used along both axes).



83. **THINK** For relatively wide slits, we consider both the interference of light from two slits, as well as the diffraction of light passing through each slit.

EXPRESS The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$ is the angle that corresponds to the first diffraction minimum. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1} \left(\frac{\lambda}{a} \right) < \sin^{-1} \left(\frac{m\lambda}{d} \right) < +\sin^{-1} \left(\frac{\lambda}{a} \right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

The equation above sets the range of allowable values of m .

ANALYZE (a) Rewriting the equation as $-d/a < m < +d/a$, noting that $d/a = (14 \mu\text{m})/(2.0 \mu\text{m}) = 7$, we arrive at the result $-7 < m < +7$, or (since m must be an integer) $-6 \leq m \leq +6$,

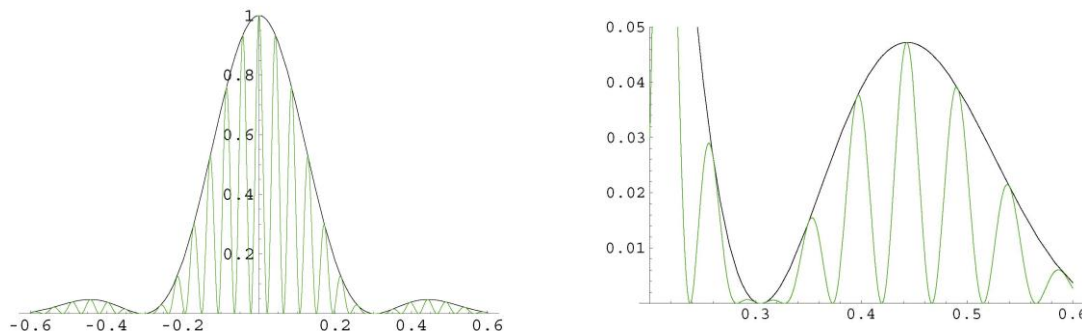
which amounts to 13 distinct values for m . Thus, thirteen maxima are within the central envelope.

(b) The range (within *one* of the first-order envelopes) is now

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{2\lambda}{a}\right),$$

which leads to $d/a < m < 2d/a$ or $7 < m < 14$. Since m is an integer, this means $8 \leq m \leq 13$ which includes 6 distinct values for m in that one envelope. If we were to include the total from both first-order envelopes, the result would be twelve, but the wording of the problem implies six should be the answer (just one envelope).

LEARN The intensity of the double-slit interference experiment is plotted below. The central diffraction envelope contains 13 maxima, and the first-order envelope has 6 on each side.



84. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where $\theta_1 = \sin^{-1}(\lambda/a)$. The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\max} < d/a \leq m_{\max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\max} + 1 = 17$ so that $m_{\max} = 8$, and the result becomes

$$8 < \frac{d}{a} \leq 9$$

where these numbers are as accurate as the experiment allows (that is, “9” means “9.000” if our measurements are that good).

85. We see that the total number of lines on the grating is $(1.8 \text{ cm})(1400/\text{cm}) = 2520 = N$. Combining Eq. 36-31 and Eq. 36-32, we find

$$\Delta\lambda = \frac{\lambda_{\text{avg}}}{Nm} = \frac{450 \text{ nm}}{(2520)(3)} = 0.0595 \text{ nm} = 59.5 \text{ pm}.$$

86. Use of Eq. 36-21 leads to $D = \frac{1.22\lambda L}{d} = 6.1 \text{ mm}$.

87. Following the method of Sample Problem — “Pointillistic paintings use the diffraction of your eye,” we have

$$\frac{1.22\lambda}{d} = \frac{D}{L}$$

where $\lambda = 550 \times 10^{-9} \text{ m}$, $D = 0.60 \text{ m}$, and $d = 0.0055 \text{ m}$. Thus we get $L = 4.9 \times 10^3 \text{ m}$.

88. We use Eq. 36-3 for $m = 2$: $m\lambda = a \sin \theta \Rightarrow \frac{a}{\lambda} = \frac{m}{\sin \theta} = \frac{2}{\sin 37^\circ} = 3.3$.

