Chapter 43

1. (a) Using Eq. 42-20 and adapting Eq. 42-21 to this sample, the number of fission-events per second is

$$R_{\text{fission}} = \frac{N \ln 2}{T_{1/2_{\text{fission}}}} = \frac{M_{\text{sam}} N_A \ln 2}{M_{\text{U}} T_{1/2_{\text{fission}}}}$$
$$= \frac{(1.0 \text{ g})(6.02 \times 10^{23} / \text{mol}) \ln 2}{(235 \text{ g}/\text{mol})(3.0 \times 10^{17} \text{ y})(365 \text{ d}/\text{y})} = 16 \text{ fissions}/\text{day}.$$

(b) Since $R \propto 1/T_{1/2}$ (see Eq. 42-20), the ratio of rates is

$$\frac{R_\alpha}{R_{\text{fission}}} = \frac{T_{1/2_{\text{fission}}}}{\mathrm{T}_{1/2_\alpha}} = \frac{3.0 \times 10^{17} \text{ y}}{7.0 \times 10^{8} \text{ y}} = 4.3 \times 10^{8}.$$

2. When a neutron is captured by ^{237}Np it gains 5.0 MeV, more than enough to offset the 4.2 MeV required for ^{238}Np to fission. Consequently, ^{237}Np is fissionable by thermal neutrons.

3. The energy transferred is

$$\begin{aligned} Q &= (m_{\text{U238}} + m_n - m_{\text{U239}})c^2 \\ &= (238.050782 \text{ u} + 1.008664 \text{ u} - 239.054287 \text{ u})(931.5 \text{ MeV/u}) \\ &= 4.8 \text{ MeV}. \end{aligned}$$

4. Adapting Eq. 42-21, there are

$$N_{\text{Pu}} = \frac{M_{\text{sam}}}{M_{\text{Pu}}} NA = \left(\frac{1000 \text{ g}}{239 \text{ g}/\text{mol}}\right)(6.02 \times 10^{23} / \text{mol}) = 2.5 \times 10^{24}$$

plutonium nuclei in the sample. If they all fission (each releasing 180 MeV), then the total energy release is 4.54×10^{26} MeV.

5. The yield of one warhead is 2.0 megatons of TNT, or

$$\text{yield} = 2(2.6 \times 10^{28} \text{ MeV}) = 5.2 \times 10^{28} \text{ MeV}.$$

Since each fission event releases about 200 MeV of energy, the number of fissions is

$$N = \frac{5.2\times10^{28}\ \text{MeV}}{200\ \text{MeV}} = 2.6\times10^{26}.$$

However, this only pertains to the 8.0% of Pu that undergoes fission, so the total number of Pu is

$$N_0 = \frac{N}{0.080} = \frac{2.6\times10^{26}}{0.080} = 3.25\times10^{27} = 5.4\times10^{3}\ \text{mol}.$$

With $M = 0.239$ kg/mol, the mass of the warhead is

$$m = (5.4\times10^{3}\ \text{mol})(0.239\ \text{kg/mol}) = 1.3\times10^{3}\ \text{kg}.$$

6. We note that the sum of superscripts (mass numbers A) must balance, as well as the sum of Z values (where reference to Appendix F or G is helpful). A neutron has $Z = 0$ and $A = 1$. Uranium has $Z = 92$.

(a) Since xenon has $Z = 54$, then "Y" must have $Z = 92 – 54 = 38$, which indicates the element strontium. The mass number of "Y" is 235 + 1 – 140 – 1 = 95, so "Y" is ^{95}Sr.

(b) Iodine has $Z = 53$, so "Y" has $Z = 92 – 53 = 39$, corresponding to the element yttrium (the symbol for which, coincidentally, is Y). Since 235 + 1 – 139 – 2 = 95, then the unknown isotope is ^{95}Y.

(c) The atomic number of zirconium is $Z = 40$. Thus, 92 – 40 – 2 = 52, which means that "X" has $Z = 52$ (tellurium). The mass number of "X" is 235 + 1 – 100 – 2 = 134, so we obtain ^{134}Te.

(d) Examining the mass numbers, we find b = 235 + 1 – 141 – 92 = 3.

7. If R is the fission rate, then the power output is $P = RQ$, where Q is the energy released in each fission event. Hence,

$$R = P/Q = (1.0\ \text{W})/(200 \times 10^{6}\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV}) = 3.1 \times 10^{10}\ \text{fissions/s}.$$

8. (a) We consider the process $^{98}\text{Mo} \to {}^{49}\text{Sc} + {}^{49}\text{Sc}$. The disintegration energy is

$$Q = (m_{\text{Mo}} - 2m_{\text{Sc}})c^2 = [97.90541\ \text{u} - 2(48.95002\ \text{u})](931.5\ \text{MeV/u}) = +5.00\ \text{MeV}.$$

(b) The fact that it is positive does not necessarily mean we should expect to find a great deal of molybdenum nuclei spontaneously fissioning; the energy barrier (see Fig. 43-3) is presumably higher and/or broader for molybdenum than for uranium.

9. (a) The mass of a single atom of ^{235}U is

$$m_0 = (235\ \text{u})(1.661 \times 10^{-27}\ \text{kg/u}) = 3.90 \times 10^{-25}\ \text{kg},$$

so the number of atoms in m = 1.0 kg is

$$N = m/m_0 = (1.0\text{ kg})/(3.90 \times 10^{-25}\text{ kg}) = 2.56 \times 10^{24} \approx 2.6 \times 10^{24}.$$

An alternate approach (but essentially the same once the connection between the "u" unit and N_A is made) would be to adapt Eq. 42-21.

(b) The energy released by N fission events is given by $E = NQ$, where Q is the energy released in each event. For 1.0 kg of ^{235}U,

$$E = (2.56 \times 10^{24})(200 \times 10^6\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}) = 8.19 \times 10^{13}\text{ J} \approx 8.2 \times 10^{13}\text{ J}.$$

(c) If P is the power requirement of the lamp, then

$$t = E/P = (8.19 \times 10^{13}\text{ J})/(100\text{ W}) = 8.19 \times 10^{11}\text{ s} = 2.6 \times 10^4\text{ y}.$$

The conversion factor 3.156×10^7 s/y is used to obtain the last result.

10. The energy released is

$$\begin{aligned} Q &= (m_U + m_n - m_{Cs} - m_{Rb} - 2m_n)c^2 \\ &= (235.04392\text{ u} - 1.00867\text{ u} - 140.91963\text{ u} - 92.92157\text{ u})(931.5\text{ MeV/u}) \\ &= 181\text{ MeV}. \end{aligned}$$

11. If M_{Cr} is the mass of a ^{52}Cr nucleus and M_{Mg} is the mass of a ^{26}Mg nucleus, then the disintegration energy is

$$Q = (M_{Cr} - 2M_{Mg})c^2 = [51.94051\text{ u} - 2(25.98259\text{ u})](931.5\text{ MeV/u}) = -23.0\text{ MeV}.$$

12. (a) Consider the process $^{239}\text{U} + \text{n} \to {}^{140}\text{Ce} + {}^{99}\text{Ru} + \text{Ne}.$ We have

$$Z_f - Z_i = Z_{Ce} + Z_{Ru} - Z_U = 58 + 44 - 92 = 10.$$

Thus the number of beta-decay events is 10.

