Chapter 32

1. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -\left(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}\right) = +3\,\text{Wb}\ .$$

2. (a) The flux through the top is +(0.30 T)πr^2 where r = 0.020 m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. **THINK** Gauss' law for magnetism states that the net magnetic flux through any closed surface is zero.

EXPRESS Mathematically, Gauss' law for magnetism is expressed as $\oint \vec{B}\cdot d\vec{A} = 0$. Now, our Gaussian surface has the shape of a right circular cylinder with two end caps and a curved surface. Thus,

$$\oint \vec{B}\cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C,$$

where Φ_1 is the magnetic flux through the first end cap, Φ_2 is the magnetic flux through the second end cap, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is Φ_1 = –25.0 μWb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder.

ANALYZE (a) Substituting the values given, the flux through the second end is

$$\Phi_2 = \pi\left(0.120\,\text{m}\right)^2\left(1.60\times10^{-3}\,\text{T}\right) = +7.24\times10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}.$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}.$$

Thus, the magnitude is $|\Phi_C| = 47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

LEARN Gauss' law for magnetism implies that magnetic monopoles do not exist; the simplest magnetic structure is a magnetic dipole (having a north pole and a south pole).

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the *xz* plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1) = \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx = 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx = 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx = \frac{\mu_0 iL}{\pi}\ln 3\,.$$

5. **THINK** Changing electric flux induces a magnetic field.

EXPRESS Consider a circle of radius r between the plates, with its center on the axis of the capacitor. Since there is no current between the capacitor plates, the Ampere-Maxwell's law reduces to

$$\oint \vec{B}\cdot d\vec{A} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}\,,$$

where $\vec{B}$ is the magnetic field at points on the circle, and Φ_E is the electric flux through the circle. Since the $\vec{B}$ field on the circle is in the tangential direction, and $\Phi_E = AE = \pi R^2 E,$ where R is the radius of the capacitor, we have

$$2\pi rB = \mu_0\varepsilon_0\pi R^2\frac{dE}{dt}$$

or

$$B = \frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt} \qquad (r \ge R).$$

ANALYZE Solving for dE/dt, we obtain

$$\frac{dE}{dt} = \frac{2Br}{\mu_0\varepsilon_0 R^2} = \frac{2\left(2.0\times10^{-7}\ \text{T}\right)\left(6.0\times10^{-3}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.0\times10^{-3}\text{m}\right)^2} = 2.4\times10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

LEARN Outside the capacitor, the induced magnetic field decreases with increased radial distance r, from a maximum value at the plate edge $r = R$.

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d\left(\frac{\text{enclosed area}}{\text{total area}}\right) = \mu_0(0.75\ \text{A})\frac{(4.0\ \text{cm})(2.0\ \text{cm})}{12\ \text{cm}^2} = 52\ \text{nT}\cdot\text{m}\,.$$

7. (a) Inside we have (by Eq. 32-16) $B=\mu_0 i_d r_1/2\pi R^2$, where $r_1 = 0.0200$ m, $R = 0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s}) = 2.66\times10^{-14}\ \text{A}\ .$$

Thus, we find

$$B = \frac{\mu_0 i_d r_1}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2} = 1.18\times10^{-19}\ \text{T}\ .$$

(b) Outside we have (by Eq. 32-17) $B=\mu_0 i_d/2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B = \frac{\mu_0 i_d}{2\pi r_2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})} = 1.06\times10^{-19}\ \text{T}$$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}\ .$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0300\ \text{m})}$$
$$= 3.54\times10^{-17}\ \text{T}\ .$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set r = 0.0500 m and solve. We now find

$$B = \frac{\varepsilon_0\mu_0(0.60\ \text{V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.60\ \text{V}\cdot\text{m/s})}{2\pi(0.0500\ \text{m})}$$
$$= 2.13\times10^{-17}\ \text{T}\ .$$

9. (a) Application of Eq. 32-7 with $A = \pi r^2$ (and taking the derivative of the field expression given in the problem) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi r^2\left(0.00450\ \text{V/m}\cdot\text{s}\right).$$

For r = 0.0200 m, this gives

$$\begin{aligned}B &= \frac{1}{2}\varepsilon_0\mu_0 r(0.00450\text{ V/m}\cdot\text{s})\\ &= \frac{1}{2}(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.0200\text{ m})(0.00450\text{ V/m}\cdot\text{s})\\ &= 5.01\times10^{-22}\text{ T}.\end{aligned}$$

(b) With $r > R$, the expression above must replaced by

$$B(2\pi r) = \varepsilon_0\mu_0\pi R^2\left(0.00450\text{ V/m}\cdot\text{s}\right).$$

Substituting r = 0.050 m and R = 0.030 m, we obtain $B = 4.51 \times 10^{-22}$ T.

10. (a) Here, the enclosed electric flux is found by integrating

$$\Phi_E = \int_0^r E\,2\pi r dr = t(0.500\text{ V/m}\cdot\text{s})(2\pi)\int_0^r\left(1-\frac{r}{R}\right)r dr = t\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right)$$

with SI units understood. Then (after taking the derivative with respect to time) Eq. 32-3 leads to

$$B(2\pi r) = \varepsilon_0\mu_0\pi\left(\frac{1}{2}r^2 - \frac{r^3}{3R}\right).$$

For r = 0.0200 m and R = 0.0300 m, this gives $B = 3.09 \times 10^{-20}$ T.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$\Phi_E = t\pi\left(\frac{1}{2}R^2 - \frac{R^3}{3R}\right) = \frac{1}{6}t\pi R^2.$$

Consequently, Eq. 32-3 becomes

$$B(2\pi r) = \frac{1}{6}\varepsilon_0\mu_0\pi R^2$$

which for r = 0.0500 m, yields

$$B = \frac{\varepsilon_0\mu_0 R^2}{12r} = \frac{(8.85\times10^{-12})(4\pi\times10^{-7})(0.030)^2}{12(0.0500)} = 1.67\times10^{-20}\text{ T}.$$

11. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where d = 5.0 mm), we use the result of part (a) in Sample Problem 32.01 – “Magnetic field induced by changing electric field:”

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d}\frac{dV}{dt} \qquad (r \le R).$$

We also use the fact that the time derivative of sin (ωt) (where $\omega = 2\pi f = 2\pi(60) \approx 377$/s in this problem) is $\omega\cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \le R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\max}\omega\cos(\omega t) \quad \Rightarrow \quad B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max}\omega}{2d}$$

where $V_{\max}$ = 150 V. This grows with r until reaching its highest value at $r = R$ = 30 mm:

$$B_{\max}\Big|_{r=R} = \frac{\mu_0 \varepsilon_0 R V_{\max}\omega}{2d} = \frac{(4\pi\times10^{-7}\ \mathrm{H/m})(8.85\times10^{-12}\ \mathrm{F/m})(30\times10^{-3}\mathrm{m})(150\,\mathrm{V})(377/\mathrm{s})}{2(5.0\times10^{-3}\mathrm{m})}$$
$$=1.9\times10^{-12}\,\mathrm{T}.$$

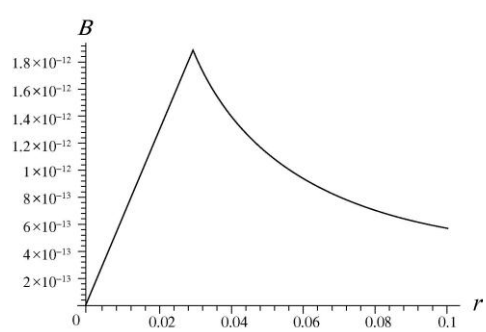
(b) For $r \le 0.03$ m, we use the expression

$$B_{\max} = \mu_0\varepsilon_0 r V_{\max}\omega/2d$$

found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03$ m we perform a similar calculation starting with the result of part (b) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}V_{\max}\omega\cos(\omega t)\right)_{\max}$$
$$= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \quad (\text{for } r \ge R)$$

(note the $B \propto r^{-1}$ dependence — see also Eqs. 32-16 and 32-17). The plot, with SI units understood, is shown to the right.



12. From Sample Problem 32.01 — "Magnetic field induced by changing electric field," we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\, B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75\, R = 0.75$ (40 mm) =30 mm.

(b) Outside the capacitor, $0.75\, B_{\max}/B_{\max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40 \text{ mm}) = 53 \text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(6.0\ \text{A}\right)}{2\pi\left(0.040\ \text{m}\right)} = 3.0\times10^{-5}\ \text{T}.$$

13. Let the area plate be A and the plate separation be d. We use Eq. 32-10:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(AE\right) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\ \text{A}}{2.0\times10^{-6}\ \text{F}} = 7.5\times10^5\ \text{V/s}.$$

Therefore, we need to change the voltage difference across the capacitor at the rate of 7.5×10^5 V/s.

14. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation, $i_d = \varepsilon_0 A(dE/dt)$, so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt}.$$

15. **THINK** The displacement current is related to the changing electric flux by $i_d = \varepsilon_0 (d\Phi_E/dt)$.

EXPRESS Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation.

ANALYZE Thus, the displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt}.$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt}.$$

LEARN The real current charging the capacitor is

$$i=\frac{dq}{dt}=\frac{d}{dt}\left(CV\right)=C\frac{dV}{dt}.$$

Thus, we see that $i=i_d$.

16. We use Eq. 32-14: $i_d=\varepsilon_0 A(dE/dt)$. Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt}=\frac{i_d}{\varepsilon_0 A}=\frac{i_d}{\varepsilon_0\pi R^2}=\frac{2.0\,\text{A}}{\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.10\,\text{m}\right)^2}=7.2\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

17. (a) Using Eq. 27-10, we find $E=\rho J=\dfrac{\rho i}{A}=\dfrac{\left(1.62\times10^{-8}\,\Omega\cdot\text{m}\right)\left(100\,\text{A}\right)}{5.00\times10^{-6}\,\text{m}^2}=0.324\,\text{V/m}.$

(b) The displacement current is

$$i_d=\varepsilon_0\frac{d\Phi_E}{dt}=\varepsilon_0 A\frac{dE}{dt}=\varepsilon_0 A\frac{d}{dt}\left(\frac{\rho i}{A}\right)=\varepsilon_0\rho\frac{di}{dt}=\left(8.85\times10^{-12}\,\text{F/m}\right)\left(1.62\times10^{-8}\,\Omega\right)\left(2000\,\text{A/s}\right)$$
$$=2.87\times10^{-16}\,\text{A}.$$

(c) The ratio of fields is $\dfrac{B\left(\text{due to}\,i_d\right)}{B\left(\text{due to}\,i\right)}=\dfrac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r}=\dfrac{i_d}{i}=\dfrac{2.87\times10^{-16}\,\text{A}}{100\,\text{A}}=2.87\times10^{-18}.$

18. From Eq. 28-11, we have $i = (\varepsilon / R)\, e^{-t/\tau}$ since we are ignoring the self-inductance of the capacitor. Equation 32-16 gives

$$B=\frac{\mu_0 i_d r}{2\pi R^2}.$$

Furthermore, Eq. 25-9 yields the capacitance

$$C=\frac{\varepsilon_0\pi(0.05\text{ m})^2}{0.003\text{ m}}=2.318\times10^{-11}\,\text{F},$$

so that the capacitive time constant is

$$\tau = (20.0\times10^{6}\,\Omega)(2.318\times10^{-11}\,\text{F}) = 4.636\times10^{-4}\,\text{s}.$$

At $t = 250\times10^{-6}$ s, the current is

$$i=\frac{12.0\text{ V}}{20.0\times10^{6}\,\Omega}\;e^{-t/\tau}=3.50\times10^{-7}\,\text{A}.$$

