

Chapter 41

1. According to Eq. 41-9, the Fermi energy is given by

$$E_F = \left(\frac{3}{16\sqrt{2}\pi} \right)^{2/3} \frac{h^2}{m} n^{2/3}$$

where n is the number of conduction electrons per unit volume, m is the mass of an electron, and h is the Planck constant. This can be written $E_F = An^{2/3}$, where

$$A = \left(\frac{3}{16\sqrt{2}\pi} \right)^{2/3} \frac{h^2}{m} = \left(\frac{3}{16\sqrt{2}\pi} \right)^{2/3} \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{9.109 \times 10^{-31} \text{ kg}} = 5.842 \times 10^{-38} \text{ J}^2 \cdot \text{s}^2 / \text{kg}.$$

Since $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2 / \text{s}^2$, the units of A can be taken to be $\text{m}^2 \cdot \text{J}$. Dividing by $1.602 \times 10^{-19} \text{ J/eV}$, we obtain $A = 3.65 \times 10^{-19} \text{ m}^2 \cdot \text{eV}$.

2. Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$\begin{aligned} C &= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi (9.109 \times 10^{-31} \text{ kg})^{3/2}}{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^3} = 1.062 \times 10^{56} \text{ kg}^{3/2} / \text{J}^3 \cdot \text{s}^3 \\ &= 6.81 \times 10^{27} \text{ m}^{-3} \cdot (\text{eV})^{-2/3}. \end{aligned}$$

Thus,

$$N(E) = CE^{1/2} = [6.81 \times 10^{27} \text{ m}^{-3} \cdot (\text{eV})^{-2/3}] (8.0 \text{ eV})^{1/2} = 1.9 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}.$$

This is consistent with that shown in Fig. 41-6.

3. The number of atoms per unit volume is given by $n = d / M$, where d is the mass density of copper and M is the mass of a single copper atom. Since each atom contributes one conduction electron, n is also the number of conduction electrons per unit volume. Since the molar mass of copper is $A = 63.54 \text{ g/mol}$,

$$M = A / N_A = (63.54 \text{ g/mol}) / (6.022 \times 10^{23} \text{ mol}^{-1}) = 1.055 \times 10^{-22} \text{ g}.$$

Thus,

$$n = \frac{8.96 \text{ g/cm}^3}{1.055 \times 10^{-22} \text{ g}} = 8.49 \times 10^{22} \text{ cm}^{-3} = 8.49 \times 10^{28} \text{ m}^{-3}.$$

4. Let $E_1 = 63 \text{ meV} + E_F$ and $E_2 = -63 \text{ meV} + E_F$. Then according to Eq. 41-6,

$$P_1 = \frac{1}{e^{(E_1 - E_F)/kT} + 1} = \frac{1}{e^x + 1}$$

where $x = (E_1 - E_F)/kT$. We solve for e^x :

$$e^x = \frac{1}{P_1} - 1 = \frac{1}{0.090} - 1 = \frac{91}{9}.$$

Thus,

$$P_2 = \frac{1}{e^{(E_2 - E_F)/kT} + 1} = \frac{1}{e^{-(E_1 - E_F)/kT} + 1} = \frac{1}{e^{-x} + 1} = \frac{1}{(91/9)^{-1} + 1} = 0.91,$$

where we use $E_2 - E_F = -63 \text{ meV} = E_F - E_1 = -(E_1 - E_F)$.

5. (a) Equation 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$N(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi (9.109 \times 10^{-31} \text{ kg})^{3/2}}{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^3} = 1.062 \times 10^{56} \text{ kg}^{3/2} / \text{J}^3 \cdot \text{s}^3.$$

(b) Now, $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2 / \text{s}^2$ (think of the equation for kinetic energy $K = \frac{1}{2}mv^2$), so $1 \text{ kg} = 1 \text{ J} \cdot \text{s}^2 \cdot \text{m}^{-2}$. Thus, the units of C can be written as

$$(\text{J} \cdot \text{s}^2)^{3/2} \cdot (\text{m}^{-2})^{3/2} \cdot \text{J}^{-3} \cdot \text{s}^{-3} = \text{J}^{-3/2} \cdot \text{m}^{-3}.$$

This means

$$C = (1.062 \times 10^{56} \text{ J}^{-3/2} \cdot \text{m}^{-3})(1.602 \times 10^{-19} \text{ J/eV})^{3/2} = 6.81 \times 10^{27} \text{ m}^{-3} \cdot \text{eV}^{-3/2}.$$

(c) If $E = 5.00 \text{ eV}$, then

$$N(E) = (6.81 \times 10^{27} \text{ m}^{-3} \cdot \text{eV}^{-3/2})(5.00 \text{ eV})^{1/2} = 1.52 \times 10^{28} \text{ eV}^{-1} \cdot \text{m}^{-3}.$$

6. We note that $n = 8.43 \times 10^{28} \text{ m}^{-3} = 84.3 \text{ nm}^{-3}$. From Eq. 41-9,

$$E_F = \frac{0.121(hc)^2}{m_e c^2} n^{2/3} = \frac{0.121(1240 \text{ eV} \cdot \text{nm})^2}{511 \times 10^3 \text{ eV}} (84.3 \text{ nm}^{-3})^{2/3} = 7.0 \text{ eV}$$

where we have used $hc = 1240 \text{ eV} \cdot \text{nm}$.

7. **THINK** This problem deals with occupancy probability $P(E)$, the probability that an energy level will be occupied by an electron.

EXPRESS A plot of $P(E)$ as a function of E is shown in Fig. 41-7. From the figure, we see that at $T = 0 \text{ K}$, $P(E)$ is unity for $E \leq E_F$, where E_F is the Fermi energy, and zero for $E > E_F$. On the other hand, the probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) At absolute temperature $T = 0$, the probability is zero that any state with energy above the Fermi energy is occupied.

(b) Now, $E - E_F = 0.0620 \text{ eV}$, and

$$(E - E_F) / kT = (0.0620 \text{ eV}) / (8.62 \times 10^{-5} \text{ eV / K})(320 \text{ K}) = 2.248.$$

We find $P(E)$ to be

$$P(E) = \frac{1}{e^{2.248} + 1} = 0.0955.$$

See Appendix B for the value of k .

LEARN When $E = E_F$, the occupancy probability is $P(E_F) = 0.5$. Thus, one may think of the Fermi energy as the energy of a quantum state that has a probability 0.5 of being occupied by an electron.

8. We note that there is one conduction electron per atom and that the molar mass of gold is 197 g / mol . Therefore, combining Eqs. 41-2, 41-3, and 41-4 leads to

$$n = \frac{(19.3 \text{ g / cm}^3)(10^6 \text{ cm}^3 / \text{m}^3)}{(197 \text{ g / mol}) / (6.02 \times 10^{23} \text{ mol}^{-1})} = 5.90 \times 10^{28} \text{ m}^{-3}.$$

9. **THINK** According to Appendix F the molar mass of silver is $M = 107.870$ g/mol and the density is $\rho = 10.49$ g/cm³. Silver is monovalent.