89. We solve Eq. 36-25 for d :

$$d = \frac{m\lambda}{\sin \theta} = \frac{2(600 \times 10^{-9} \text{ m})}{\sin 33^\circ} = 2.203 \times 10^{-6} \text{ m} = 2.203 \times 10^{-4} \text{ cm}$$

which is typically expressed in reciprocal form as the “number of lines per centimeter” (or per millimeter, or per inch):

$$\frac{1}{d} = 4539 \text{ lines/cm}.$$

The full width is 3.00 cm, so the number of lines is $(4539/\text{cm})(3.00 \text{ cm}) = 1.36 \times 10^4$.

90. Although the angles in this problem are not particularly big (so that the small angle approximation could be used with little error), we show the solution appropriate for large as well as small angles (that is, we do not use the small angle approximation here). Equation 36-3 gives

$$m\lambda = a \sin \theta \Rightarrow \theta = \sin^{-1}(m\lambda/a) = \sin^{-1}[2(0.42 \mu\text{m})/(5.1 \mu\text{m})] = 9.48^\circ.$$

The geometry of Figure 35-10(a) is a useful reference (even though it shows a double slit instead of the single slit that we are concerned with here). We see in that figure the relation between y , D , and θ :

$$y = D \tan \theta = (3.2 \text{ m}) \tan(9.48^\circ) = 0.534 \text{ m}.$$

91. The problem specifies $d = 12/8900$ using the mm unit, and we note there are no refraction angles greater than 90° . We convert $\lambda = 500 \text{ nm}$ to $5 \times 10^{-4} \text{ mm}$ and solve Eq. 36-25 for " m_{max} " (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d \sin 90^\circ}{\lambda} = \frac{12}{(8900)(5 \times 10^{-4})} \approx 2$$

where we have rounded down. There are no values of m (for light of wavelength λ) greater than $m = 2$.

92. We denote the Earth-Moon separation as L . The energy of the beam of light that is projected onto the Moon is concentrated in a circular spot of diameter d_1 , where $d_1/L = 2\theta_R = 2(1.22\lambda/d_0)$, with d_0 the diameter of the mirror on Earth. The fraction of energy picked up by the reflector of diameter d_2 on the Moon is then $\eta' = (d_2/d_1)^2$. This reflected light, upon reaching the Earth, has a circular cross section of diameter d_3 satisfying

$$d_3/L = 2\theta_R = 2(1.22\lambda/d_2).$$

The fraction of the reflected energy that is picked up by the telescope is then $\eta'' = (d_0/d_3)^2$. Consequently, the fraction of the original energy picked up by the detector is

$$\begin{aligned} \eta = \eta' \eta'' &= \left(\frac{d_0}{d_3}\right)^2 \left(\frac{d_2}{d_1}\right)^2 = \left[\frac{d_0 d_2}{(2.44\lambda d_{em}/d_0)(2.44\lambda d_{em}/d_2)} \right]^2 = \left(\frac{d_0 d_2}{2.44\lambda d_{em}} \right)^4 \\ &= \left[\frac{(2.6 \text{ m})(0.10 \text{ m})}{2.44(0.69 \times 10^{-6} \text{ m})(3.82 \times 10^8 \text{ m})} \right]^4 \approx 4 \times 10^{-13}. \end{aligned}$$

93. Since we are considering the *diameter* of the central diffraction maximum, then we are working with *twice* the Rayleigh angle. Using notation similar to that in Sample Problem — “Pointillistic paintings use the diffraction of your eye,” we have $2(1.22\lambda/d) = D/L$. Therefore,

$$d = 2 \frac{1.22 \lambda L}{D} = 2 \frac{(1.22)(500 \times 10^{-9} \text{ m})(3.54 \times 10^5 \text{ m})}{9.1 \text{ m}} = 0.047 \text{ m}.$$

94. Letting $d \sin \theta = (L/N) \sin \theta = m\lambda$, we get

$$\lambda = \frac{(L/N) \sin \theta}{m} = \frac{(1.0 \times 10^7 \text{ nm})(\sin 30^\circ)}{(1)(10000)} = 500 \text{ nm} .$$

95. **THINK** We use phasors to explore how doubling slit width changes the intensity of the central maximum of diffraction and the energy passing through the slit.