Since $i = i_d$ (see Eq. 32-15) and r = 0.0300 m, then (with plate radius R = 0.0500 m) we find

$$B=\frac{\mu_0 i_d r}{2\pi R^2}=\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(3.50\times10^{-7}\ \mathrm{A})(0.030\ \mathrm{m})}{2\pi(0.050\ \mathrm{m})^2}=8.40\times10^{-13}\ \mathrm{T}.$$

19. (a) Equation 32-16 (with Eq. 26-5) gives, with $A = \pi R^2$,

$$B=\frac{\mu_0 i_d r}{2\pi R^2}=\frac{\mu_0 J_d A r}{2\pi R^2}=\frac{\mu_0 J_d(\pi R^2)r}{2\pi R^2}=\frac{1}{2}\mu_0 J_d r$$
$$=\frac{1}{2}(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(6.00\ \mathrm{A/m^2})(0.0200\ \mathrm{m})=75.4\ \mathrm{nT}.$$

(b) Similarly, Eq. 32-17 gives $B=\dfrac{\mu_0 i_d}{2\pi r}=\dfrac{\mu_0 J_d \pi R^2}{2\pi r}=67.9\ \mathrm{nT}$.

20. (a) Equation 32-16 gives $B=\dfrac{\mu_0 i_d r}{2\pi R^2}=2.22\ \mu\mathrm{T}$.

(b) Equation 32-17 gives $B=\dfrac{\mu_0 i_d}{2\pi r}=2.00\ \mu\mathrm{T}$.

21. (a) Equation 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\mathrm{enc}}$. It is the enclosed portion of the displacement current, and if we related this to the displacement current density J_d, then

$$i_{d\ \mathrm{enc}}=\int_0^r J_d\,2\pi r\,dr=(4.00\ \mathrm{A/m^2})(2\pi)\int_0^r(1-r/R)r\,dr=8\pi\left(\frac{1}{2}r^2-\frac{r^3}{3R}\right)$$

with SI units understood. Now, we apply Eq. 32-17 (with i_d replaced with $i_{d,\mathrm{enc}}$) or start from scratch with Eq. 32-11, to get $B=\dfrac{\mu_0 i_{d\ \mathrm{enc}}}{2\pi r}=27.9\ \mathrm{nT}$.

(b) The integral shown above will no longer (since now $r > R$) have r as the upper limit; the upper limit is now R. Thus,

$$i_{d\ \mathrm{enc}}=i_d=8\pi\left(\frac{1}{2}R^2-\frac{R^3}{3R}\right)=\frac{4}{3}\pi R^2.$$

Now Eq. 32-17 gives $B=\dfrac{\mu_0 i_d}{2\pi r}=15.1\ \mathrm{nT}$.

22. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\mathrm{enc}}$. It is the enclosed portion of the displacement current. Thus Eq. 32-17 (which derives from Eq. 32-11) becomes, with i_d replaced with $i_{d,\mathrm{enc}}$,

$$B = \frac{\mu_0 i_{d\,\text{enc}}}{2\pi r} = \frac{\mu_0 (3.00\text{ A})(r/R)}{2\pi r}$$

which yields (after canceling r, and setting R = 0.0300 m) B = 20.0 μT.

(b) Here i_d = 3.00 A, and we get $B = \dfrac{\mu_0 i_d}{2\pi r} = 12.0\ \mu\text{T}$.

23. **THINK** The electric field between the plates in a parallel-plate capacitor is changing, so there is a nonzero displacement current $i_d = \varepsilon_0 (d\Phi_E / dt)$ between the plates.

EXPRESS Let A be the area of a plate and E be the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. The current into the positive plate of the capacitor is

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\frac{d(Ed)}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt},$$

which is the same as the displacement current.

ANALYZE (a) Thus, at any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires: $i_d = i$ = 2.0 A.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\text{A}}{(8.85\times10^{-12}\text{ F/m})(1.0\,\text{m})^2} = 2.3\times10^{11}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

(c) The displacement current through the indicated path is

$$i_d' = i_d\left(\frac{d^2}{L^2}\right) = (2.0\text{ A})\left(\frac{0.50\text{m}}{1.0\text{m}}\right)^2 = 0.50\text{ A}.$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_d' = (1.26\times10^{-16}\text{ H/m})(0.50\,\text{A}) = 6.3\times10^{-7}\text{ T}\cdot\text{m}.$$

LEARN the displacement through the dashed path is proportional to the area encircled by the path since the displacement current is uniformly distributed over the full plate area.

24. (a) From Eq. 32-10,

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} \varepsilon_0 A \frac{d}{dt}\left[\left(4.0\times10^5\right)-\left(6.0\times10^4 t\right)\right] = -\varepsilon_0 A\left(6.0\times10^4 \text{ V/m}\cdot\text{s}\right)$$
$$= -\left(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(4.0\times10^{-2}\text{m}^2\right)\left(6.0\times10^4 \text{ V/m}\cdot\text{s}\right)$$
$$= -2.1\times10^{-8}\,\text{A}.$$

Thus, the magnitude of the displacement current is $|i_d| = 2.1\times10^{-8}\,\text{A}$.

(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop *s* around the plates, then according to Eq. 32-18,

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

25. (a) We use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ to find

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0\left(J_d \pi r^2\right)}{2\pi r} = \frac{1}{2}\mu_0 J_d r = \frac{1}{2}\left(1.26\times10^{-6}\text{ H/m}\right)\left(20\,\text{A/m}^2\right)\left(50\times10^{-3}\text{m}\right)$$
$$= 6.3\times10^{-7}\,\text{T}.$$

(b) From $i_d = J_d \pi r^2 = \varepsilon_0 \dfrac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \dfrac{dE}{dt}$, we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\,\text{A/m}^2}{8.85\times10^{-12}\text{ F/m}} = 2.3\times10^{12}\,\frac{\text{V}}{\text{m}\cdot\text{s}}.$$

26. (a) Since $i = i_d$ (Eq. 32-15) then the portion of displacement current enclosed is

$$i_{d,\text{enc}} = i\,\frac{\pi\left(R/3\right)^2}{\pi R^2} = \frac{i}{9} = 1.33\,\text{A}.$$

(b) We see from Sample Problem 32.01 — "Magnetic field induced by changing electric field" that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to *r*. Therefore,

$$\frac{B}{B_{\max}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{r}{R}$$

which yields $r = R/4 = (1.20\text{ cm})/4 = 0.300$ cm.