EXPRESS The mass of a silver atom is, dividing the molar mass by Avogadro's number:

$$M_0 = \frac{M}{N_A} = \frac{107.870 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 1.791 \times 10^{-25} \text{ kg}.$$

Since silver is monovalent, there is one valence electron per atom (see Eq. 41-2).

ANALYZE (a) The number density is

$$n = \frac{\rho}{M_0} = \frac{10.49 \times 10^{-3} \text{ kg/m}^3}{1.791 \times 10^{-25} \text{ kg}} = 5.86 \times 10^{28} \text{ m}^{-3}.$$

This is the same as the number density of conduction electrons.

(b) The Fermi energy is

$$\begin{aligned} E_F &= \frac{0.121h^2}{m} n^{2/3} = \frac{(0.121)(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{9.109 \times 10^{-31} \text{ kg}} = (5.86 \times 10^{28} \text{ m}^{-3})^{2/3} \\ &= 8.80 \times 10^{-19} \text{ J} = 5.49 \text{ eV}. \end{aligned}$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80 \times 10^{-19} \text{ J})}{9.109 \times 10^{-31} \text{ kg}}} = 1.39 \times 10^6 \text{ m/s}.$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{mv_F} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.109 \times 10^{-31} \text{ kg})(1.39 \times 10^6 \text{ m/s})} = 5.22 \times 10^{-10} \text{ m}.$$

LEARN Once the number density of conduction electrons is known, the Fermi energy for a particular metal can be calculated using Eq. 41-9.

10. The probability P_h that a state is occupied by a hole is the same as the probability the state is *unoccupied* by an electron. Since the total probability that a state is either occupied or unoccupied is 1, we have $P_h + P = 1$. Thus,

$$P_h = 1 - \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{e^{(E-E_F)/kT}}{1 + e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT} + 1}.$$

11. We use

$$N_o(E) = N(E)P(E) = CE^{1/2} \left[e^{(E-E_F)/kT} + 1 \right]^{-1},$$

where

$$\begin{aligned} C &= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109 \times 10^{-31} \text{ kg})^{3/2}}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^3} = 1.062 \times 10^{56} \text{ kg}^{3/2} / \text{J}^3 \cdot \text{s}^3 \\ &= 6.81 \times 10^{27} \text{ m}^{-3} \cdot (\text{eV})^{-3/2}. \end{aligned}$$

(a) At $E = 4.00 \text{ eV}$,

$$N_o = \frac{(6.81 \times 10^{27} \text{ m}^{-3} \cdot (\text{eV})^{-3/2})(4.00 \text{ eV})^{1/2}}{\exp((4.00 \text{ eV} - 7.00 \text{ eV}) / [(8.62 \times 10^{-5} \text{ eV} / \text{K})(1000 \text{ K})]) + 1} = 1.36 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}.$$

(b) At $E = 6.75 \text{ eV}$,

$$N_o = \frac{(6.81 \times 10^{27} \text{ m}^{-3} \cdot (\text{eV})^{-3/2})(6.75 \text{ eV})^{1/2}}{\exp((6.75 \text{ eV} - 7.00 \text{ eV}) / [(8.62 \times 10^{-5} \text{ eV} / \text{K})(1000 \text{ K})]) + 1} = 1.68 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}.$$

(c) Similarly, at $E = 7.00 \text{ eV}$, the value of $N_o(E)$ is $9.01 \times 10^{27} \text{ m}^{-3} \cdot \text{eV}^{-1}$.

(d) At $E = 7.25 \text{ eV}$, the value of $N_o(E)$ is $9.56 \times 10^{26} \text{ m}^{-3} \cdot \text{eV}^{-1}$.

(e) At $E = 9.00 \text{ eV}$, the value of $N_o(E)$ is $1.71 \times 10^{18} \text{ m}^{-3} \cdot \text{eV}^{-1}$.

12. The molar mass of carbon is $m = 12.01115 \text{ g/mol}$ and the mass of the Earth is $M_e = 5.98 \times 10^{24} \text{ kg}$. Thus, the number of carbon atoms in a diamond as massive as the Earth is $N = (M_e/m)N_A$, where N_A is the Avogadro constant. From the result of Sample Problem – “Probability of electron excitation in an insulator,” the probability in question is given by

$$\begin{aligned} P &= N_e^{-E_g/kT} = \left(\frac{M_e}{m} \right) N_A e^{-E_g/kT} = \left(\frac{5.98 \times 10^{24} \text{ kg}}{12.01115 \text{ g/mol}} \right) (6.02 \times 10^{23} / \text{mol})(3 \times 10^{-93}) \\ &= 9 \times 10^{-43} \approx 10^{-42}. \end{aligned}$$

13. (a) Equation 41-6 leads to

$$E = E_F + kT \ln(P^{-1} - 1) = 7.00 \text{ eV} + (8.62 \times 10^{-5} \text{ eV} / \text{K})(1000 \text{ K}) \ln\left(\frac{1}{0.900} - 1\right) = 6.81 \text{ eV}.$$

(b) $N(E) = CE^{1/2} = (6.81 \times 10^{27} \text{ m}^{-3} \cdot \text{eV}^{-3/2})(6.81 \text{ eV})^{1/2} = 1.77 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}.$

(c) $N_o(E) = P(E)N(E) = (0.900)(1.77 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}) = 1.59 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}$.

14. (a) The volume per cubic meter of sodium occupied by the sodium ions is

$$V_{\text{Na}} = \frac{(971 \text{ kg})(6.022 \times 10^{23} / \text{mol})(4\pi/3)(98.0 \times 10^{-12} \text{ m})^3}{(23.0 \text{ g/mol})} = 0.100 \text{ m}^3,$$

so the fraction available for conduction electrons is $1 - (V_{\text{Na}} / 1.00 \text{ m}^3) = 1 - 0.100 = 0.900$, or 90.0%.

(b) For copper, we have

$$V_{\text{Cu}} = \frac{(8960 \text{ kg})(6.022 \times 10^{23} / \text{mol})(4\pi/3)(135 \times 10^{-12} \text{ m})^3}{(63.5 \text{ g/mol})} = 0.1876 \text{ m}^3.$$

Thus, the fraction is $1 - (V_{\text{Cu}} / 1.00 \text{ m}^3) = 1 - 0.1876 = 0.8124$, or 81.2%.

(c) Sodium, because the electrons occupy a greater portion of the space available.

15. **THINK** The Fermi-Dirac occupation probability is given by $P_{\text{FD}} = 1 / (e^{\Delta E/kT} + 1)$, and the Boltzmann occupation probability is given by $P_{\text{B}} = e^{-\Delta E/kT}$.