EXPRESS We imagine dividing the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor. Then, at the central maximum in the diffraction pattern, we would add the N phasors, all in the same direction and each with the same amplitude. We would find that the intensity there is proportional to N^2 .

ANALYZE If we double the slit width, we need $2N$ phasors if they are each to have the amplitude of the phasors we used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

LEARN From the discussion above, we see that the intensity of the central maximum increases as N^2 . The dependence arises from the following two considerations: (1) The total power reaching the screen is proportional to N , and (2) the width of each maximum (distance between two adjacent minima) is proportional to $1/N$.

96. The condition for a minimum in a single-slit diffraction pattern is given by Eq. 36-3, which we solve for the wavelength:

$$\lambda = \frac{a \sin \theta}{m} = \frac{(0.022 \text{ mm}) \sin 1.8^\circ}{1} = 6.91 \times 10^{-4} \text{ mm} = 691 \text{ nm} .$$

97. Equation 36-14 gives the Rayleigh angle (in radians):

$$\theta_R = \frac{1.22\lambda}{d} = \frac{D}{L}$$

where the rationale behind the second equality is given in Sample Problem — “Pointillistic paintings use the diffraction of your eye.” We are asked to solve for d and are given $\lambda = 550 \times 10^{-9} \text{ m}$, $D = 30 \times 10^{-2} \text{ m}$, and $L = 160 \times 10^3 \text{ m}$. Consequently, we obtain $d = 0.358 \text{ m} \approx 36 \text{ cm}$.

98. Following Sample Problem — “Pointillistic paintings use the diffraction of your eye,”

we use Eq. 36-17 and obtain $L = \frac{Dd}{1.22\lambda} = 164 \text{ m}$.

99. (a) Use of Eq. 36-25 for the limit-wavelengths ($\lambda_1 = 700 \text{ nm}$ and $\lambda_2 = 550 \text{ nm}$) leads to the condition

$$m_1 \lambda_1 \geq m_2 \lambda_2$$

for $m_1 + 1 = m_2$ (the low end of a high-order spectrum is what is overlapping with the high end of the next-lower-order spectrum). Assuming equality in the above equation, we can solve for “ m_1 ” (realizing it might not be an integer) and obtain $m_1 \approx 4$ where we have rounded *up*. It is the fourth-order spectrum that is the lowest-order spectrum to overlap with the next higher spectrum.

(b) The problem specifies $d = (1/200) \text{ mm}$, and we note there are no refraction angles greater than 90° . We concentrate on the largest wavelength $\lambda = 700 \text{ nm} = 7 \times 10^{-4} \text{ mm}$ and solve Eq. 36-25 for “ m_{max} ” (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d \sin 90^\circ}{\lambda} = \frac{(1/200) \text{ mm}}{7 \times 10^{-4} \text{ mm}} \approx 7$$

where we have rounded down. There are no values of m (for the appearance of the full spectrum) greater than $m = 7$.

100. (a) Maxima of a diffraction grating pattern occur at angles θ given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. With $\theta = 30^\circ$, and $d = (1 \text{ mm})/200 = 5.0 \times 10^{-6} \text{ m}$, the wavelengths for the m th order maxima are given by

$$\lambda = \frac{d \sin \theta}{m} = \frac{(5.0 \times 10^{-6} \text{ m}) \sin 30^\circ}{m} = \frac{2.5 \times 10^{-6} \text{ m}}{m} = \frac{2500 \text{ nm}}{m}$$

For the light to be in the visible spectrum ($400 - 750 \text{ nm}$), the values of m are $m = 4, 5$, and 6 . The wavelengths are: $\lambda_4 = (2500 \text{ nm})/4 = 625 \text{ nm}$, $\lambda_5 = (2500 \text{ nm})/5 = 500 \text{ nm}$, and $\lambda_6 = (2500 \text{ nm})/6 = 417 \text{ nm}$.