(b) Using Table 37-3, the energy released in this fission process is

$$\begin{aligned} Q &= (m_U + m_n - m_{Ce} - m_{Ru} - 10m_e)c^2 \\ &= (238.05079\text{ u} + 1.00867\text{ u} - 139.90543\text{ u} - 98.90594\text{ u})(931.5\text{ MeV/u}) - 10(0.511\text{ MeV}) \\ &= 226\text{ MeV}. \end{aligned}$$

13. (a) The electrostatic potential energy is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Z_{\text{Xe}}Z_{\text{Sr}}e^2}{r_{\text{Xe}} + r_{\text{Sr}}}$$

where Z_{Xe} is the atomic number of xenon, Z_{Sr} is the atomic number of strontium, r_{Xe} is the radius of a xenon nucleus, and r_{Sr} is the radius of a strontium nucleus. Atomic numbers can be found either in Appendix F or Appendix G. The radii are given by r = (1.2 fm)$A^{1/3}$, where A is the mass number, also found in Appendix F. Thus,

$$r_{\text{Xe}} = (1.2\text{ fm})(140)^{1/3} = 6.23\text{ fm} = 6.23 \times 10^{-15}\text{ m}$$

and

$$r_{\text{Sr}} = (1.2\text{ fm})(96)^{1/3} = 5.49\text{ fm} = 5.49 \times 10^{-15}\text{ m}.$$

Hence, the potential energy is

$$U = (8.99\times10^{9}\text{ V}\cdot\text{m/C})\frac{(54)(38)(1.60\times10^{-19}\text{ C})^2}{6.23\times10^{-15}\text{ m} + 5.49\times10^{-15}\text{ m}} = 4.08\times10^{-11}\text{ J}$$
$$= 251\text{ MeV}.$$

(b) The energy released in a typical fission event is about 200 MeV, roughly the same as the electrostatic potential energy when the fragments are touching. The energy appears as kinetic energy of the fragments and neutrons produced by fission.

14. (a) The surface area a of a nucleus is given by

$$a \simeq 4\pi R^2 \simeq 4\pi\left(R_0 A^{1/3}\right)^2 \propto A^{2/3}.$$

Thus, the fractional change in surface area is

$$\frac{\Delta a}{a_i} = \frac{a_f - a_i}{a_i} = \frac{(140)^{2/3} + (96)^{2/3}}{(236)^{2/3}} - 1 = +0.25.$$

(b) Since $V \propto R^3 \propto (A^{1/3})^3 = A$, we have

$$\frac{\Delta V}{V} = \frac{V_f}{V_i} - 1 = \frac{140 + 96}{236} - 1 = 0.$$

(c) The fractional change in potential energy is

$$\frac{\Delta U}{U} = \frac{U_f}{U_i} - 1 = \frac{Q_{\text{Xe}}^2 / R_{\text{Xe}} + Q_{\text{Sr}}^2 / R_{\text{Sr}}}{Q_{\text{U}}^2 / R_{\text{U}}} - 1 = \frac{(54)^2(140)^{-1/3} + (38)^2(96)^{-1/3}}{(92)^2(236)^{-1/3}} - 1$$
$$= -0.36.$$

15. **THINK** One megaton of TNT releases 2.6×10^{28} MeV of energy. The energy released in each fission event is about 200 MeV.

EXPRESS The energy yield of the bomb is

$$E = (66 \times 10^{-3}\text{ megaton})(2.6 \times 10^{28}\text{ MeV/ megaton}) = 1.72 \times 10^{27}\text{ MeV}.$$

At 200 MeV per fission event, the total number of fission events taking place is

$$(1.72 \times 10^{27}\text{ MeV})/(200\text{ MeV}) = 8.58 \times 10^{24}.$$

Now, since only 4.0% of the ^{235}U nuclei originally present undergo fission, there must have been $(8.58 \times 10^{24})/(0.040) = 2.14 \times 10^{26}$ nuclei originally present.

ANALYZE (a) The mass of ^{235}U originally present was

$$(2.14 \times 10^{26})(235\text{ u})(1.661 \times 10^{-27}\text{ kg/u}) = 83.7\text{ kg} \approx 84\text{ kg}.$$

(b) Two fragments are produced in each fission event, so the total number of fragments is

$$2(8.58 \times 10^{24}) = 1.72 \times 10^{25} \approx 1.7 \times 10^{25}.$$

(c) One neutron produced in a fission event is used to trigger the next fission event, so the average number of neutrons released to the environment in each event is 1.5. The total number released is

$$(8.58 \times 10^{24})(1.5) = 1.29 \times 10^{25} \approx 1.3 \times 10^{25}.$$

LEARN When one ^{235}U nucleus undergoes fission, the neutrons it produces (an average number of 2.5 neutrons per fission) can trigger other ^{235}U nuclei to fission, thereby setting up a chain reaction that allows an enormous amount of energy to be released.

16. (a) Using the result of Problem 43-4, the TNT equivalent is

$$\frac{(2.50\text{ kg})(4.54 \times 10^{26}\text{ MeV / kg})}{2.6 \times 10^{28}\text{ MeV} / 10^{6}\text{ ton}} = 4.4 \times 10^{4}\text{ ton} = 44\text{ kton}.$$

(b) Assuming that this is a fairly inefficiently designed bomb, then much of the remaining 92.5 kg is probably “wasted” and was included perhaps to make sure the bomb did not “fizzle.” There is also an argument for having more than just the critical mass based on the short assembly time of the material during the implosion, but this so-called “super-critical mass,” as generally quoted, is much less than 92.5 kg, and does not necessarily have to be purely plutonium.

17. **THINK** We represent the unknown fragment as ${}^{A}_{Z}X$, where A and Z are its mass number and atomic number, respectively. Charge and mass number are conserved in the neutron-capture process.

EXPRESS The reaction can be written as

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{82}_{32}\mathrm{Ge} + {}^{A}_{Z}X\ .$$

Conservation of charge yields 92 + 0 = 32 + Z, so Z = 60. Conservation of mass number yields 235 + 1 = 83 + A, so A = 153.

ANALYZE (a) Looking in Appendix F or G for nuclides with Z = 60, we find that the unknown fragment is ${}^{153}_{60}\mathrm{Nd}$.

(b) We neglect the small kinetic energy and momentum carried by the neutron that triggers the fission event. Then,

$$Q = K_{\mathrm{Ge}} + K_{\mathrm{Nd}},$$

where K_{Ge} is the kinetic energy of the germanium nucleus and K_{Nd} is the kinetic energy of the neodymium nucleus. Conservation of momentum yields $\vec{p}_{\mathrm{Ge}} + \vec{p}_{\mathrm{Nd}} = 0$. Now, we can write the classical formula for kinetic energy in terms of the magnitude of the momentum vector:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that

$$K_{\mathrm{Nd}} = \frac{p_{\mathrm{Nd}}^2}{2M_{\mathrm{Nd}}} = \frac{p_{\mathrm{Ge}}^2}{2M_{\mathrm{Nd}}} = \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}\frac{p_{\mathrm{Ge}}^2}{2M_{\mathrm{Ge}}} = \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}}\ .$$

Thus, the energy equation becomes

$$Q = K_{\mathrm{Ge}} + \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}} = \frac{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}{M_{\mathrm{Nd}}}K_{\mathrm{Ge}}$$

and

$$K_{\mathrm{Ge}} = \frac{M_{\mathrm{Nd}}}{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}Q = \frac{153\ \mathrm{u}}{153\ \mathrm{u} + 83\ \mathrm{u}}(170\ \mathrm{MeV}) = 110\ \mathrm{MeV}.$$

(c) Similarly,

$$K_{\mathrm{Nd}} = \frac{M_{\mathrm{Ge}}}{M_{\mathrm{Nd}} + M_{\mathrm{Ge}}}Q = \frac{83\ \mathrm{u}}{153\ \mathrm{u} + 83\ \mathrm{u}}(170\ \mathrm{MeV}) = 60\ \mathrm{MeV}.$$

(d) The initial speed of the germanium nucleus is

$$v_{\mathrm{Ge}} = \sqrt{\frac{2K_{\mathrm{Ge}}}{M_{\mathrm{Ge}}}} = \sqrt{\frac{2(110\times10^{6}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})}{(83\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u})}} = 1.60\times10^{7}\ \mathrm{m/s}.$$

(e) The initial speed of the neodymium nucleus is

$$v_{\mathrm{Nd}} = \sqrt{\frac{2K_{\mathrm{Nd}}}{M_{\mathrm{ND}}}} = \sqrt{\frac{2(60\times10^{6}\ \mathrm{eV})(1.60\times10^{-19}\ \mathrm{J/eV})}{(153\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u})}} = 8.69\times10^{6}\ \mathrm{m/s}.$$

LEARN By momentum conservation, the two fragments fly apart in opposite directions.