EXPRESS Let f be the fractional difference. Then

$$f = \frac{P_{\text{B}} - P_{\text{FD}}}{P_{\text{B}}} = \frac{e^{-\Delta E/kT} - \frac{1}{e^{\Delta E/kT} + 1}}{e^{-\Delta E/kT}}.$$

Using a common denominator and a little algebra yields $f = \frac{e^{-\Delta E/kT}}{e^{-\Delta E/kT} + 1}$. The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f}.$$

We take the natural logarithm of both sides and solve for T . The result is

$$T = \frac{\Delta E}{k \ln \left(\frac{f}{1-f} \right)}.$$

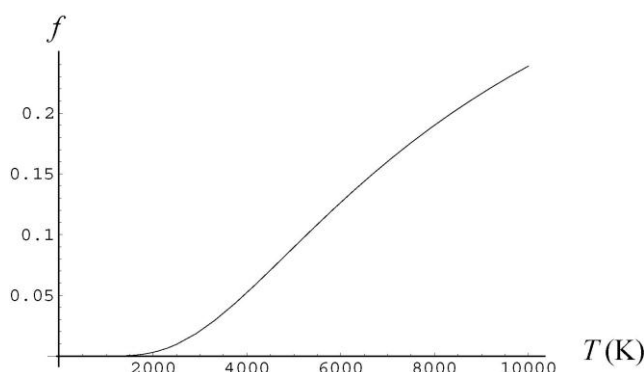
ANALYZE (a) Letting f equal 0.01, we evaluate the expression for T :

$$T = \frac{(1.00\text{eV})(1.60 \times 10^{-19}\text{J/eV})}{(1.38 \times 10^{-23}\text{J/K}) \ln\left(\frac{0.010}{1-0.010}\right)} = 2.50 \times 10^3\text{K}.$$

(b) We set f equal to 0.10 and evaluate the expression for T :

$$T = \frac{(1.00\text{eV})(1.60 \times 10^{-19}\text{J/eV})}{(1.38 \times 10^{-23}\text{J/K}) \ln\left(\frac{0.10}{1-0.10}\right)} = 5.30 \times 10^3\text{K}.$$

LEARN The fractional difference as a function of T is plotted below:



With a given ΔE , the difference increases with T .

16. (a) The ideal gas law in the form of Eq. 20-9 leads to $p = NkT/V = n_0kT$. Thus, we solve for the molecules per cubic meter:

$$n_0 = \frac{p}{kT} = \frac{(1.0\text{atm})(1.0 \times 10^5\text{Pa/atm})}{(1.38 \times 10^{-23}\text{J/K})(273\text{K})} = 2.7 \times 10^{25}\text{m}^{-3}.$$

(b) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in copper:

$$n = \frac{8.96 \times 10^3\text{kg/m}^3}{(63.54)(1.67 \times 10^{-27}\text{kg})} = 8.43 \times 10^{28}\text{m}^{-3}.$$

(c) The ratio is $n/n_0 = (8.43 \times 10^{28}\text{m}^{-3})/(2.7 \times 10^{25}\text{m}^{-3}) = 3.1 \times 10^3$.

(d) We use $d_{\text{avg}} = n^{-1/3}$. For case (a), $d_{\text{avg},0} = (2.7 \times 10^{25}\text{m}^{-3})^{-1/3} = 3.3\text{nm}$.

(e) For case (b), $d_{\text{avg}} = (8.43 \times 10^{28}\text{m}^{-3})^{-1/3} = 0.23\text{nm}$.

17. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N . We use the result of Problem 41-1 to find n : $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19} \text{ m}^2 \cdot \text{eV}$. Thus,

$$n = \left(\frac{E_F}{A} \right)^{3/2} = \left(\frac{11.6 \text{ eV}}{3.65 \times 10^{-19} \text{ m}^2 \cdot \text{eV}} \right)^{3/2} = 1.79 \times 10^{29} \text{ m}^{-3}.$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now,

$$M = (27.0 \text{ g/mol}) / (6.022 \times 10^{23} \text{ mol}^{-1}) = 4.48 \times 10^{-23} \text{ g},$$

so

$$N = (2.70 \text{ g/cm}^3) / (4.48 \times 10^{-23} \text{ g}) = 6.03 \times 10^{22} \text{ cm}^{-3} = 6.03 \times 10^{28} \text{ m}^{-3}.$$

Thus, the number of free electrons per atom is

$$\frac{n}{N} = \frac{1.79 \times 10^{29} \text{ m}^{-3}}{6.03 \times 10^{28} \text{ m}^{-3}} = 2.97 \approx 3.$$

18. The mass of the sample is

$$m = \rho V = (9.0 \text{ g/cm}^3)(40.0 \text{ cm}^3) = 360 \text{ g},$$

which is equivalent to

$$n = \frac{m}{M} = \frac{360 \text{ g}}{60 \text{ g/mol}} = 6.0 \text{ mol}.$$

Since the atoms are bivalent (each contributing two electrons), there are 12.0 moles of conduction electrons, or

$$N = nN_A = (12.0 \text{ mol})(6.02 \times 10^{23} / \text{mol}) = 7.2 \times 10^{24}.$$

19. (a) We evaluate $P(E) = 1/(e^{(E-E_F)/kT} + 1)$ for the given value of E , using

$$kT = \frac{(1.381 \times 10^{-23} \text{ J/K})(273 \text{ K})}{1.602 \times 10^{-19} \text{ J/eV}} = 0.02353 \text{ eV}.$$

For $E = 4.4 \text{ eV}$, $(E - E_F)/kT = (4.4 \text{ eV} - 5.5 \text{ eV})/(0.02353 \text{ eV}) = -46.25$ and

$$P(E) = \frac{1}{e^{-46.25} + 1} = 1.0.$$

(b) Similarly, for $E = 5.4 \text{ eV}$, $P(E) = 0.986 \approx 0.99$.

(c) For $E = 5.5 \text{ eV}$, $P(E) = 0.50$.

(d) For $E = 5.6 \text{ eV}$, $P(E) = 0.014$.

(e) For $E = 6.4 \text{ eV}$, $P(E) = 2.447 \times 10^{-17} \approx 2.4 \times 10^{-17}$.