(c) The three wavelengths correspond to orange, blue-green, and violet, respectively.

101. The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ . The angular position and wavelength are related by $\mathbf{d} \sin \theta = m\lambda$, where \mathbf{d} is the slit separation (which we made boldfaced in order not to confuse it with the d used in the derivative, below) and m is an integer. We differentiate this expression with respect to θ to obtain

$$\frac{d\theta}{d\lambda} \mathbf{d} \cos \theta = m,$$

or

$$D = \frac{d\theta}{d\lambda} = \frac{m}{d \cos \theta}.$$

Now $m = (d/\lambda) \sin \theta$, so $D = \frac{d \sin \theta}{d \lambda \cos \theta} = \frac{\tan \theta}{\lambda}$.

102. (a) Employing Eq. 36-3 with the small angle approximation ($\sin \theta \approx \tan \theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with $m = 1$)

$$D = \frac{ya}{m\lambda} = \frac{(0.90 \text{ mm})(0.40 \text{ mm})}{4.50 \times 10^{-4} \text{ mm}} = 800 \text{ mm} = 80 \text{ cm}$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for $m = 3$)

$$y = \frac{(3)\lambda D}{a} = \frac{(3)(4.50 \times 10^{-4} \text{ mm})(800 \text{ mm})}{(0.40 \text{ mm})} = 2.7 \text{ mm}.$$

Subtracting this from the first minimum position $y = 0.9 \text{ mm}$, we find the result $\Delta y = 1.8 \text{ mm}$.

103. (a) We require that $\sin \theta = m\lambda_{1,2}/d \leq \sin 30^\circ$, where $m = 1, 2$ and $\lambda_1 = 500 \text{ nm}$. This gives

$$d \geq \frac{2\lambda_s}{\sin 30^\circ} = \frac{2(600 \text{ nm})}{\sin 30^\circ} = 2400 \text{ nm} = 2.4 \mu\text{m}.$$

For a grating of given total width L we have $N = L/d \propto d^{-1}$, so we need to minimize d to maximize $R = mN \propto d^{-1}$. Thus we choose $d = 2400 \text{ nm} = 2.4 \mu\text{m}$.

(b) Let the third-order maximum for $\lambda_2 = 600 \text{ nm}$ be the first minimum for the single-slit diffraction profile. This requires that $d \sin \theta = 3\lambda_2 = a \sin \theta$, or

$$a = d/3 = 2400 \text{ nm}/3 = 800 \text{ nm} = 0.80 \mu\text{m}.$$

(c) Letting $\sin \theta = m_{\max} \lambda_2 / d \leq 1$, we obtain

$$m_{\max} \leq \frac{d}{\lambda_2} = \frac{2400 \text{ nm}}{800 \text{ nm}} = 3.$$

Since the third order is missing the only maxima present are the ones with $m = 0, 1$ and 2 . Thus, the largest order of maxima produced by the grating is $m = 2$.

104. For $\lambda = 0.10$ nm, we have scattering for order m , and for $\lambda' = 0.075$ nm, we have scattering for order m' . From Eq. 36-34, we see that we must require $m\lambda = m'\lambda'$, which suggests (looking for the smallest integer solutions) that $m = 3$ and $m' = 4$. Returning with this result and with $d = 0.25$ nm to Eq. 36-34, we obtain

$$\theta = \sin^{-1} \frac{m\lambda}{2d} = 37^\circ.$$

Studying Figure 36-30, we conclude that the angle between incident and scattered beams is $180^\circ - 2\theta = 106^\circ$.