18. If P is the power output, then the energy E produced in the time interval Δt (= 3 y) is

$$\begin{aligned} E &= P\,\Delta t = (200\times10^{6}\ \mathrm{W})(3\ \mathrm{y})(3.156\times10^{7}\ \mathrm{s/y}) = 1.89\times10^{16}\ \mathrm{J} \\ &= (1.89\times10^{16}\ \mathrm{J})/(1.60\times10^{-19}\ \mathrm{J/eV}) = 1.18\times10^{35}\ \mathrm{eV} \\ &= 1.18\times10^{29}\ \mathrm{MeV}. \end{aligned}$$

At 200 MeV per event, this means $(1.18\times10^{29})/200 = 5.90\times10^{26}$ fission events occurred. This must be half the number of fissionable nuclei originally available. Thus, there were $2(5.90\times10^{26}) = 1.18\times10^{27}$ nuclei. The mass of a ^{235}U nucleus is

$$(235\ \mathrm{u})(1.661\times10^{-27}\ \mathrm{kg/u}) = 3.90\times10^{-25}\ \mathrm{kg},$$

so the total mass of ^{235}U originally present was $(1.18\times10^{27})(3.90\times10^{-25}\ \mathrm{kg}) = 462$ kg.

19. After each time interval t_{gen} the number of nuclides in the chain reaction gets multiplied by k. The number of such time intervals that has gone by at time t is t/t_{gen}. For example, if the multiplication factor is 5 and there were 12 nuclei involved in the reaction to start with, then after one interval 60 nuclei are involved. And after another interval 300 nuclei are involved. Thus, the number of nuclides engaged in the chain reaction at time t is $N(t) = N_0 k^{t/t_{\mathrm{gen}}}$. Since $P \propto N$ we have

$$P(t) = P_0 k^{t/t_{\mathrm{gen}}}.$$

20. We use the formula from Problem 43-19:

$$P(t) = P_0 k^{t/t_{\mathrm{gen}}} = (400\,\mathrm{MW})(1.0003)^{(5.00\ \mathrm{min})(60\ \mathrm{s/min})/(0.00300\,\mathrm{s})} = 8.03\times10^{3}\,\mathrm{MW}.$$

21. If R is the decay rate then the power output is $P = RQ$, where Q is the energy produced by each alpha decay. Now

$$R = \lambda N = N \ln 2/T_{1/2},$$

where λ is the disintegration constant and $T_{1/2}$ is the half-life. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. If M is the total mass of material and m is the mass of a single ^{238}Pu nucleus, then

$$N = \frac{M}{m} = \frac{1.00\ \text{kg}}{(238\ \text{u})(1.661\times10^{-27}\ \text{kg/u})} = 2.53\times10^{24}.$$

Thus,

$$P = \frac{NQ\ln 2}{T_{1/2}} = \frac{(2.53\times10^{24})(5.50\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})(\ln 2)}{(87.7\ \text{y})(3.156\times10^{7}\ \text{s/y})} = 557\ \text{W}.$$

22. We recall Eq. 43-6:

$$Q \approx 200\ \text{MeV} = 3.2\times10^{-11}\ \text{J}.$$

It is important to bear in mind that watts multiplied by seconds give joules. From $E = Pt_{\text{gen}} = NQ$ we get the number of free neutrons:

$$N = \frac{Pt_{\text{gen}}}{Q} = \frac{(500\times10^{6}\ \text{W})(1.0\times10^{-3}\ \text{s})}{3.2\times10^{-11}\ \text{J}} = 1.6\times10^{16}.$$

23. **THINK** The neutron generation time t_{gen} in a reactor is the average time needed for a fast neutron emitted in a fission event to be slowed to thermal energies by the moderator and then initiate another fission event.

EXPRESS Let P_0 be the initial power output, P be the final power output, k be the multiplication factor, t be the time for the power reduction, and t_{gen} be the neutron generation time. Then, according to the result of Problem 43-19,

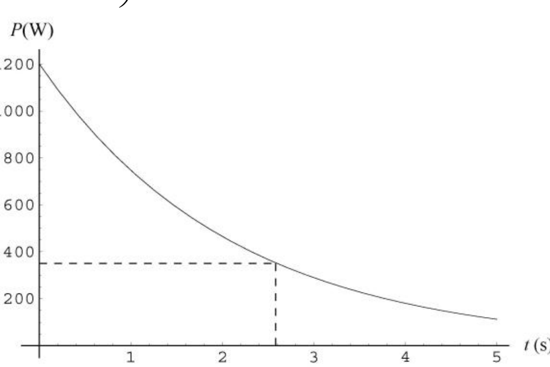
$$P = P_0 k^{t/t_{\text{gen}}}.$$

ANALYZE We divide by P_0, take the natural logarithm of both sides of the equation and solve for $\ln k$:

$$\ln k = \frac{t_{\text{gen}}}{t}\ln\left(\frac{P}{P_0}\right) = \frac{1.3\times10^{-3}\ \text{s}}{2.6\ \text{s}}\ln\left(\frac{350\ \text{MW}}{1200\ \text{MW}}\right) = -0.0006161.$$

Hence, $k = e^{-0.0006161} = 0.99938$.

LEARN The power output as a function of time is shown to the right. Since the multiplication factor k is smaller than 1, the output decreases with time.



24. (a) We solve Q_{eff} from $P = RQ_{\text{eff}}$:

$$\begin{aligned} Q_{\text{eff}} &= \frac{P}{R} = \frac{P}{N\lambda} = \frac{mPT_{1/2}}{M\ln 2} \\ &= \frac{(90.0\ \text{u})(1.66\times10^{-27}\ \text{kg/u})(0.93\ \text{W})(29\ \text{y})(3.15\times10^{7}\ \text{s/y})}{(1.00\times10^{-3}\ \text{kg})(\ln 2)(1.60\times10^{-13}\ \text{J/MeV})} \\ &= 1.2\ \text{MeV}. \end{aligned}$$

(b) The amount of ^{90}Sr needed is

$$M = \frac{150\ \text{W}}{(0.050)(0.93\ \text{W/g})} = 3.2\ \text{kg}.$$

25. **THINK** Momentum is conserved in the collision process. In addition, energy is also conserved since the collision is elastic.

EXPRESS Let v_{ni} be the initial velocity of the neutron, v_{nf} be its final velocity, and v_f be the final velocity of the target nucleus. Then, since the target nucleus is initially at rest, conservation of momentum yields

$$m_n v_{ni} = m_n v_{nf} + m v_f$$

and conservation of energy yields

$$\frac{1}{2}m_n v_{ni}^2 = \frac{1}{2}m_n v_{nf}^2 + \frac{1}{2}m v_f^2.$$

We solve these two equations simultaneously for v_f. This can be done, for example, by using the conservation of momentum equation to obtain an expression for v_{nf} in terms of v_f and substituting the expression into the conservation of energy equation. We solve the resulting equation for v_f. We obtain $v_f = 2m_n v_{ni}/(m + m_n)$.