(f) Solving $P = 1/(e^{\Delta E/kT} + 1)$ for $e^{\Delta E/kT}$, we get

$$e^{\Delta E/kT} = \frac{1}{P} - 1.$$

Now, we take the natural logarithm of both sides and solve for T . The result is

$$T = \frac{\Delta E}{k \ln\left(\frac{1}{P} - 1\right)} = \frac{(5.6 \text{ eV} - 5.5 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(1.381 \times 10^{-23} \text{ J/K}) \ln\left(\frac{1}{0.16} - 1\right)} = 699 \text{ K} \approx 7.0 \times 10^2 \text{ K}.$$

20. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy. Now,

$$E - E_F = 6.10 \text{ eV} - 5.00 \text{ eV} = 1.10 \text{ eV}$$

and

$$\frac{E - E_F}{kT} = \frac{1.10 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(1500 \text{ K})} = 8.51,$$

so

$$P(E) = \frac{1}{e^{8.51} + 1} = 2.01 \times 10^{-4}.$$

From Fig. 41-6, we find the density of states at 6.0 eV to be about $N(E) = 1.7 \times 10^{28} / \text{m}^3 \cdot \text{eV}$. Thus, using Eq. 41-7, the density of occupied states is

$$N_o(E) = N(E)P(E) = (1.7 \times 10^{28} / \text{m}^3 \cdot \text{eV})(2.01 \times 10^{-4}) = 3.42 \times 10^{24} / \text{m}^3 \cdot \text{eV}.$$

Within energy range of $\Delta E = 0.0300 \text{ eV}$ and a volume $V = 5.00 \times 10^{-8} \text{ m}^3$, the number of occupied states is

$$\begin{aligned} \left(\frac{\text{number}}{\text{states}} \right) &= N_o(E)V\Delta E = (3.42 \times 10^{24} / \text{m}^3 \cdot \text{eV})(5.00 \times 10^{-8} \text{ m}^3)(0.0300 \text{ eV}) \\ &= 5.1 \times 10^{15}. \end{aligned}$$

21. (a) At $T = 300$ K, $f = \frac{3kT}{2E_F} = \frac{3(8.62 \times 10^{-5} \text{ eV / K})(300 \text{ K})}{2(7.0 \text{ eV})} = 5.5 \times 10^{-3}$.

(b) At $T = 1000$ K, $f = \frac{3kT}{2E_F} = \frac{3(8.62 \times 10^{-5} \text{ eV / K})(1000 \text{ K})}{2(7.0 \text{ eV})} = 1.8 \times 10^{-2}$.

(c) Many calculators and most math software packages (here we use MAPLE) have built-in numerical integration routines. Setting up ratios of integrals of Eq. 41-7 and canceling common factors, we obtain

$$frac = \frac{\int_{E_F}^{\infty} \sqrt{E} / (e^{(E-E_F)/kT} + 1) dE}{\int_0^{\infty} \sqrt{E} / (e^{(E-E_F)/kT} + 1) dE}$$

where $k = 8.62 \times 10^{-5}$ eV/K. We use the Fermi energy value for copper ($E_F = 7.0$ eV) and evaluate this for $T = 300$ K and $T = 1000$ K; we find $frac = 0.00385$ and $frac = 0.0129$, respectively.

22. The fraction f of electrons with energies greater than the Fermi energy is (approximately) given in Problem 41-21:

$$f = \frac{3kT/2}{E_F}$$

where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy. We solve for T :

$$T = \frac{2fE_F}{3k} = \frac{2(0.013)(4.70 \text{ eV})}{3(8.62 \times 10^{-5} \text{ eV / K})} = 472 \text{ K}.$$

23. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n} \int_0^{\infty} EN(E)P(E)dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n} \int_0^{E_F} E^{3/2} dE = \frac{2C}{5n} E_F^{5/2} .$$

Now

$$n = \int_0^\infty N(E)P(E)dE = C \int_0^{E_F} E^{1/2} dE = \frac{2C}{3} E_F^{3/2}.$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}} = \left(\frac{2C}{5} \right) E_F^{5/2} \left(\frac{3}{2CE_F^{3/2}} \right) = \frac{3}{5} E_F.$$

24. From Eq. 41-9, we find the number of conduction electrons per unit volume to be

$$\begin{aligned} n &= \frac{16\sqrt{2}\pi}{3} \left(\frac{m_e E_F}{h^2} \right)^{3/2} = \frac{16\sqrt{2}\pi}{3} \left(\frac{(m_e c^2) E_F}{(hc)^2} \right)^{3/2} = \frac{16\sqrt{2}\pi}{3} \left(\frac{(0.511 \times 10^6 \text{ eV})(5.0 \text{ eV})}{(1240 \text{ eV} \cdot \text{nm})^2} \right)^{3/2} \\ &= 50.9 / \text{nm}^3 = 5.09 \times 10^{28} / \text{m}^3 \\ &\approx 8.4 \times 10^4 \text{ mol/m}^3. \end{aligned}$$

Since the atom is bivalent, the number density of the atom is

$$n_{\text{atom}} = n / 2 = 4.2 \times 10^4 \text{ mol/m}^3.$$

Thus, the mass density of the atom is

$$\rho = n_{\text{atom}} M = (4.2 \times 10^4 \text{ mol/m}^3)(20.0 \text{ g/mol}) = 8.4 \times 10^5 \text{ g/m}^3 = 0.84 \text{ g/cm}^3.$$

25. (a) Using Eq. 41-4, the energy released would be

$$\begin{aligned} E &= NE_{\text{avg}} = \frac{(3.1 \text{ g})}{(63.54 \text{ g/mol}) / (6.02 \times 10^{23} / \text{mol})} \left(\frac{3}{5} \right) (7.0 \text{ eV}) (1.6 \times 10^{-19} \text{ J/eV}) \\ &= 1.97 \times 10^4 \text{ J}. \end{aligned}$$

(b) Keeping in mind that a watt is a joule per second, we have

$$t = \frac{E}{P} = \frac{1.97 \times 10^4 \text{ J}}{100 \text{ J/s}} = 197 \text{ s}.$$

26. Let the energy of the state in question be an amount ΔE above the Fermi energy E_F . Then, Eq. 41-6 gives the occupancy probability of the state as

$$P = \frac{1}{e^{(E_F + \Delta E - E_F)/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1}.$$

We solve for ΔE to obtain

$$\Delta E = kT \ln \left(\frac{1}{P} - 1 \right) = (1.38 \times 10^{23} \text{ J/K})(300 \text{ K}) \ln \left(\frac{1}{0.10} - 1 \right) = 9.1 \times 10^{-21} \text{ J} ,$$

which is equivalent to $5.7 \times 10^{-2} \text{ eV} = 57 \text{ meV}$.