(c) We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-17):

$$\frac{B}{B_{\max}} = \frac{3.00\,\text{mT}}{12.0\,\text{mT}} = \frac{R}{r}$$

which yields $r = 4R = 4(1.20\text{ cm}) = 4.80\text{ cm}$.

27. (a) In region a of the graph,

$$|i_d| = \varepsilon_0\left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\text{ F/m}\right)\left(1.6\,\text{m}^2\right)\left|\frac{4.5\times10^5\text{ N/C}-6.0\times10^5\text{ N/C}}{4.0\times10^{-6}\text{s}}\right| = 0.71\text{A}.$$

(b) $i_d \propto dE/dt = 0$.

(c) In region c of the graph,

$$|i_d| = \varepsilon_0 A\left|\frac{dE}{dt}\right| = \left(8.85\times10^{-12}\text{ F/m}\right)\left(1.6\text{m}^2\right)\left|\frac{-4.0\times10^5\text{ N/C}}{2.0\times10^{-6}\text{s}}\right| = 2.8\text{A}.$$

28. (a) Figure 32-35 indicates that $i = 4.0$ A when $t = 20$ ms. Thus,

$$B_i = \mu_0 i/2\pi r = 0.089\text{ mT}.$$

(b) Figure 32-35 indicates that $i = 8.0$ A when $t = 40$ ms. Thus, $B_i \approx 0.18$ mT.

(c) Figure 32-35 indicates that $i = 10$ A when $t > 50$ ms. Thus, $B_i \approx 0.220$ mT.

(d) Equation 32-4 gives the displacement current in terms of the time-derivative of the electric field: $i_d = \varepsilon_0 A(dE/dt)$, but using Eq. 26-5 and Eq. 26-10 we have $E = \rho i/A$ (in terms of the real current); therefore, $i_d = \varepsilon_0\rho(di/dt)$. For $0 < t < 50$ ms, Fig. 32-35 indicates that $di/dt = 200$ A/s. Thus,

$$B_{id} = \mu_0 i_d/2\pi r = 6.4\times10^{-22}\text{ T}.$$

(e) As in (d), $B_{id} = \mu_0 i_d/2\pi r = 6.4\times10^{-22}$ T.

(f) Here $di/dt = 0$, so (by the reasoning in the previous step) $B = 0$.

(g) By the right-hand rule, the direction of $\vec{B}_i$ at $t = 20$ s is out of the page.

(h) By the right-hand rule, the direction of $\vec{B}_{id}$ at $t = 20$ s is out of the page.

29. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\max} = i_{d\,\max} = 7.60\ \mu$A.

(b) Since $i_d = \varepsilon_0\,(d\Phi_E/dt)$, we have

$$\left(\frac{d\Phi_E}{dt}\right)_{\max}=\frac{i_{d\,\max}}{\varepsilon_0}=\frac{7.60\times10^{-6}\,\text{A}}{8.85\times10^{-12}\,\text{F/m}}=8.59\times10^{5}\,\text{V}\cdot\text{m/s}.$$

(c) Let the area plate be A and the plate separation be d. The displacement current is

$$i_d=\varepsilon_0\frac{d\Phi_E}{dt}=\varepsilon_0\frac{d}{dt}(AE)=\varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right)=\frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right).$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \varepsilon_m \sin \omega t$ and $dV/dt = \omega\varepsilon_m \cos \omega t$. Thus, $i_d=(\varepsilon_0 A\omega\varepsilon_m/d)\cos\omega t$ and $i_{d\,\max}=\varepsilon_0 A\omega\varepsilon_m/d$. This means

$$d=\frac{\varepsilon_0 A\omega\varepsilon_m}{i_{d\,\max}}=\frac{(8.85\times10^{-12}\,\text{F/m})\pi(0.180\,\text{m})^2(130\,\text{rad/s})(220\,\text{V})}{7.60\times10^{-6}\,\text{A}}=3.39\times10^{-3}\,\text{m},$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint\vec{B}\cdot d\vec{s}=\mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates, $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint\vec{B}\cdot d\vec{s}=2\pi rB$. Thus,

$$2\pi rB=\mu_0\left(\frac{r^2}{R^2}\right)i_d \quad\Rightarrow\quad B=\frac{\mu_0 i_d r}{2\pi R^2}.$$

The maximum magnetic field is given by

$$B_{\max}=\frac{\mu_0 i_{d\,\max} r}{2\pi R^2}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(7.6\times10^{-6}\,\text{A})(0.110\text{m})}{2\pi(0.180\,\text{m})^2}=5.16\times10^{-12}\,\text{T}.$$

30. (a) The flux through Arizona is

$$\Phi=-B_r A=-(43\times10^{-6}\,\text{T})(295{,}000\,\text{km}^2)(10^3\,\text{m/km})^2=-1.3\times10^{7}\,\text{Wb},$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3 \times 10^7$ Wb.

(b) The direction is outward.

31. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T}\ .$$

32. (a) The potential energy of the atom in association with the presence of an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-31 and 32-32:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_\ell \mu_B B_{\text{ext}}\,.$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_\ell$ = 0, meaning that $m_\ell = 0$ for this level.

(b) For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_ℓ. The middle one in the triplet is unshifted from the original value of E_2 so its m_ℓ must be equal to 0. The other two in the triplet then correspond to $m_\ell = -1$ and $m_\ell = +1$, respectively.