ANALYZE (a) The energy lost by the neutron is the same as the energy gained by the target nucleus, so

$$\Delta K = \frac{1}{2}m v_f^2 = \frac{1}{2}\frac{4m_n^2 m}{(m+m_n)^2}v_{ni}^2.$$

The initial kinetic energy of the neutron is $K = \frac{1}{2}m_n v_{ni}^2$, so

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m+m_n)^2}.$$

(b) The mass of a neutron is 1.0 u and the mass of a hydrogen atom is also 1.0 u. (Atomic masses can be found in Appendix G.) Thus,

$$\frac{\Delta K}{K}=\frac{4(1.0\ \text{u})(1.0\ \text{u})}{(1.0\ \text{u}+1.0\ \text{u})^2}=1.0.$$

(c) Similarly, the mass of a deuterium atom is 2.0 u, so

$$(\Delta K)/K = 4(1.0\ \text{u})(2.0\ \text{u})/(2.0\ \text{u} + 1.0\ \text{u})^2 = 0.89.$$

(d) The mass of a carbon atom is 12 u, so

$$(\Delta K)/K = 4(1.0\ \text{u})(12\ \text{u})/(12\ \text{u} + 1.0\ \text{u})^2 = 0.28.$$

(e) The mass of a lead atom is 207 u, so

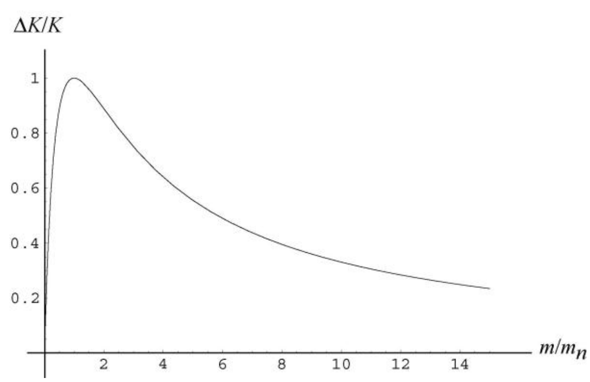
$$(\Delta K)/K = 4(1.0\ \text{u})(207\ \text{u})/(207\ \text{u} + 1.0\ \text{u})^2 = 0.019.$$

(f) During each collision, the energy of the neutron is reduced by the factor 1 – 0.89 = 0.11. If E_i is the initial energy, then the energy after n collisions is given by $E = (0.11)^n E_i$. We take the natural logarithm of both sides and solve for n. The result is

$$n=\frac{\ln(E/E_i)}{\ln 0.11}=\frac{\ln(0.025\ \text{eV}/1.00\ \text{eV})}{\ln 0.11}=7.9\approx 8.$$

The energy first falls below 0.025 eV on the eighth collision.

LEARN The fractional kinetic energy loss as a function of the mass of the stationary atom (in units of m/m_n) is plotted below.



From the plot, it is clear that the energy loss is greatest ($\Delta K/K = 1$) when the atom has the same as the neutron.

26. The ratio is given by

$$\frac{N_5(t)}{N_8(t)} = \frac{N_5(0)}{N_8(0)} e^{-(\lambda_5 - \lambda_8)t},$$

or

$$t = \frac{1}{\lambda_8 - \lambda_5} \ln\left[\left(\frac{N_5(t)}{N_8(t)}\right)\left(\frac{N_8(0)}{N_5(0)}\right)\right] = \frac{1}{(1.55 - 9.85)10^{-10}\ \text{y}^{-1}} \ln[(0.0072)(0.15)^{-1}]$$
$$= 3.6 \times 10^9\ \text{y}.$$

27. (a) $P_{avg} = (15 \times 10^9\ \text{W}\cdot\text{y})/(200{,}000\ \text{y}) = 7.5 \times 10^4\ \text{W} = 75\ \text{kW}$.

(b) Using the result of Eq. 43-6, we obtain

$$M = \frac{m_{\text{U}} E_{\text{total}}}{Q} = \frac{(235\text{u})(1.66\times10^{-27}\,\text{kg/u})(15\times10^9\,\text{W}\cdot\text{y})(3.15\times10^7\,\text{s/y})}{(200\,\text{MeV})(1.6\times10^{-13}\,\text{J/MeV})} = 5.8\times10^3\,\text{kg}.$$

28. The nuclei of ^{238}U can capture neutrons and beta-decay. With a large amount of neutrons available due to the fission of ^{235}U, the probability for this process is substantially increased, resulting in a much higher decay rate for ^{238}U and causing the depletion of ^{238}U (and relative enrichment of ^{235}U).

29. **THINK** With a shorter half-life, ^{235}U has a greater decay rate than ^{238}U. Thus, if the ore contains only 0.72% of ^{235}U today, then the concentration must be higher in the far distant past.

EXPRESS Let t be the present time and $t = 0$ be the time when the ratio of ^{235}U to ^{238}U was 3.0%. Let N_{235} be the number of ^{235}U nuclei present in a sample now and $N_{235,0}$ be the number present at $t = 0$. Let N_{238} be the number of ^{238}U nuclei present in the sample now and $N_{238,0}$ be the number present at $t = 0$. The law of radioactive decay holds for each species, so

$$N_{235} = N_{235,0} e^{-\lambda_{235} t}$$

and

$$N_{238} = N_{238,0} e^{-\lambda_{238} t}.$$

Dividing the first equation by the second, we obtain

$$r = r_0 e^{-(\lambda_{235} - \lambda_{238})t}$$

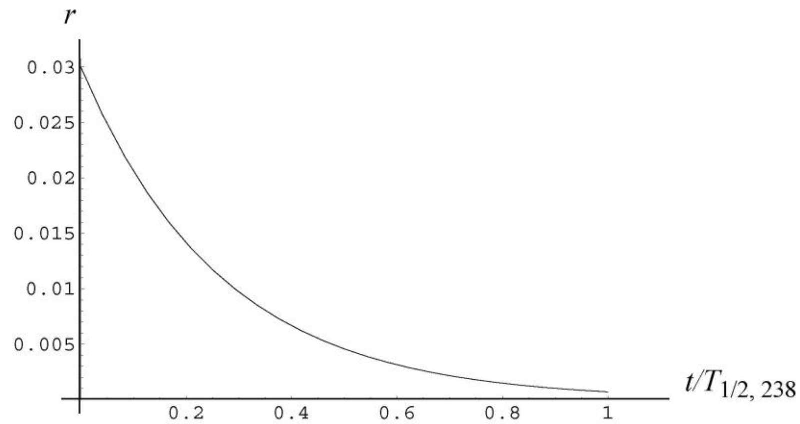
where $r = N_{235}/N_{238}$ (= 0.0072) and $r_0 = N_{235,0}/N_{238,0}$ (= 0.030). We solve for t:

$$t = -\frac{1}{\lambda_{235} - \lambda_{238}} \ln\left(\frac{r}{r_0}\right).$$

ANALYZE Now we use $\lambda_{235} = (\ln 2)/T_{1/2_{235}}$ and $\lambda_{238} = (\ln 2)/T_{1/2_{238}}$ to obtain

$$t = \frac{T_{1/2_{235}} T_{1/2_{238}}}{(T_{1/2_{238}} - T_{1/2_{235}})\ln 2} \ln\left(\frac{r}{r_0}\right) = -\frac{(7.0\times10^8\text{ y})(4.5\times10^9\text{ y})}{(4.5\times10^9\text{ y} - 7.0\times10^8\text{ y})\ln 2} \ln\left(\frac{0.0072}{0.030}\right)$$
$$= 1.7\times10^9\text{ y}.$$

LEARN How the ratio $r = N_{235}/N_{238}$ changes with time is plotted below. In the plot, we take the ratio to be 0.03 at $t = 0$. At $t = 1.7\times10^9$ y or $t/T_{1/2,238} = 0.378$, r is reduced to 0.072.