27. (a) Combining Eqs. 41-2, 41-3, and 41-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133 \text{ g/cm}^3)}{(65.37 \text{ g/mol}) / (6.02 \times 10^{23} \text{ mol})} = 1.31 \times 10^{23} \text{ cm}^{-3} = 1.31 \times 10^{29} \text{ m}^{-3} .$$

(b) From Eq. 41-9,

$$E_F = \frac{0.121 h^2}{m_e} n^{2/3} = \frac{0.121 (6.63 \times 10^{-34} \text{ J}\cdot\text{s})^2 (1.31 \times 10^{29} \text{ m}^{-3})^{2/3}}{(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ J/eV})} = 9.43 \text{ eV} .$$

(c) Equating the Fermi energy to $\frac{1}{2} m_e v_F^2$ we find (using the $m_e c^2$ value in Table 37-3)

$$v_F = \sqrt{\frac{2E_F c^2}{m_e c^2}} = \sqrt{\frac{2(9.43 \text{ eV})(2.998 \times 10^8 \text{ m/s})^2}{511 \times 10^3 \text{ eV}}} = 1.82 \times 10^6 \text{ m/s} .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_F} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(1.82 \times 10^6 \text{ m/s})} = 0.40 \text{ nm} .$$

28. Combining Eqs. 41-2, 41-3, and 41-4, the number density of conduction electrons in gold is

$$n = \frac{(19.3 \text{ g/cm}^3)(6.02 \times 10^{23} / \text{mol})}{(197 \text{ g/mol})} = 5.90 \times 10^{22} \text{ cm}^{-3} = 59.0 \text{ nm}^{-3} .$$

Now, using $hc = 1240 \text{ eV}\cdot\text{nm}$, Eq. 41-9 leads to

$$E_F = \frac{0.121 (hc)^2}{(m_e c^2)} n^{2/3} = \frac{0.121 (1240 \text{ eV}\cdot\text{nm})^2}{511 \times 10^3 \text{ eV}} (59.0 \text{ nm}^{-3})^{2/3} = 5.52 \text{ eV} .$$

29. Let the volume be $v = 1.00 \times 10^{-6} \text{ m}^3$. Then,

$$\begin{aligned}
 K_{\text{total}} &= NE_{\text{avg}} = nvE_{\text{avg}} = (8.43 \times 10^{28} \text{ m}^{-3})(1.00 \times 10^{-6} \text{ m}^3) \left(\frac{3}{5} \right) (7.00 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV}) \\
 &= 5.71 \times 10^4 \text{ J} = 57.1 \text{ kJ}.
 \end{aligned}$$

30. The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and

$$E_F = \frac{0.121h^2}{m_e} n^{2/3} = \frac{0.121(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{9.11 \times 10^{-31} \text{ kg}} (1.70 \times 10^{28} \text{ m}^{-3})^{2/3} = 3.855 \times 10^{-19} \text{ J}$$

is the Fermi energy. Now,

$$E - E_F = 4.00 \times 10^{-19} \text{ J} - 3.855 \times 10^{-19} \text{ J} = 1.45 \times 10^{-20} \text{ J}$$

and

$$\frac{E - E_F}{kT} = \frac{1.45 \times 10^{-20} \text{ J}}{(1.38 \times 10^{-23} \text{ J/K})(200 \text{ K})} = 5.2536,$$

so

$$P(E) = \frac{1}{e^{5.2536} + 1} = 5.20 \times 10^{-3}.$$

Next, for the density of states associated with the conduction electrons of a metal, Eq. 41-5 gives

$$\begin{aligned}
 N(E) &= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} = \frac{8\sqrt{2}\pi (9.109 \times 10^{-31} \text{ kg})^{3/2}}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^3} (4.00 \times 10^{-19} \text{ J})^{1/2} \\
 &= (1.062 \times 10^{56} \text{ kg}^{3/2} / \text{J}^3 \cdot \text{s}^3) (4.00 \times 10^{-19} \text{ J})^{1/2} \\
 &= 6.717 \times 10^{46} / \text{m}^3 \cdot \text{J}
 \end{aligned}$$

where we have used $1 \text{ kg} = 1 \text{ J} \cdot \text{s}^2 \cdot \text{m}^{-2}$ for unit conversion. Thus, using Eq. 41-7, the density of occupied states is

$$N_o(E) = N(E)P(E) = (6.717 \times 10^{46} / \text{m}^3 \cdot \text{J})(5.20 \times 10^{-3}) = 3.49 \times 10^{44} / \text{m}^3 \cdot \text{J}.$$

Within energy range of $\Delta E = 3.20 \times 10^{-20} \text{ J}$ and a volume $V = 6.00 \times 10^{-6} \text{ m}^3$, the number of occupied states is

$$\left(\begin{array}{c} \text{number} \\ \text{states} \end{array} \right) = N_o(E)V\Delta E = (3.49 \times 10^{44} / \text{m}^3 \cdot \text{J})(6.00 \times 10^{-6} \text{ m}^3)(3.20 \times 10^{-20} \text{ J}) \\ = 6.7 \times 10^{19}.$$

31. **THINK** The valence band and the conduction band are separated by an energy gap.

EXPRESS Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength.

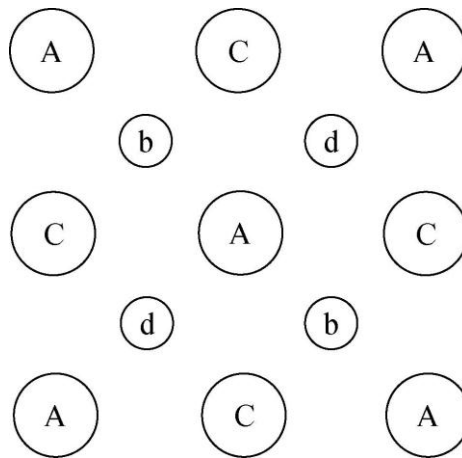
ANALYZE (a) Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{(5.5 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 2.26 \times 10^{-7} \text{ m} = 226 \text{ nm}.$$

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

LEARN Note that photons from other transitions have a greater energy, so their waves have shorter wavelengths.

32. Each arsenic atom is connected (by covalent bonding) to four gallium atoms, and each gallium atom is similarly connected to four arsenic atoms. The “depth” of their very nontrivial lattice structure is, of course, not evident in a flattened-out representation such as shown for silicon in Fig. 41-10.



Still we try to convey some sense of this (in the $[1, 0, 0]$ view shown — for those who might be familiar with Miller indices) by using letters to indicate the depth: A for the closest atoms (to the observer), b for the next layer deep, C for further into the page, d for the last layer seen, and E (not shown) for the atoms that are at the deepest layer (and are behind the A's) needed for our description of the structure. The capital letters are used for the gallium atoms, and the small letters for the arsenic.

Consider the arsenic atom (with the letter b) near the upper left; it has covalent bonds with the two A's and the two C's near it. Now consider the arsenic atom (with the letter d) near the upper right; it has covalent bonds with the two C's, which are near it, and with the two E's (which are behind the A's which are near :+).

(a) The 3p, 3d, and 4s subshells of both arsenic and gallium are filled. They both have partially filled 4p subshells. An isolated, neutral arsenic atom has three electrons in the 4p subshell, and an isolated, neutral gallium atom has one electron in the 4p subshell. To supply the total of eight shared electrons (for the four bonds connected to each ion in the lattice), not only the electrons from 4p must be shared but also the electrons from 4s. The core of the gallium ion has charge $q = +3e$ (due to the "loss" of its single 4p and two 4s electrons).

(b) The core of the arsenic ion has charge $q = +5e$ (due to the "loss" of the three 4p and two 4s electrons).

(c) As remarked in part (a), there are two electrons shared in each of the covalent bonds. This is the same situation that one finds for silicon (see Fig. 41-10).