(c) For any pair of adjacent levels in the triplet, $|\Delta m_\ell| = 1$. Thus, the spacing is given by

$$\Delta U = |\Delta(-m_\ell \mu_B B)| = |\Delta m_\ell|\,\mu_B B = \mu_B B = (9.27\times10^{-24}\ \text{J/T})(0.50\text{T}) = 4.64\times10^{-24}\ \text{J}.$$

33. **THINK** An electron in an atom has both orbital angular momentum and spin angular momentum; the *z* components of the angular momenta are quantized.

EXPRESS The *z* component of the orbital angular momentum is give by

$$L_{\text{orb},z} = \frac{m_\ell h}{2\pi}$$

where h is the Planck constant and m_ℓ is the orbital magnetic quantum number. The corresponding *z* component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_{\text{B}}$$

where $\mu_B = eh/4\pi m$ is the Bohr magneton. When placed in an external field $\vec{B}_{ext}$, the energy associated with the orientation of $\vec{\mu}_{orb}$ is given by

$$U = -\vec{\mu}_{orb} \cdot \vec{B}_{ext}.$$

ANALYZE (a) Since $m_\ell = 0$, $L_{orb,z} = m_\ell h/2\pi = 0$.

(b) Since $m_\ell = 0$, $\mu_{orb,z} = -m_\ell \mu_B = 0$.

(c) Since $m_\ell = 0$, then from Eq. 32-32, $U = -\mu_{orb,z}B_{ext} = -m_\ell \mu_B B_{ext} = 0$.

(d) Regardless of the value of m_ℓ, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times10^{-24}\ \text{J/T}\right)\left(35\,\text{mT}\right) = \pm3.2\times10^{-25}\ \text{J}\ .$$

(e) Now $m_\ell = -3$, so

$$L_{orb,z} = \frac{m_\ell h}{2\pi} = \frac{(-3)\left(6.63\times10^{-27}\ \text{J}\cdot\text{s}\right)}{2\pi} = -3.16\times10^{-34}\ \text{J}\cdot\text{s} \approx -3.2\times10^{-34}\ \text{J}\cdot\text{s}$$

(f) and $\mu_{orb,z} = -m_\ell \mu_B = -(-3)\left(9.27\times10^{-24}\ \text{J/T}\right) = 2.78\times10^{-23}\ \text{J/T} \approx 2.8\times10^{-23}\ \text{J/T}$.

(g) The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{orb,z}B_{ext} = -\left(2.78\times10^{-23}\ \text{J/T}\right)\left(35\times10^{-3}\ \text{T}\right) = -9.7\times10^{-25}\ \text{J}.$$

(h) On the other hand, the potential energy associated with the electron spin, being independent of m_ℓ, remains the same: $\pm3.2 \times 10^{-25}$ J.

LEARN Spin is an intrinsic angular momentum that is not associated with the motion of the electron. Its z component is quantized, and can be written as

$$S_z = \frac{m_s h}{2\pi}$$

where $m_s = \pm 1/2$ is the spin magnetic quantum number.

34. We use Eq. 32-27 to obtain

$$\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z},$$

where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-24 and 32-25). Thus,

$$\Delta U = -B\ \mu_B - (-\mu_B)\ = 2\mu_B B = 2(9.27\times10^{-24}\ \text{J/T})(0.25\text{T}) = 4.6\times10^{-24}\ \text{J}\ .$$

35. We use Eq. 32-31: $\mu_{\text{orb},\,z} = -m_\ell\,\mu_B$.

(a) For $m_\ell = 1$, $\mu_{\text{orb},z} = -(1)\ (9.3\times10^{-24}\ \text{J/T}) = -9.3\times10^{-24}$ J/T.

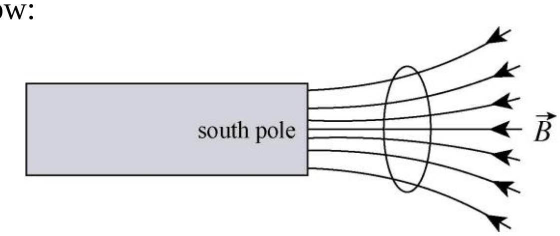
(b) For $m_\ell = -2$, $\mu_{\text{orb},z} = -(-2)\ (9.3\times10^{-24}\ \text{J/T}) = 1.9\times10^{-23}$ J/T.

36. Combining Eq. 32-27 with Eqs. 32-22 and 32-23, we see that the energy difference is

$$\Delta U = 2\mu_B B$$

where μ_B is the Bohr magneton (given in Eq. 32-25). With $\Delta U = 6.00\times10^{-25}$ J, we obtain B = 32.3 mT.

37. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of Section 32-9 is two-fold: $\vec{u}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material toward regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch, or in the $+x$ direction.

(c) The direction of the current is clockwise (from the perspective of the bar magnet).

(d) Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the right in our sketch, or in the $+x$ direction.

38. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t}\ ,$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e} .$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr .$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e} .$$

39. For the measurements carried out, the largest ratio of the magnetic field to the temperature is (0.50 T)/(10 K) = 0.050 T/K. Look at Fig. 32-14 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

40. (a) From Fig. 32-14 we estimate a slope of B/T = 0.50 T/K when $M/M_{\max}$ = 50%. So

$$B = 0.50 \text{ T} = (0.50 \text{ T/K})(300 \text{ K}) = 1.5\times10^2 \text{ T}.$$

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 6.0\times10^2$ T.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

41. **THINK** As defined in Eq. 32-38, magnetization is the dipole moment per unit volume.

EXPRESS Let M be the magnetization and $\mathcal{V}$ be the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). The dipole moment is given by $\mu = M\mathcal{V}$.

ANALYZE Substituting the values given, we obtain

$$\mu = M\pi r^2 L = \left(5.30\times10^3 \text{ A/m}\right)\pi\left(0.500\times10^{-2} \text{ m}\right)^2\left(5.00\times10^{-2} \text{ m}\right) = 2.08\times10^{-2} \text{ J/T} .$$

LEARN In a sample with N atoms, the magnetization reaches maximum, or saturation, when all the dipoles are completely aligned, leading to $M_{\max} = N\mu/\mathcal{V}$.

42. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - \left(-\vec{\mu}\cdot\vec{B}\right)\right| = 2\mu B$$

which leads to

$$T = \frac{4\mu B}{3k} = \frac{4\left(1.0\times10^{-23}\ \text{J/T}\right)\left(0.50\,\text{T}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = 0.48\,\text{K}\ .$$

43. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r}.$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}.$$

Since the magnetic force with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so $r = m_e v/eB$. Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B}.$$

The magnetic force $-e\vec{v}\times\vec{B}$ must point toward the center of the circular path. If the magnetic field is directed out of the page (defined to be $+z$ direction), the electron will travel counterclockwise around the circle. Since the electron is negative, the current is in the opposite direction, clockwise and, by the right-hand rule for dipole moments, the dipole moment is into the page, or in the $-z$ direction. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion.

(c) The direction of the dipole moment is $-z$, opposite to the magnetic field.

(d) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}\left(K_e + K_i\right) = \frac{5.3\times10^{21}\text{m}^{-3}}{1.2\text{T}}\left(6.2\times10^{-20}\text{J}+7.6\times10^{-21}\text{J}\right) = 3.1\times10^{2}\ \text{A/m}.$$

44. Section 32-10 explains the terms used in this problem and the connection between M and μ. The graph in Fig. 32-39 gives a slope of

$$\frac{M/M_{\text{max}}}{B_{\text{ext}}/T} = \frac{0.15}{0.20\ \text{T/K}} = 0.75\ \text{K/T}\ .$$

Thus we can write

$$\frac{\mu}{\mu_{\text{max}}} = (0.75\ \text{K/T})\frac{0.800\ \text{T}}{2.00\ \text{K}} = 0.30\,.$$

45. **THINK** According to statistical mechanics, the probability of a magnetic dipole moment placed in an external magnetic field having energy U is $P = e^{-U/kT}$, where k is the Boltzmann's constant.

EXPRESS The orientation energy of a dipole in a magnetic field is given by $U = -\vec{\mu}\cdot\vec{B}$. So if a dipole is parallel with $\vec{B}$, then $U = -\mu B$; however, $U = +\mu B$ if the alignment is anti-parallel. We use the notation $P(\mu) = e^{\mu B/kT}$ for the probability of a dipole that is parallel to $\vec{B}$, and $P(-\mu) = e^{-\mu B/kT}$ for the probability of a dipole that is anti-parallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities.

ANALYZE (a) With N atoms per unit volume, we find the magnetization to be

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu)+P(-\mu)} = \frac{N\mu\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{e^{\mu B/kT} + e^{-\mu B/kT}} = N\mu \tanh\left(\frac{\mu B}{kT}\right).$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu\,(1+\mu B/kT)-(1-\mu B/kT)}{(1+\mu B/kT)+(1-\mu B/kT)} = \frac{N\mu^2 B}{kT}\ .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so $M = N\mu \tanh\left(\dfrac{\mu B}{kT}\right) \approx N\mu$.

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-14. By adjusting the parameters used in one's plot, the curve in Fig. 32-14 can reliably be fit with a tanh function.

LEARN As can be seen from Fig. 32-14, the magnetization M is linear in B/kT in the regime $B/T \ll 1$. On the other hand, when $B \gg T$, M approaches a constant.

46. From Eq. 29-37 (see also Eq. 29-36) we write the torque as $\tau = -\mu B_h \sin\theta$ where the minus indicates that the torque opposes the angular displacement θ (which we will assume is small and in radians). The small angle approximation leads to $\tau \approx -\mu B_h \theta$, which is an indicator for simple harmonic motion (see section 16-5, especially Eq. 16-22). Comparing with Eq. 16-23, we then find the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{\mu B_h}}$$

where I is the rotational inertial that we asked to solve for. Since the frequency is given as 0.312 Hz, then the period is $T = 1/f = 1/(0.312\text{ Hz}) = 3.21$ s. Similarly, $B_h = 18.0 \times 10^{-6}$ T and $\mu = 6.80 \times 10^{-4}$ J/T. The above relation then yields $I = 3.19 \times 10^{-9}\ \text{kg}\cdot\text{m}^2$.

47. **THINK** In this problem, we model the Earth's magnetic dipole moment with a magnetized iron sphere.

EXPRESS If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}.$$

We substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} \quad\Rightarrow\quad R = \left(\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right)^{1/3}.$$

ANALYZE (a) The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\ \text{kg/u}) = 9.30\times10^{-26}\ \text{kg}.$$

Therefore, the radius of the iron sphere is

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\ \text{J/T})}{4\pi(14\times10^{3}\ \text{kg/m}^3)(2.1\times10^{-23}\ \text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}.$$

(b) The volume of the sphere is $V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(1.82\times10^5\,\text{m}\right)^3 = 2.53\times10^{16}\,\text{m}^3$ and the volume of the Earth is

$$V_E = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37\times10^6\,\text{m}\right)^3 = 1.08\times10^{21}\,\text{m}^3,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{V_s}{V_E} = \frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}.$$

LEARN The finding that $V_s \ll V_E$ makes it unlikely that our simple model of a magnetized iron sphere could explain the origin of Earth's magnetization.

48. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\text{g/cm}^3\right)\left(5.0\,\text{cm}\right)\left(1.0\,\text{cm}^2\right)}{\left(55.847\,\text{g/mol}\right)/\left(6.022\times10^{23}/\text{mol}\right)} = 4.3\times10^{23}.$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1\times10^{-23}\,\text{J/T}\right)\left(4.3\times10^{23}\right) = 8.9\,\text{A}\cdot\text{m}^2.$$

(b) $\tau = \mu B \sin 90° = (8.9\ \text{A}\cdot\text{m}^2)(1.57\ \text{T}) = 13\ \text{N}\cdot\text{m}$.

49. **THINK** Exchange coupling is a quantum phenomenon in which electron spins of one atom interact with those of neighboring atoms.