30. We are given the energy release per fusion (Q = 3.27 MeV = 5.24 × 10^{-13} J) and that a pair of deuterium atoms is consumed in each fusion event. To find how many pairs of deuterium atoms are in the sample, we adapt Eq. 42-21:

$$N_{d\text{ pairs}} = \frac{M_{\text{sam}}}{2M_d} N_{\text{A}} = \left(\frac{1000\text{ g}}{2(2.0\text{ g/mol})}\right)(6.02\times10^{23}\text{/mol}) = 1.5\times10^{26}.$$

Multiplying this by Q gives the total energy released: 7.9 × 10^{13} J. Keeping in mind that a watt is a joule per second, we have

$$t = \frac{7.9\times10^{13}\text{ J}}{100\text{ W}} = 7.9\times10^{11}\text{ s} = 2.5\times10^4\text{ y}.$$

31. **THINK** Coulomb repulsion acts to prevent two charged particles from coming close enough to be within the range of their attractive nuclear force.

EXPRESS We take the height of the Coulomb barrier to be the value of the kinetic energy K each deuteron must initially have if they are to come to rest when their surfaces touch. If r is the radius of a deuteron, conservation of energy yields

$$2K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}.$$

ANALYZE With r = 2.1 fm, we have

$$K = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{4r} = (8.99\times10^9 \text{ V}\cdot\text{m/C})\frac{(1.60\times10^{-19} \text{ C})^2}{4(2.1\times10^{-15} \text{ m})} = 2.74\times10^{-14} \text{ J} = 170 \text{ keV}.$$

LEARN The height of the Coulomb barrier depends on the charges and radii of the two interacting nuclei. Increasing the charge raises the barrier.

32. (a) Our calculation is identical to that in Sample Problem — "Fusion in a gas of protons and required temperature" except that we are now using R appropriate to two deuterons coming into "contact," as opposed to the R = 1.0 fm value used in the Sample Problem. If we use R = 2.1 fm for the deuterons, then our K is simply the K calculated in the Sample Problem, divided by 2.1:

$$K_{d+d} = \frac{K_{p+p}}{2.1} = \frac{360 \text{ keV}}{2.1} \approx 170 \text{ keV}.$$

Consequently, the voltage needed to accelerate each deuteron from rest to that value of K is 170 kV.

(b) Not all deuterons that are accelerated toward each other will come into "contact" and not all of those that do so will undergo nuclear fusion. Thus, a great many deuterons must be repeatedly encountering other deuterons in order to produce a macroscopic energy release. An accelerator needs a fairly good vacuum in its beam pipe, and a very large number flux is either impractical and/or very expensive. Regarding expense, there are other factors that have dissuaded researchers from using accelerators to build a controlled fusion "reactor," but those factors may become less important in the future — making the feasibility of accelerator "add-ons" to magnetic and inertial confinement schemes more cost-effective.

33. Our calculation is very similar to that in Sample Problem – "Fusion in a gas of protons and required temperature" except that we are now using R appropriate to two lithium-7 nuclei coming into "contact," as opposed to the R = 1.0 fm value used in the Sample Problem. If we use

$$R = r = r_0 A^{1/3} = (1.2 \text{ fm})\sqrt[3]{7} = 2.3 \text{ fm}$$

and $q = Ze = 3e$, then our K is given by (see the Sample Problem)

$$K = \frac{Z^2e^2}{16\pi\varepsilon_0 r} = \frac{3^2(1.6\times10^{-19} \text{ C})^2}{16\pi(8.85\times10^{-12} \text{ F/m})(2.3\times10^{15} \text{ m})}$$

which yields 2.25×10^{-13} J = 1.41 MeV. We interpret this as the answer to the problem, though the term "Coulomb barrier height" as used here may be open to other interpretations.

34. From the expression for $n(K)$ given we may write $n(K) \propto K^{1/2}e^{-K/kT}$. Thus, with

$$k = 8.62 \times 10^{-5} \text{ eV/K} = 8.62 \times 10^{-8} \text{ keV/K},$$

we have

$$\frac{n(K)}{n(K_{\text{avg}})} = \left(\frac{K}{K_{\text{avg}}}\right)^{1/2} e^{-(K-K_{\text{avg}})/kT} = \left(\frac{5.00\,\text{keV}}{1.94\,\text{keV}}\right)^{1/2} \exp\left(-\frac{5.00\,\text{keV}-1.94\,\text{keV}}{(8.62\times10^{-8}\,\text{keV})(1.50\times10^{7}\,\text{K})}\right)$$
$$= 0.151.$$

35. The kinetic energy of each proton is

$$K = k_B T = (1.38\times10^{-23} \text{ J/K})(1.0\times10^{7} \text{ K}) = 1.38\times10^{-16} \text{ J}.$$

At the closest separation, r_{min}, all the kinetic energy is converted to potential energy:

$$K_{\text{tot}} = 2K = U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_{\text{min}}}\ .$$

Solving for r_{min}, we obtain

$$r_{\text{min}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2K} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{2(1.38\times10^{-16}\,\text{J})} = 8.33\times10^{-13}\ \text{m} \approx 1\ \text{pm}.$$

36. The energy released is

$$\begin{aligned} Q &= -\Delta mc^2 = -(m_{\text{He}} - m_{\text{H2}} - m_{\text{H1}})c^2 \\ &= -(3.016029\ \text{u} - 2.014102\ \text{u} - 1.007825\ \text{u})(931.5\ \text{MeV/u}) \\ &= 5.49\ \text{MeV}. \end{aligned}$$

37. (a) Let M be the mass of the Sun at time t and E be the energy radiated to that time. Then, the power output is

$$P = dE/dt = (dM/dt)c^2,$$

where $E = Mc^2$ is used. At the present time,

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\,\text{W}}{\left(2.998\times10^{8}\ \text{m/s}\right)^2} = 4.3\times10^{9}\ \text{kg/s}\ .$$

(b) We assume the rate of mass loss remained constant. Then, the total mass loss is

$$\begin{aligned} \Delta M &= (dM/dt)\,\Delta t = (4.33\times10^{9}\ \text{kg/s})\,(4.5\times10^{9}\ \text{y})\,(3.156\times10^{7}\ \text{s/y}) \\ &= 6.15\times10^{26}\ \text{kg}. \end{aligned}$$

The fraction lost is

$$\frac{\Delta M}{M+\Delta M}=\frac{6.15\times10^{26}\,\text{kg}}{2.0\times10^{30}\,\text{kg}+6.15\times10^{26}\,\text{kg}}=3.1\times10^{-4}\,.$$

38. In Fig. 43-10, let Q_1 = 0.42 MeV, Q_2 = 1.02 MeV, Q_3 = 5.49 MeV, and Q_4 = 12.86 MeV. For the overall proton-proton cycle

$$\begin{aligned}Q&=2Q_1+2Q_2+2Q_3+Q_4\\&=2(0.42\ \text{MeV}+1.02\ \text{MeV}+5.49\ \text{MeV})+12.86\ \text{MeV}=26.7\ \text{MeV}.\end{aligned}$$

39. If M_{He} is the mass of an atom of helium and M_{C} is the mass of an atom of carbon, then the energy released in a single fusion event is

$$Q=\left(3M_{\text{He}}-M_{\text{C}}\right)c^2=[3(4.0026\ \text{u})-(12.0000\ \text{u})](931.5\ \text{MeV/u})=7.27\ \text{MeV}.$$

Note that $3M_{\text{He}}$ contains the mass of six electrons and so does M_{C}. The electron masses cancel and the mass difference calculated is the same as the mass difference of the nuclei.