33. (a) At the bottom of the conduction band $E = 0.67$ eV. Also $E_F = 0.67$ eV/2 = 0.335 eV. So the probability that the bottom of the conduction band is occupied is

$$P(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1} = \frac{1}{\exp\left(\frac{0.67\text{eV} - 0.335\text{eV}}{(8.62 \times 10^{-5} \text{ eV/K})(290\text{K})}\right) + 1} = 1.5 \times 10^{-6}.$$

(b) At the top of the valence band $E = 0$, so the probability that the state is *unoccupied* is given by

$$1 - P(E) = 1 - \frac{1}{e^{(E - E_F)/kT} + 1} = \frac{1}{e^{-(E - E_F)/kT} + 1} = \frac{1}{e^{-(0 - 0.335\text{eV})/[(8.62 \times 10^{-5} \text{ eV/K})(290\text{K})]} + 1} \\ = 1.5 \times 10^{-6}.$$

34. (a) The number of electrons in the valence band is

$$N_{\text{ev}} = N_v P(E_v) = \frac{N_v}{e^{(E_v - E_F)/kT} + 1}.$$

Since there are a total of N_v states in the valence band, the number of holes in the valence band is

$$N_{\text{hv}} = N_v - N_{\text{ev}} = N_v \left[1 - \frac{1}{e^{(E_v - E_F)/kT} + 1} \right] = \frac{N_v}{e^{-(E_v - E_F)/kT} + 1}.$$

Now, the number of electrons in the conduction band is

$$N_{\text{ec}} = N_c P(E_c) = \frac{N_c}{e^{(E_c - E_F)/kT} + 1},$$

Hence, from $N_{\text{ev}} = N_{\text{hc}}$, we get

$$\frac{N_v}{e^{-(E_v - E_F)/kT} + 1} = \frac{N_c}{e^{(E_c - E_F)/kT} + 1}.$$

(b) In this case, $e^{(E_c - E_F)/kT} \gg 1$ and $e^{-(E_v - E_F)/kT} \gg 1$. Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c - E_F)/kT}} \approx \frac{N_v}{e^{-(E_v - E_F)/kT}},$$

or $e^{(E_v - E_c + 2E_F)/kT} \approx N_v / N_c$. We solve for E_F :

$$E_F \approx \frac{1}{2}(E_c + E_v) + \frac{1}{2}kT \ln\left(\frac{N_v}{N_c}\right).$$

35. **THINK** Doping silicon with phosphorus increases the number of electrons in the conduction band.

EXPRESS Sample Problem — “Doping silicon with phosphorus” gives the fraction of silicon atoms that must be replaced by phosphorus atoms. We find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is $M_{\text{Si}} = 28.086 \text{ g/mol}$, so the mass of one silicon atom is

$$m_{0,\text{Si}} = M_{\text{Si}} / N_A = (28.086 \text{ g/mol}) / (6.022 \times 10^{23} \text{ mol}^{-1}) = 4.66 \times 10^{-23} \text{ g}$$

and the number of atoms in 1.0 g is

$$N_{\text{Si}} = m_{\text{Si}} / m_{0,\text{Si}} = (1.0 \text{ g}) / (4.66 \times 10^{-23} \text{ g}) = 2.14 \times 10^{22}.$$

According to the Sample Problem, one of every 5×10^6 silicon atoms is replaced with a phosphorus atom. This means there will be

$$N_{\text{P}} = (2.14 \times 10^{22}) / (5 \times 10^6) = 4.29 \times 10^{15}$$

phosphorus atoms in 1.0 g of silicon.

ANALYZE The molar mass of phosphorus is $M_{\text{P}} = 30.9758 \text{ g/mol}$ so the mass of a phosphorus atom is

$$m_{0,P} = M_P / N_A = (30.9758 \text{ g/mol}) / (6.022 \times 10^{23} \text{ mol}^{-1}) = 5.14 \times 10^{-23} \text{ g}.$$

The mass of phosphorus that must be added to 1.0 g of silicon is

$$m_P = N_P m_{0,P} = (4.29 \times 10^{15})(5.14 \times 10^{-23} \text{ g}) = 2.2 \times 10^{-7} \text{ g}.$$

LEARN The phosphorus atom is a *donor* atom since it donates an electron to the conduction band. Semiconductors doped with donor atoms are called *n*-type semiconductors.

36. (a) The Fermi level is above the top of the silicon valence band.

(b) Measured from the top of the valence band, the energy of the donor state is

$$E = 1.11 \text{ eV} - 0.11 \text{ eV} = 1.0 \text{ eV}.$$

We solve E_F from Eq. 41-6:

$$\begin{aligned} E_F = E - kT \ln[P^{-1} - 1] &= 1.0 \text{ eV} - (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) \ln[(5.00 \times 10^{-5})^{-1} - 1] \\ &= 0.744 \text{ eV}. \end{aligned}$$

(c) Now $E = 1.11 \text{ eV}$, so

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{(1.11 \text{ eV} - 0.744 \text{ eV}) / [(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})]} + 1} = 7.13 \times 10^{-7}.$$

37. (a) The probability that a state with energy E is occupied is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is $E = 1.11 \text{ eV}$. Furthermore,

$$kT = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) = 0.02586 \text{ eV}.$$

For pure silicon, $E_F = 0.555 \text{ eV}$ and

$$(E - E_F)/kT = (0.555 \text{ eV}) / (0.02586 \text{ eV}) = 21.46.$$

Thus,

$$P(E) = \frac{1}{e^{21.46} + 1} = 4.79 \times 10^{-10}.$$

(b) For the doped semiconductor,

$$(E - E_F)/kT = (0.11 \text{ eV})/(0.02586 \text{ eV}) = 4.254$$

and

$$P(E) = \frac{1}{e^{4.254} + 1} = 1.40 \times 10^{-2}.$$

(c) The energy of the donor state, relative to the top of the valence band, is $1.11 \text{ eV} - 0.15 \text{ eV} = 0.96 \text{ eV}$. The Fermi energy is $1.11 \text{ eV} - 0.11 \text{ eV} = 1.00 \text{ eV}$. Hence,

$$(E - E_F)/kT = (0.96 \text{ eV} - 1.00 \text{ eV})/(0.02586 \text{ eV}) = -1.547$$

and

$$P(E) = \frac{1}{e^{-1.547} + 1} = 0.824.$$

38. (a) The semiconductor is *n*-type, since each phosphorus atom has one more valence electron than a silicon atom.

(b) The added charge carrier density is

$$n_P = 10^{-7} n_{\text{Si}} = 10^{-7} (5 \times 10^{28} \text{ m}^{-3}) = 5 \times 10^{21} \text{ m}^{-3}.$$

(c) The ratio is

$$(5 \times 10^{21} \text{ m}^{-3})/[2(5 \times 10^{15} \text{ m}^{-3})] = 5 \times 10^5.$$

Here the factor of 2 in the denominator reflects the contribution to the charge carrier density from *both* the electrons in the conduction band *and* the holes in the valence band.