EXPRESS The field of a dipole along its axis is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3},$$

where μ is the dipole moment and z is the distance from the dipole. The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by

$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi,$$

where ϕ is the angle between the dipole moment and the field.

ANALYZE (a) Thus, the magnitude of the magnitude field at a distance 10 nm away from the atom is

$$B = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.5\times10^{-23}\ \text{J/T}\right)}{2\pi\left(10\times10^{-9}\ \text{m}\right)} = 3.0\times10^{-6}\ \text{T}.$$

(b) The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2\left(1.5\times10^{-23}\ \text{J/T}\right)\left(3.0\times10^{-6}\text{T}\right) = 9.0\times10^{-29}\,\text{J} = 5.6\times10^{-10}\ \text{eV}.$$

(c) The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

LEARN The persistent alignment of magnetic dipole moments despite the randomizing tendency due to thermal agitation is what gives the ferromagnetic materials their permanent magnetism.

50. (a) Equation 29-36 gives

$$\tau = \mu_{\text{rod}} B \sin\theta = (2700\ \text{A/m})(0.06\ \text{m})\pi(0.003\ \text{m})^2(0.035\ \text{T})\sin(68°) = 1.49\times10^{-4}\ \text{N}\cdot\text{m}.$$

We have used the fact that the volume of a cylinder is its length times its (circular) cross sectional area.

(b) Using Eq. 29-38, we have

$$\begin{aligned}\Delta U &= -\,\mu_{\text{rod}} B(\cos\theta_f - \cos\theta_i)\\ &= -(2700\ \text{A/m})(0.06\ \text{m})\pi(0.003\text{m})^2(0.035\text{T})[\cos(34°) - \cos(68°)]\\ &= -72.9\ \mu\text{J}.\end{aligned}$$

51. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\text{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$\begin{aligned}n &= \frac{\rho N_A}{M} = \frac{\left(8.90\,\text{g/cm}^3\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)}{58.71\,\text{g/mol}} = 9.126\times10^{22}\ \text{atoms/cm}^3\\ &= 9.126\times10^{28}\ \text{atoms/m}^3.\end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\text{sat}}}{n} = \frac{4.70\times10^5\ \text{A/m}}{9.126\times10^{28}\,\text{m}^3} = 5.15\times10^{-24}\,\text{A}\cdot\text{m}^2.$$

52. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then 10°C + (30°C/km)x = 770°C. Therefore,

$$x = \frac{770°\,\mathrm{C} - 10°\,\mathrm{C}}{30°\,\mathrm{C/km}} = 25\,\mathrm{km}.$$

53. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius (r_{avg} = 5.5 cm) to calculate n:

$$n = \frac{N}{2\pi r_{\text{avg}}} = \frac{400 \text{ turns}}{2\pi(5.5\times10^{-2}\,\mathrm{m})} = 1.16\times10^{3} \text{ turns/m} .$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20\times10^{-3}\ \mathrm{T}}{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(1.16\times10^{3}\,/\,\mathrm{m})} = 0.14\ \mathrm{A} .$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\varepsilon = N(d\Phi/dt)$ and the current in the secondary is $i_s = \varepsilon/R$, where R is the resistance of the coil. Thus,

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt}.$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\, dt = \frac{N}{R}\int \frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^{\Phi} d\Phi = \frac{N\Phi}{R} .$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus,

$$\Phi = 801\pi r^2 B_0 .$$

The radius r is (6.0 cm – 5.0 cm)/2 = 0.50 cm and

$$\Phi = 801\pi(0.50\times10^{-2}\ \mathrm{m})^2(0.20\times10^{-3}\mathrm{T}) = 1.26\times10^{-5}\ \mathrm{Wb} .$$

Consequently, $q = \dfrac{50(1.26\times10^{-5}\ \mathrm{Wb})}{8.0\ \Omega} = 7.9\times10^{-5}\ \mathrm{C}$.

54. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1}=\frac{r_1^3}{r_2^3}\ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2}=\frac{R_e^3}{(R_e+h)^3}\ .$$

Taking the cube root of both sides and solving for h, we get

$$h=\left(2^{1/3}-1\right)R_e=\left(2^{1/3}-1\right)(6370\,\text{km})=1.66\times10^3\,\text{km}.$$

(b) For maximum B, we set $\sin\lambda_m = 1.00$. Also, r = 6370 km – 2900 km = 3470 km. Thus,

$$B_{\max}=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.00\times10^{22}\,\text{A}\cdot\text{m}^2\right)}{4\pi\left(3.47\times10^6\,\text{m}\right)^3}\sqrt{1+3(1.00)^2}$$
$$=3.83\times10^{-4}\,\text{T}.$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so λ_m = 90.0° – 11.5° = 78.5° at Earth's geographic north pole. Also $r = R_e$ = 6370 km. Thus,

$$B=\frac{\mu_0\mu}{4\pi R_E^3}\sqrt{1+3\sin^2\lambda_m}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.0\times10^{22}\ \text{J/T}\right)\sqrt{1+3\sin^2 78.5^\circ}}{4\pi\left(6.37\times10^6\,\text{m}\right)^3}$$
$$=6.11\times10^{-5}\,\text{T}.$$

(d) $\phi_i=\tan^{-1}\left(2\tan 78.5^\circ\right)=84.2^\circ$.

(e) A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we used are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

55. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\,\text{J/T}}{\pi(6.37\times10^6\ \text{m})^2} = 6.3\times10^8\,\text{A}\ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

56. (a) The period of rotation is $T = 2\pi/\omega$, and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\pi r^2 = \frac{1}{2}q\omega r^2\ .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

57. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is

$$U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e\cos\theta,$$

where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ,

$$U(\theta) = -\mu B_e\cos\theta \approx -\mu B_e\left(1-\frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\ ,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}} \ ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\ \text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\ \text{T})} = 8.4\times10^2\ \text{J/T} \ .$$

58. (a) Equation 30-22 gives $B = \dfrac{\mu_0 ir}{2\pi R^2} = 222\ \mu\text{T}$.