40. (a) We are given the energy release per fusion (Q = 26.7 MeV = 4.28×10^{-12} J) and that four protons are consumed in each fusion event. To find how many sets of four protons are in the sample, we adapt Eq. 42-21:

$$N_{4p}=\frac{M_{\text{sam}}}{4M_{\text{H}}}N_{\text{A}}=\left(\frac{1000\,\text{g}}{4\left(1.0\,\text{g/mol}\right)}\right)\left(6.02\times10^{23}/\text{mol}\right)=1.5\times10^{26}\ .$$

Multiplying this by Q gives the total energy released: 6.4×10^{14} J. It is not required that the answer be in SI units; we could have used MeV throughout (in which case the answer is 4.0×10^{27} MeV).

(b) The number of ^{235}U nuclei is

$$N_{235}=\left(\frac{1000\,\text{g}}{235\,\text{g/mol}}\right)\left(6.02\times10^{23}/\text{mol}\right)=2.56\times10^{24}\ .$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_{\text{fission}}=\left(2.56\times10^{22}\right)\left(200\,\text{MeV}\right)=5.1\times10^{26}\ \text{MeV}=8.2\times10^{13}\,\text{J}\ .$$

We see that the fusion process (with regard to a unit mass of fuel) produces a larger amount of energy (despite the fact that the Q value per event is smaller).

41. Since the mass of a helium atom is

$$(4.00\ \text{u})(1.661 \times 10^{-27}\ \text{kg/u}) = 6.64 \times 10^{-27}\ \text{kg},$$

the number of helium nuclei originally in the star is

$$(4.6 \times 10^{32}\ \text{kg})/(6.64 \times 10^{-27}\ \text{kg}) = 6.92 \times 10^{58}.$$

Since each fusion event requires three helium nuclei, the number of fusion events that can take place is

$$N = 6.92 \times 10^{58}/3 = 2.31 \times 10^{58}.$$

If Q is the energy released in each event and t is the conversion time, then the power output is $P = NQ/t$ and

$$t = \frac{NQ}{P} = \frac{\left(2.31\times10^{58}\right)\left(7.27\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{5.3\times10^{30}\ \text{W}} = 5.07\times10^{15}\ \text{s}$$
$$= 1.6\times10^{8}\ \text{y}\ .$$

42. We assume the neutrino has negligible mass. The photons, of course, are also taken to have zero mass.

$$\begin{aligned}
Q_1 &= \left(2m_p - m_2 - m_e\right)c^2 = \left[2\left(m_1 - m_e\right) - \left(m_2 - m_e\right) - m_e\right]c^2 \\
&= \left[2\left(1.007825\,\text{u}\right) - 2.014102\,\text{u} - 2\left(0.0005486\,\text{u}\right)\right]\left(931.5\,\text{MeV/u}\right) \\
&= 0.42\,\text{MeV} \\
Q_2 &= \left(m_2 + m_p - m_3\right)c^2 = \left(m_2 + m_p - m_3\right)c^2 \\
&= \left(2.014102\,\text{u} + 1.007825\,\text{u} - 3.016029\,\text{u}\right)\left(931.5\,\text{MeV/u}\right) \\
&= 5.49\ \text{MeV} \\
Q_3 &= \left(2m_3 - m_4 - 2m_p\right)c^2 = \left(2m_3 - m_4 - 2m_p\right)c^2 \\
&= \left[2\left(3.016029\,\text{u}\right) - 4.002603\,\text{u} - 2\left(1.007825\,\text{u}\right)\right]\left(931.5\,\text{MeV/u}\right) \\
&= 12.86\,\text{MeV}\ .
\end{aligned}$$

43. (a) The energy released is

$$\begin{aligned}
Q &= \left(5m_{^2\text{H}} - m_{^3\text{He}} - m_{^4\text{He}} - m_{^1\text{H}} - 2m_n\right)c^2 \\
&= \left[5\left(2.014102\,\text{u}\right) - 3.016029\,\text{u} - 4.002603\,\text{u} - 1.007825\,\text{u} - 2\left(1.008665\,\text{u}\right)\right]\left(931.5\,\text{MeV/u}\right) \\
&= 24.9\,\text{MeV}.
\end{aligned}$$

(b) Assuming 30.0% of the deuterium undergoes fusion, the total energy released is

$$E = NQ = \left(\frac{0.300\,M}{5m_{^2\mathrm{H}}}\right)Q\ .$$

Thus, the rating is

$$\begin{aligned} R &= \frac{E}{2.6\times 10^{28}\ \mathrm{MeV/megaton\,TNT}} \\ &= \frac{(0.300)(500\,\mathrm{kg})(24.9\,\mathrm{MeV})}{5(2.0\,\mathrm{u})\left(1.66\times 10^{-27}\ \mathrm{kg/u}\right)\left(2.6\times 10^{28}\ \mathrm{MeV/megaton\,TNT}\right)} \\ &= 8.65\,\mathrm{megaton\,TNT}\ . \end{aligned}$$

44. The mass of the hydrogen in the Sun's core is $m_{\mathrm{H}} = 0.35\left(\frac{1}{8}\,M_{\mathrm{Sun}}\right)$. The time it takes for the hydrogen to be entirely consumed is

$$t = \frac{M_{\mathrm{H}}}{dm/dt} = \frac{(0.35)\left(\frac{1}{8}\right)\left(2.0\times 10^{30}\ \mathrm{kg}\right)}{\left(6.2\times 10^{11}\ \mathrm{kg/s}\right)\left(3.15\times 10^{7}\ \mathrm{s/y}\right)} = 5\times 10^{9}\ \mathrm{y}\ .$$

45. (a) Since two neutrinos are produced per proton-proton cycle (see Eq. 43-10 or Fig. 43-10), the rate of neutrino production R_ν satisfies

$$R_\nu = \frac{2P}{Q} = \frac{2\left(3.9\times 10^{26}\ \mathrm{W}\right)}{(26.7\,\mathrm{MeV})\left(1.6\times 10^{-13}\ \mathrm{J/MeV}\right)} = 1.8\times 10^{38}\ \mathrm{s}^{-1}\ .$$

(b) Let d_{es} be the Earth to Sun distance, and R be the radius of Earth (see Appendix C). Earth represents a small cross section in the "sky" as viewed by a fictitious observer on the Sun. The rate of neutrinos intercepted by that area (very small, relative to the area of the full "sky") is

$$R_{\nu,\mathrm{Earth}} = R_\nu\left(\frac{\pi R_e^2}{4\pi d_{es}^2}\right) = \frac{\left(1.8\times 10^{38}\ \mathrm{s}^{-1}\right)}{4}\left(\frac{6.4\times 10^{6}\ \mathrm{m}}{1.5\times 10^{11}\ \mathrm{m}}\right)^2 = 8.2\times 10^{28}\ \mathrm{s}^{-1}\ .$$

46. (a) The products of the carbon cycle are $2e^{+} + 2\nu + {}^{4}\mathrm{He}$, the same as that of the proton-proton cycle (see Eq. 43-10). The difference in the number of photons is not significant.