105. The key trigonometric identity used in this proof is $\sin(2\theta) = 2\sin\theta \cos\theta$. Now, we wish to show that Eq. 36-19 becomes (when $d = a$) the pattern for a single slit of width $2a$ (see Eq. 36-5 and Eq. 36-6):

$$I(\theta) = I_m \left(\frac{\sin(2\pi a \sin\theta/\lambda)}{2\pi a \sin\theta/\lambda} \right)^2.$$

We note from Eq. 36-20 and Eq. 36-21, that the parameters β and α are identical in this case (when $d = a$), so that Eq. 36-19 becomes

$$I(\theta) = I_m \left(\frac{\cos(\pi a \sin\theta/\lambda) \sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right)^2.$$

Multiplying numerator and denominator by 2 and using the trig identity mentioned above, we obtain

$$I(\theta) = I_m \left(\frac{2\cos(\pi a \sin\theta/\lambda) \sin(\pi a \sin\theta/\lambda)}{2\pi a \sin\theta/\lambda} \right)^2 = I_m \left(\frac{\sin(2\pi a \sin\theta/\lambda)}{2\pi a \sin\theta/\lambda} \right)^2$$

which is what we set out to show.

106. Employing Eq. 36-3, we find (with $m = 3$ and all lengths in μm)

$$\theta = \sin^{-1} \frac{m\lambda}{a} = \sin^{-1} \frac{(3)(0.5)}{2}$$

which yields $\theta = 48.6^\circ$. Now, we use the experimental geometry ($\tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern) to find

$$y = D \tan\theta = 2.27 \text{ m}.$$

107. (a) The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1} \frac{\lambda}{a},$$

which could be further simplified *if* the small-angle approximation were justified (which it is *not*, since a is so small). The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d},$$

so that our range specification becomes

$$-\sin^{-1}\left(\frac{\lambda}{a}\right) < \sin^{-1}\left(\frac{m\lambda}{d}\right) < +\sin^{-1}\left(\frac{\lambda}{a}\right),$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a}.$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\max} < d/a \leq m_{\max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\max} + 1 = 17$ so that $m_{\max} = 8$, and the result becomes

$$8 < \frac{d}{a} \leq 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

108. We refer (somewhat sloppily) to the 400 nm wavelength as “blue” and the 700 nm wavelength as “red.” Consider Eq. 36-25 ($m\lambda = d \sin\theta$), for the 3rd order blue, and also for the 2nd order red:

$$(3) \lambda_{\text{blue}} = 400 \text{ nm} = d \sin(\theta_{\text{blue}})$$

$$(2) \lambda_{\text{red}} = 700 \text{ nm} = d \sin(\theta_{\text{red}}).$$

Since sine is an increasing function of angle (in the first quadrant) then the above set of values make clear that $\theta_{\text{red (second order)}} > \theta_{\text{blue (third order)}}$ which shows that the spectrums overlap (regardless of the value of d).

109. One strategy is to divide Eq. 36-25 by Eq. 36-3, assuming the same angle (a point we’ll come back to, later) and the same light wavelength for both:

$$\frac{m}{m'} = \frac{m\lambda}{m'\lambda} = \frac{d \sin \theta}{a \sin \theta} = \frac{d}{a}.$$

We recall that d is measured from middle of transparent strip to the middle of the next transparent strip, which in this particular setup means $d = 2a$. Thus, $m/m' = 2$, or $m = 2m'$.

Now we interpret our result. First, the division of the equations is not valid when $m = 0$ (which corresponds to $\theta = 0$), so our remarks do not apply to the $m = 0$ maximum. Second, Eq. 36-25 gives the “bright” interference results, and Eq. 36-3 gives the “dark” diffraction results (where the latter overrules the former in places where they coincide – see Figure 36-17 in the textbook). For $m' =$ any nonzero integer, the relation $m = 2m'$ implies that $m =$ any nonzero *even* integer. As mentioned above, these are occurring at the same angle, so the even integer interference maxima are eliminated by the diffraction minima.