39. **THINK** The valence band and the conduction band are separated by an energy gap E_g . An electron must acquire E_g in order to make the transition to the conduction band.

EXPRESS Since the energy received by each electron is exactly E_g , the difference in energy between the bottom of the conduction band and the top of the valence band, the number of electrons that can be excited across the gap by a single photon of energy E is

$$N = E / E_g.$$

ANALYZE With $E_g = 1.1 \text{ eV}$ and $E = 662 \text{ keV}$, we obtain

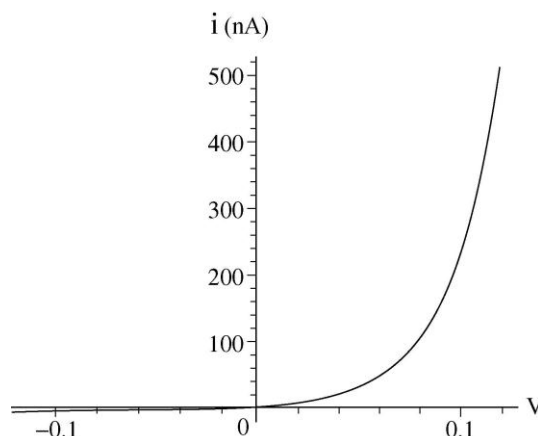
$$N = (662 \times 10^3 \text{ eV})/(1.1 \text{ eV}) = 6.0 \times 10^5.$$

Since each electron that jumps the gap leaves a hole behind, this is also the number of electron-hole pairs that can be created.

LEARN The wavelength of the photon is

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ nm} \cdot \text{eV}}{662 \times 10^3 \text{ eV}} = 1.87 \times 10^{-3} \text{ nm} = 1.87 \text{ pm}.$$

40. (a) The vertical axis in the graph below is the current in nanoamperes:



(b) The ratio is

$$\frac{I|_{V=+0.50 \text{ V}}}{I|_{V=-0.50 \text{ V}}} = \frac{I_0 \left[\exp\left(\frac{+0.50 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})}\right) - 1 \right]}{I_0 \left[\exp\left(\frac{-0.50 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})}\right) - 1 \right]} = 2.5 \times 10^8.$$

41. The valence band is essentially filled and the conduction band is essentially empty. If an electron in the valence band is to absorb a photon, the energy it receives must be sufficient to excite it across the band gap. Photons with energies less than the gap width are not absorbed and the semiconductor is transparent to this radiation. Photons with energies greater than the gap width are absorbed and the semiconductor is opaque to this radiation. Thus, the width of the band gap is the same as the energy of a photon associated with a wavelength of 295 nm. Noting that $hc = 1240 \text{ eV} \cdot \text{nm}$, we obtain

$$E_{\text{gap}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{295 \text{ nm}} = 4.20 \text{ eV}.$$

42. Since (using $hc = 1240 \text{ eV} \cdot \text{nm}$)

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{140 \text{ nm}} = 8.86 \text{ eV} > 7.6 \text{ eV},$$

the light will be absorbed by the KCl crystal. Thus, the crystal is opaque to this light.

43. We denote the maximum dimension (side length) of each transistor as ℓ_{\max} , the size of the chip as A , and the number of transistors on the chip as N . Then $A = N\ell_{\max}^2$. Therefore,

$$\ell_{\max} = \sqrt{\frac{A}{N}} = \sqrt{\frac{(1.0 \text{ in.} \times 0.875 \text{ in.})(2.54 \times 10^{-2} \text{ m/in.})^2}{3.5 \times 10^6}} = 1.3 \times 10^{-5} \text{ m} = 13 \mu\text{m}.$$

44. (a) According to Chapter 25, the capacitance is $C = \kappa\epsilon_0 A/d$. In our case $\kappa = 4.5$, $A = (0.50 \mu\text{m})^2$, and $d = 0.20 \mu\text{m}$, so

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{(4.5)(8.85 \times 10^{-12} \text{ F/m})(0.50 \mu\text{m})^2}{0.20 \mu\text{m}} = 5.0 \times 10^{-17} \text{ F}.$$

(b) Let the number of elementary charges in question be N . Then, the total amount of charges that appear in the gate is $q = Ne$. Thus, $q = Ne = CV$, which gives

$$N = \frac{CV}{e} = \frac{(5.0 \times 10^{-17} \text{ F})(1.0 \text{ V})}{1.6 \times 10^{-19} \text{ C}} = 3.1 \times 10^2.$$

45. **THINK** We differentiate the occupancy probability $P(E)$ with respect to E to explore the properties of $P(E)$.

EXPRESS The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy.

ANALYZE (a) The derivative of $P(E)$ is

$$\frac{dP}{dE} = \frac{-1}{[e^{(E-E_F)/kT} + 1]^2} \frac{d}{dE} e^{(E-E_F)/kT} = \frac{-1}{[e^{(E-E_F)/kT} + 1]^2} \frac{1}{kT} e^{(E-E_F)/kT}.$$

For $E = E_F$, we readily obtain the desired result:

$$\left. \frac{dP}{dE} \right|_{E=E_F} = \frac{-1}{[e^{(E_F-E_F)/kT} + 1]^2} \frac{1}{kT} e^{(E_F-E_F)/kT} = -\frac{1}{4kT}.$$

(b) The equation of a line may be written as $y = m(x - x_0)$ where $m = -1/4kT$ is the slope, and x_0 is the x -intercept (which is what we are asked to solve for). It is clear that $P(E_F) = 1/2$, so our equation of the line, evaluated at $x = E_F$, becomes

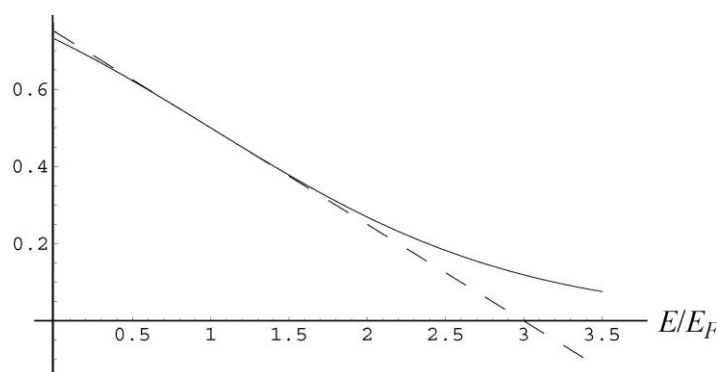
$$1/2 = (-1/4kT)(E_F - x_0),$$

which leads to $x_0 = E_F + 2kT$.

LEARN The straight line can be rewritten as

$$y = \frac{1}{2} - \frac{1}{4kT}(E - E_F).$$

A plot of $P(E)$ (solid line) and $y(E)$ (dashed line) in units of E_F/kT . The straight line passes the horizontal axis at $E/E_F = 3$.