(b) We have

$$\begin{aligned} Q_{\mathrm{carbon}} &= Q_1 + Q_2 + \cdots + Q_6 \\ &= (1.95\times 1.19 + 7.55 + 7.30 + 1.73 + 4.97)\,\mathrm{MeV} \\ &= 24.7\,\mathrm{MeV} \end{aligned}$$

which is the same as that for the proton-proton cycle (once we subtract out the electron-positron annihilations; see Fig. 43-10):

$$Q_{p-p} = 26.7 \text{ MeV} - 2(1.02 \text{ MeV}) = 24.7 \text{ MeV}.$$

47. **THINK** The energy released by burning 1 kg of carbon is 3.3×10^7 J.

EXPRESS The mass of a carbon atom is $(12.0 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 1.99 \times 10^{-26}$ kg, so the number of carbon atoms in 1.00 kg of carbon is

$$(1.00 \text{ kg})/(1.99 \times 10^{-26} \text{ kg}) = 5.02 \times 10^{25}.$$

ANALYZE (a) The heat of combustion per atom is

$$(3.3 \times 10^7 \text{ J/kg})/(5.02 \times 10^{25} \text{ atom/kg}) = 6.58 \times 10^{-19} \text{ J/atom}.$$

This is 4.11 eV/atom.

(b) In each combustion event, two oxygen atoms combine with one carbon atom, so the total mass involved is 2(16.0 u) + (12.0 u) = 44 u. This is

$$(44 \text{ u})(1.661 \times 10^{-27} \text{ kg/u}) = 7.31 \times 10^{-26} \text{ kg}.$$

Each combustion event produces 6.58×10^{-19} J so the energy produced per unit mass of reactants is $(6.58 \times 10^{-19} \text{ J})/(7.31 \times 10^{-26} \text{ kg}) = 9.00 \times 10^6$ J/kg.

(c) If the Sun were composed of the appropriate mixture of carbon and oxygen, the number of combustion events that could occur before the Sun burns out would be

$$(2.0 \times 10^{30} \text{ kg})/(7.31 \times 10^{-26} \text{ kg}) = 2.74 \times 10^{55}.$$

The total energy released would be

$$E = (2.74 \times 10^{55})(6.58 \times 10^{-19} \text{ J}) = 1.80 \times 10^{37} \text{ J}.$$

If P is the power output of the Sun, the burn time would be

$$t = \frac{E}{P} = \frac{1.80 \times 10^{37} \text{ J}}{3.9 \times 10^{26} \text{ W}} = 4.62 \times 10^{10} \text{ s} = 1.46 \times 10^3 \text{ y},$$

or 1.5×10^3 y, to two significant figures.

LEARN The Sun burns not coal but hydrogen via the proton-proton cycle in which the fusion of hydrogen nuclei into helium nuclei take place. The mechanism of thermonuclear fusion reactions allows the Sun to radiate energy at a rate of 3.9×10^{26} W for several billion years.

48. In Eq. 43-13,

$$Q = \left(2m_{^2\mathrm{H}} - m_{^3\mathrm{He}} - m_n\right)c^2 = \left[2\left(2.014102\,\mathrm{u}\right) - 3.016049\,\mathrm{u} - 1.008665\,\mathrm{u}\right]\left(931.5\,\mathrm{MeV/u}\right)$$
$$= 3.27\mathrm{MeV}\ .$$

In Eq. 43-14,

$$Q = \left(2m_{^2\mathrm{H}} - m_{^3\mathrm{H}} - m_{^1\mathrm{H}}\right)c^2 = \left[2\left(2.014102\,\mathrm{u}\right) - 3.016049\,\mathrm{u} - 1.007825\,\mathrm{u}\right]\left(931.5\,\mathrm{MeV/u}\right)$$
$$= 4.03\,\mathrm{MeV}\ .$$

Finally, in Eq. 43-15,

$$\begin{aligned} Q &= \left(m_{^2\mathrm{H}} + m_{^3\mathrm{H}} - m_{^4\mathrm{He}} - m_n\right)c^2 \\ &= \ 2.014102\,\mathrm{u} + 3.016049\,\mathrm{u} - 4.002603\,\mathrm{u} - 1.008665\,\mathrm{u}\ \left(931.5\,\mathrm{MeV/u}\right) \\ &= 17.59\,\mathrm{MeV}\ . \end{aligned}$$

49. Since 1.00 L of water has a mass of 1.00 kg, the mass of the heavy water in 1.00 L is 0.0150×10^{-2} kg = 1.50×10^{-4} kg. Since a heavy water molecule contains one oxygen atom, one hydrogen atom and one deuterium atom, its mass is

$$(16.0\ \mathrm{u} + 1.00\ \mathrm{u} + 2.00\ \mathrm{u}) = 19.0\ \mathrm{u} = (19.0\ \mathrm{u})(1.661 \times 10^{-27}\ \mathrm{kg/u}) = 3.16 \times 10^{-26}\ \mathrm{kg}.$$

The number of heavy water molecules in a liter of water is

$$(1.50 \times 10^{-4}\ \mathrm{kg})/(3.16 \times 10^{-26}\ \mathrm{kg}) = 4.75 \times 10^{21}.$$

Since each fusion event requires two deuterium nuclei, the number of fusion events that can occur is $N = 4.75 \times 10^{21}/2 = 2.38 \times 10^{21}$. Each event releases energy

$$Q = (3.27 \times 10^{6}\ \mathrm{eV})(1.60 \times 10^{-19}\ \mathrm{J/eV}) = 5.23 \times 10^{-13}\ \mathrm{J}.$$

Since all events take place in a day, which is 8.64×10^4 s, the power output is

$$P = \frac{NQ}{t} = \frac{\left(2.38\times10^{21}\right)\left(5.23\times10^{-13}\ \mathrm{J}\right)}{8.64\times10^{4}\ \mathrm{s}} = 1.44\times10^{4}\ \mathrm{W} = 14.4\,\mathrm{kW}\ .$$

50. (a) From $E = NQ = (M_{\mathrm{sam}}/4m_p)Q$ we get the energy per kilogram of hydrogen consumed:

$$\frac{E}{M_{\text{sam}}} = \frac{Q}{4m_p} = \frac{(26.2\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})}{4(1.67\times10^{-27}\ \text{kg})} = 6.3\times10^{14}\ \text{J/kg}\ .$$

(b) Keeping in mind that a watt is a joule per second, the rate is

$$\frac{dm}{dt} = \frac{3.9\times10^{26}\ \text{W}}{6.3\times10^{14}\ \text{J/kg}} = 6.2\times10^{11}\ \text{kg/s}\ .$$

This agrees with the computation shown in Sample Problem — "Consumption rate of hydrogen in the Sun."

(c) From the Einstein relation $E = Mc^2$ we get $P = dE/dt = c^2 dM/dt$, or

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\ \text{W}}{(3.0\times10^{8}\ \text{m/s})^2} = 4.3\times10^{9}\ \text{kg/s}\ .$$

(d) This finding, that $dm/dt > dM/dt$, is in large part due to the fact that, as the protons are consumed, their mass is mostly turned into alpha particles (helium), which remain in the Sun.

(e) The time to lose 0.10% of its total mass is

$$t = \frac{0.0010\,M}{dM/dt} = \frac{(0.0010)(2.0\times10^{30}\ \text{kg})}{(4.3\times10^{9}\ \text{kg/s})(3.15\times10^{7}\ \text{s/y})} = 1.5\times10^{10}\ \text{y}\ .$$

51. Since plutonium has Z = 94 and uranium has Z = 92, we see that (to conserve charge) two electrons must be emitted so that the nucleus can gain a $+2e$ charge. In the beta decay processes described in Chapter 42, electrons and neutrinos are emitted. The reaction series is as follows:

$$^{238}\text{U} + \text{n} \rightarrow {}^{239}\text{Np} + {}^{239}\text{U} + e + \nu$$
$$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + e + \nu$$

52. Conservation of energy gives $Q = K_\alpha + K_\text{n}$, and conservation of linear momentum (due to the assumption of negligible initial velocities) gives $|p_\alpha| = |p_\text{n}|$. We can write the classical formula for kinetic energy in terms of momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that $K_\text{n} = (m_\alpha/m_\text{n})K_\alpha$.