110. The derivation is similar to that used to obtain Eq. 36-27. At the first minimum beyond the m th principal maximum, two waves from adjacent slits have a phase difference of $\Delta\phi = 2\pi m + (2\pi/N)$, where N is the number of slits. This implies a difference in path length of

$$\Delta L = (\Delta\phi/2\pi)\lambda = m\lambda + (\lambda/N).$$

If θ_m is the angular position of the m th maximum, then the difference in path length is also given by $\Delta L = d \sin(\theta_m + \Delta\theta)$. Thus

$$d \sin(\theta_m + \Delta\theta) = m\lambda + (\lambda/N).$$

We use the trigonometric identity

$$\sin(\theta_m + \Delta\theta) = \sin \theta_m \cos \Delta\theta + \cos \theta_m \sin \Delta\theta.$$

Since $\Delta\theta$ is small, we may approximate $\sin \Delta\theta$ by $\Delta\theta$ in radians and $\cos \Delta\theta$ by unity. Thus,

$$d \sin \theta_m + d \Delta\theta \cos \theta_m = m\lambda + (\lambda/N).$$

We use the condition $d \sin \theta_m = m\lambda$ to obtain $d \Delta\theta \cos \theta_m = \lambda/N$ and

$$\Delta\theta = \frac{\lambda}{N d \cos \theta_m}.$$

111. There are two unknowns, the x-ray wavelength λ and the plane separation d , so data for scattering at two angles from the same planes should suffice. The observations obey Bragg’s law, so

$$2d \sin \theta_1 = m_1 \lambda, \quad 2d \sin \theta_2 = m_2 \lambda.$$

However, these cannot be solved for the unknowns. For example, we can use the first equation to eliminate λ from the second. We obtain

$$m_2 \sin \theta_1 = m_1 \sin \theta_2,$$

an equation that does not contain either of the unknowns.

112. The problem specifies $d = (1 \text{ mm})/500 = 2.00 \text{ } \mu\text{m}$ unit, and we note there are no refraction angles greater than 90° . We concentrate on the largest wavelength $\lambda = 700 \text{ nm} = 0.700 \text{ } \mu\text{m}$ and solve Eq. 36-25 for " m_{max} " (realizing it might not be an integer):

$$m_{\text{max}} = \frac{d \sin 90^\circ}{\lambda} = \frac{d}{\lambda} = \frac{2.00 \text{ } \mu\text{m}}{0.700 \text{ } \mu\text{m}} \approx 2$$

where we have rounded down. There are no values of m (for appearance of the full spectrum) greater than $m = 2$.

113. When the speaker phase difference is $\pi \text{ rad}$ (180°), we expect to see the "reverse" of Fig. 36-15 [translated into the acoustic context, so that "bright" becomes "loud" and "dark" becomes "quiet"]. That is, with 180° phase difference, all the peaks in Fig. 36-15 become valleys and all the valleys become peaks. As the phase changes from zero to 180° (and similarly for the change from 180° back to $360^\circ = \text{original pattern}$), the peaks should shift (and change height) in a continuous fashion – with the most dramatic feature being a large "dip" in the center diffraction envelope which deepens until it seems to split the central maximum into smaller diffraction maxima which (once the phase difference reaches $\pi \text{ rad}$) will be located at angles given by $a \sin \theta = \pm \lambda$. How many interference fringes would actually "be inside" each of these smaller diffraction maxima would, of course, depend on the particular values of a , λ and d .

114. From $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer, we write

$$d \sin(\theta + \Delta\theta) = m(\lambda + \Delta\lambda)$$

Subtracting the first equation from the second gives

$$d [\sin(\theta + \Delta\theta) - \sin \theta] = m(\lambda + \Delta\lambda) - m\lambda = m\Delta\lambda.$$

Noting that

$$\lim_{\Delta\theta \rightarrow 0} \frac{\sin(\theta + \Delta\theta) - \sin \theta}{\Delta\theta} = \cos \theta,$$

the above expression simplifies to

$$\cos \theta = \frac{m\Delta\lambda}{d\Delta\theta}.$$

Thus,

$$\Delta\theta = \frac{m\Delta\lambda}{d \cos \theta} = \frac{m\Delta\lambda}{d \sqrt{1 - \sin^2 \theta}} = \frac{m\Delta\lambda}{d \sqrt{1 - (m\lambda/d)^2}} = \frac{m\Delta\lambda}{\sqrt{d^2 - (m\lambda)^2}} = \frac{\Delta\lambda}{\sqrt{(d/m)^2 - \lambda^2}}.$$