46. (a) For copper, Eq. 41-10 leads to

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Cu}} = (2 \times 10^{-8} \Omega \cdot \text{m})(4 \times 10^{-3} \text{K}^{-1}) = 8 \times 10^{-11} \Omega \cdot \text{m} / \text{K}.$$

(b) For silicon,

$$\frac{d\rho}{dT} = [\rho\alpha]_{\text{Si}} = (3 \times 10^3 \Omega \cdot \text{m})(-70 \times 10^{-3} \text{K}^{-1}) = -2.1 \times 10^2 \Omega \cdot \text{m} / \text{K}.$$

47. The description in the problem statement implies that an atom is at the center point C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as $a = 388$ pm. Since each face is an equilateral triangle, the “altitude” of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the “slant height” in the solid geometry literature). At a certain location along the line segment representing the “slant height” of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the 60° angle of the equilateral face, it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to the farthest point on the

tetrahedron (this will land on an atom we label B), then this new line is the altitude h of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}}.$$

Now we include coordinates: imagine atom B is on the $+y$ axis at $y_b = h = a\sqrt{2/3}$, and atom A is on the $+x$ axis at $x_a = AC' = a/\sqrt{3}$. Then point C' is the origin. The tetrahedron center point C is on the y axis at some value y_c , which we find as follows: C must be equidistant from A and B , so

$$y_b - y_c = \sqrt{x_a^2 + y_c^2} \Rightarrow a\sqrt{\frac{2}{3}} - y_c = \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at C and going to A as

$$\vec{r}_{ac} = x_a \hat{i} + (-y_c) \hat{j} = \frac{a}{\sqrt{3}} \hat{i} - \frac{a}{2\sqrt{6}} \hat{j}.$$

Similarly, the vector starting at C and going to B is

$$\vec{r}_{bc} = (y_b - y_c) \hat{j} = \frac{a}{2} \sqrt{3/2} \hat{j}.$$

Therefore, using Eq. 3-20,

$$\theta = \cos^{-1} \left(\frac{\vec{r}_{ac} \cdot \vec{r}_{bc}}{|\vec{r}_{ac}| |\vec{r}_{bc}|} \right) = \cos^{-1} \left(-\frac{1}{3} \right)$$

which yields $\theta = 109.5^\circ$ for the angle between adjacent bonds.

(b) The length of vector \vec{r}_{bc} (which is, of course, the same as the length of \vec{r}_{ac}) is

$$|\vec{r}_{bc}| = \frac{a}{2} \sqrt{\frac{3}{2}} = \frac{388 \text{ pm}}{2} \sqrt{\frac{3}{2}} = 237.6 \text{ pm} \approx 238 \text{ pm}.$$

We note that in the solid geometry literature, the distance $\frac{a}{2} \sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.

48. According to Eq. 41-6,

$$P(E_F + \Delta E) = \frac{1}{e^{(E_F + \Delta E - E_F)/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1} = \frac{1}{e^x + 1}$$

where $x = \Delta E / kT$. Also,

$$P(E_F - \Delta E) = \frac{1}{e^{(E_F - \Delta E - E_F)/kT} + 1} = \frac{1}{e^{-\Delta E/kT} + 1} = \frac{1}{e^{-x} + 1}.$$

Thus,

$$P(E_F + \Delta E) + P(E_F - \Delta E) = \frac{1}{e^x + 1} + \frac{1}{e^{-x} + 1} = \frac{e^x + 1 + e^{-x} + 1}{(e^{-x} + 1)(e^x + 1)} = 1.$$

A special case of this general result can be found in Problem 41-4, where $\Delta E = 63 \text{ meV}$ and

$$P(E_F + 63 \text{ meV}) + P(E_F - 63 \text{ meV}) = 0.090 + 0.91 = 1.0.$$

49. (a) Setting $E = E_F$ (see Eq. 41-9), Eq. 41-5 becomes

$$N(E_F) = \frac{8\pi m \sqrt{2m}}{h^3} \left(\frac{3}{16\pi\sqrt{2}} \right)^{1/3} \frac{h}{\sqrt{m}} n^{1/3}.$$

Noting that $16\sqrt{2} = 2^4 2^{1/2} = 2^{9/2}$ so that the cube root of this is $2^{3/2} = 2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F) = \frac{4m}{h^2} \sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 37-3 as well as the value $hc = 1240 \text{ eV} \cdot \text{nm}$:

$$N(E_F) = \left(\frac{4mc^2}{(hc)^2} \sqrt[3]{3\pi^2} \right) n^{1/3} = \left(\frac{4(511 \times 10^3 \text{ eV})}{(1240 \text{ eV} \cdot \text{nm})^2} \sqrt[3]{3\pi^2} \right) n^{1/3} = (4.11 \text{ nm}^{-2} \cdot \text{eV}^{-1}) n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of Problem 41-3 may be written as

$$n = 8.49 \times 10^{28} \text{ m}^{-3} = 84.9 \text{ nm}^{-3}.$$

The cube root of this is $n^{1/3} \approx 4.4/\text{nm}$. Hence, the expression in part (a) leads to

$$N(E_F) = (4.11 \text{ nm}^{-2} \cdot \text{eV}^{-1})(4.4 \text{ nm}^{-1}) = 18 \text{ nm}^{-3} \cdot \text{eV}^{-1} = 1.8 \times 10^{28} \text{ m}^{-3} \cdot \text{eV}^{-1}.$$

If we multiply this by $10^{27} \text{ m}^3/\text{nm}^3$, we see this compares very well with the curve in Fig. 41-6 evaluated at 7.0 eV.

50. If we use the approximate formula discussed in Problem 41-21, we obtain

$$\text{frac} = \frac{3(8.62 \times 10^{-5} \text{ eV / K})(961 + 273 \text{ K})}{2(5.5 \text{ eV})} \approx 0.03 .$$

The numerical approach is briefly discussed in part (c) of Problem 41-21. Although the problem does not ask for it here, we remark that numerical integration leads to a fraction closer to 0.02.

51. We equate E_F with $\frac{1}{2}m_e v_F^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 37-3:

$$v_F = \sqrt{\frac{2E_F}{m}} = c \sqrt{\frac{2E_F}{mc^2}} = (3.0 \times 10^5 \text{ km/s}) \sqrt{\frac{2(7.0 \text{ eV})}{5.11 \times 10^5 \text{ eV}}} = 1.6 \times 10^3 \text{ km/s} .$$

52. The numerical factor $\left(\frac{3}{16\sqrt{2\pi}}\right)^{2/3}$ is approximately equal to 0.121.

53. We use the ideal gas law in the form of Eq. 20-9:

$$p = nkT = (8.43 \times 10^{28} \text{ m}^{-3})(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K}) = 3.49 \times 10^8 \text{ Pa} = 3.49 \times 10^3 \text{ atm} .$$