(a) Consequently, conservation of energy and momentum allows us to solve for kinetic energy of the alpha particle, which results from the fusion:

$$K_\alpha = \frac{Q}{1+(m_\alpha/m_n)} = \frac{17.59\,\text{MeV}}{1+(4.0015\,\text{u}/1.008665\,\text{u})} = 3.541\,\text{MeV}$$

where we have found the mass of the alpha particle by subtracting two electron masses from the ^{4}He mass (quoted several times in this Chapter 42).

(b) Then, $K_n = Q - K_\alpha$ yields 14.05 MeV for the neutron kinetic energy.

53. At T = 300 K, the average kinetic energy of the neutrons is (using Eq. 20-24)

$$K_{avg} = \frac{3}{2}KT = \frac{3}{2}(8.62\times10^{-5}\ \text{eV/K})(300\ \text{K}) \approx 0.04\ \text{eV}.$$

54. First, we figure out the mass of U-235 in the sample (assuming "3.0%" refers to the proportion by weight as opposed to proportion by number of atoms):

$$\begin{aligned} M_{\text{U}-235} &= (3.0\%)M_{\text{sam}}\left(\frac{(97\%)m_{238}+(3.0\%)m_{235}}{(97\%)m_{238}+(3.0\%)m_{235}+2m_{16}}\right) \\ &= (0.030)(1000\ \text{g})\left(\frac{0.97(238)+0.030(235)}{0.97(238)+0.030(235)+2(16.0)}\right) \\ &= 26.4\ \text{g}. \end{aligned}$$

Next, the number of ^{235}U nuclei is

$$N_{235} = \frac{(26.4\ \text{g})(6.02\times10^{23}/\text{mol})}{235\ \text{g/mol}} = 6.77\times10^{22}.$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 43-6) is

$$N_{235}Q_{\text{fission}} = (6.77\times10^{22})(200\ \text{MeV}) = 1.35\times10^{25}\ \text{MeV} = 2.17\times10^{12}\ \text{J}.$$

Keeping in mind that a watt is a joule per second, the time that this much energy can keep a 100-W lamp burning is found to be

$$t = \frac{2.17\times10^{12}\ \text{J}}{100\ \text{W}} = 2.17\times10^{10}\ \text{s} \approx 690\ \text{y}.$$

If we had instead used the Q = 208 MeV value from Sample Problem — "Q value in a fission of uranium-235," then our result would have been 715 y, which perhaps suggests that our result is meaningful to just one significant figure ("roughly 700 years").

55. (a) From $\rho_H = 0.35\rho = n_p m_p$, we get the proton number density n_p:

$$n_p = \frac{0.35\rho}{m_p} = \frac{(0.35)(1.5\times10^5\ \text{kg/m}^3)}{1.67\times10^{-27}\ \text{kg}} = 3.1\times10^{31}\,\text{m}^{-3}.$$

(b) From Chapter 19 (see Eq. 19-9), we have

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5\ \text{Pa}}{(1.38\times10^{-23}\ \text{J/K})(273\,\text{K})} = 2.68\times10^{25}\ \text{m}^{-3}$$

for an ideal gas under "standard conditions." Thus,

$$\frac{n_p}{(N/V)} = \frac{3.14\times10^{31}\ \text{m}^{-3}}{2.44\times10^{25}\ \text{m}^{-3}} = 1.2\times10^6\ .$$

56. (a) Rather than use $P(v)$ as it is written in Eq. 19-27, we use the more convenient nK expression given in Problem 43-34. The $n(K)$ expression can be derived from Eq. 19-27, but we do not show that derivation here. To find the most probable energy, we take the derivative of $n(K)$ and set the result equal to zero:

$$\left.\frac{dn(K)}{dK}\right|_{K=K_p} = \frac{1.13n}{(kT)^{3/2}}\left(\frac{1}{2K^{1/2}} - \frac{K^{3/2}}{kT}\right)e^{-K/kT}\Bigg|_{K=K_p} = 0,$$

which gives $K_p = \frac{1}{2}kT$. Specifically, for $T = 1.5\times10^7$ K we find

$$K_p = \frac{1}{2}kT = \frac{1}{2}(8.62\times10^{-5}\ \text{eV/K})(1.5\times10^7\ \text{K}) = 6.5\times10^2\ \text{eV}$$

or 0.65 keV, in good agreement with Fig. 43-10.

(b) Equation 19-35 gives the most probable speed in terms of the molar mass M, and indicates its derivation. Since the mass m of the particle is related to M by the Avogadro constant, then using Eq. 19-7,

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{mN_A}} = \sqrt{\frac{2kT}{m}}\ .$$

With $T = 1.5\times10^7$ K and $m = 1.67\times10^{-27}$ kg, this yields $v_p = 5.0\times10^5$ m/s.

(c) The corresponding kinetic energy is

$$K_{v,p} = \frac{1}{2}mv_p^2 = \frac{1}{2}m\left(\sqrt{\frac{2kT}{m}}\right)^2 = kT$$

which is twice as large as that found in part (a). Thus, at $T = 1.5 \times 10^7$ K we have $K_{v,p}$ = 1.3 keV, which is indicated in Fig. 43-10 by a single vertical line.

57. (a) The mass of each DT pellet is

$$m = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi (20\times10^{-6}\,\text{m})^3 (200\,\text{kg/m}^3) = 6.7\times10^{-12}\,\text{kg}$$

Since there are equal number of ${}^2\text{H}$ and ${}^3\text{H}$ present, we have

$$N_{{}^2\text{H}} = N_{{}^3\text{H}} = \frac{mN_A}{M_{{}^2\text{H}} + M_{{}^3\text{H}}} = \frac{(6.7\times10^{-12}\,\text{kg})(6.02\times10^{23})}{(0.020\,\text{kg})+(0.030\,\text{kg})} = 8.07\times10^{14}$$

Each fusion reaction releases 17.59 MeV of energy, with 10% efficiency, the total energy released by the pellet is

$$E = (0.10)(8.07\times10^{14})(17.59\text{ MeV}) = 1.42\times10^{15}\text{ MeV} = 227\text{ J}$$

or about 230 J.

(b) Since 1.0 kg of TNT gives off 4.6 MJ, the TNT equivalent of the pellet is

$$m = \frac{227\text{ J}}{4.6\times10^6\text{ J}} = 4.93\times10^{-5}\text{ kg}.$$

(c) The power generated is

$$P = \left(\frac{dN}{dt}\right)E = (100\,/\,\text{s})(227\text{ J}) = 2.3\times10^4\text{ W}$$

58. (a) Equation 19-35 gives the most probable speed in terms of the molar mass M: $v_p = \sqrt{2RT/M}$. With $T = 1 \times 10^8$ K and $M = 2.0 \times 10^{-3}$ kg/mol, this yields

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.314\text{ J/mol}\cdot\text{K})(108\text{ K})}{2.0\times10^{-3}\text{ kg}}} = 9.1\times10^5\,\text{m/s}.$$

(b) The distance moved is $r = v_p \Delta t = (9.1\times10^5\,\text{m/s})(1\times10^{-12}\text{s}) = 9.1\times10^{-7}\,\text{m}$.