(b) Equation 30-19 (or Eq. 30-6) gives $B = \dfrac{\mu_0 i}{2\pi r} = 167\ \mu\text{T}$.

(c) As in part (b), we obtain a field of $B = \dfrac{\mu_0 i}{2\pi r} = 22.7\ \mu\text{T}$.

(d) Equation 32-16 (with Eq. 32-15) gives $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 1.25\ \mu\text{T}$.

(e) As in part (d), we get $B = \dfrac{\mu_0 i_d r}{2\pi R^2} = 3.75\ \mu\text{T}$.

(f) Equation 32-17 yields B = 22.7 μT.

(g) Because the displacement current in the gap is spread over a larger cross-sectional area, values of B within that area are relatively small. Outside that cross-sectional area, the two values of B are identical.

59. (a) We use the result of part (a) in Sample Problem 32.01 — "Magnetic field induced by changing electric field:"

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt} \qquad (\text{for } r \le R) \ ,$$

where r = 0.80R , and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d}e^{-t/\tau} \ .$$

Here V_0 = 100 V. Thus,

$$B(t)=\left(\frac{\mu_0\varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d}e^{-t/\tau}\right)=-\frac{\mu_0\varepsilon_0 V_0 r}{2\tau d}e^{-t/\tau}$$

$$=-\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(8.85\times10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)(100\ \mathrm{V})(0.80)(16\ \mathrm{mm})}{2\left(12\times10^{-3}\ \mathrm{s}\right)(5.0\ \mathrm{mm})}e^{-t/12\ \mathrm{ms}}$$

$$=-\left(1.2\times10^{-13}\ \mathrm{T}\right)e^{-t/12\ \mathrm{ms}}\ .$$

The magnitude is $|B(t)|=\left(1.2\times10^{-13}\ \mathrm{T}\right)e^{-t/12\ \mathrm{ms}}$.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\ \mathrm{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}$ T, with a magnitude $|B(t)|=$ 5.9×10^{-15} T.

60. (a) From Eq. 32-1, we have

$$\left(\Phi_B\right)_{\text{in}}=\left(\Phi_B\right)_{\text{out}}=0.0070\,\mathrm{Wb}+\left(0.40\,\mathrm{T}\right)\left(\pi r^2\right)=9.2\times10^{-3}\,\mathrm{Wb}.$$

Thus, the magnetic of the magnetic flux is 9.2 mWb.

(b) The flux is inward.

61. **THINK** The Earth's magnetic field at a given latitude has both horizontal and vertical components.

EXPRESS Let B_h and B_v be the horizontal and vertical components of the Earth's magnetic field, respectively. Since B_h and B_v are perpendicular to each other, the Pythagorean theorem leads to $B=\sqrt{B_h^2+B_v^2}$. The tangent of the inclination angle is given by $\tan\phi_i = B_v / B_h$.

ANALYZE (a) Substituting the expression given in the problem statement, we have

$$B=\sqrt{B_h^2+B_v^2}=\sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2+\left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2}=\frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m+4\sin^2\lambda_m}$$

$$=\frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m},$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.

(b) The inclination ϕ_i is related to λ_m by $\tan\phi_i=\dfrac{B_v}{B_h}=\dfrac{\left(\mu_0\mu/2\pi r^3\right)\sin\lambda_m}{\left(\mu_0\mu/4\pi r^3\right)\cos\lambda_m}=2\tan\lambda_m.$

LEARN At the magnetic equator ($\lambda_m = 0$), $\phi_i = 0°$, and the field is

$$B = \frac{\mu_0 \mu}{4\pi r^3} = \frac{\left(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}\right)\left(8.00 \times 10^{22}\ \text{A} \cdot \text{m}^2\right)}{4\pi\left(6.37 \times 10^6\ \text{m}\right)^3} = 3.10 \times 10^{-5}\ \text{T}.$$

62. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B = \frac{\mu_0 \mu}{4\pi r^3} = \frac{\left(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}\right)\left(8.00 \times 10^{22}\ \text{A} \cdot \text{m}^2\right)}{4\pi\left(6.37 \times 10^6\ \text{m}\right)^3} = 3.10 \times 10^{-5}\ \text{T}.$$

(b) $\phi_i = \tan^{-1}(2 \tan \lambda_m) = \tan^{-1}(0) = 0°$.

(c) At $\lambda_m = 60.0°$, we find

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1 + 3\sin^2 \lambda_m} = \left(3.10 \times 10^{-5}\right)\sqrt{1 + 3\sin^2 60.0°} = 5.59 \times 10^{-5}\ \text{T}.$$

(d) $\phi_i = \tan^{-1}(2 \tan 60.0°) = 73.9°$.

(e) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B = \frac{\mu_0 \mu}{4\pi r^3}\sqrt{1 + 3\sin^2 \lambda_m} = \left(3.10 \times 10^{-5}\right)\sqrt{1 + 3(1.00)^2} = 6.20 \times 10^{-5}\ \text{T}.$$

(f) $\phi_i = \tan^{-1}(2 \tan 90.0°) = 90.0°$.

63. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt},$$

and for $r \ge R$, it is

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r}\frac{dE}{dt}.$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\text{max}} = \frac{\mu_0 \varepsilon_0 R}{2}\frac{dE}{dt}.$$

There are two values of r for which $B = B_{\text{max}}/2$: one less than R and one greater.

(a) To find the one that is less than *R*, we solve

$$\frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for *r*. The result is $r = R/2 = (55.0\text{ mm})/2 = 27.5\text{ mm}$.

(b) To find the one that is greater than *R*, we solve

$$\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}=\frac{\mu_0\varepsilon_0 R}{4}\frac{dE}{dt}$$

for *r*. The result is $r = 2R = 2(55.0\text{ mm}) = 110\text{ mm}$.

64. (a) Again from Fig. 32-14, for $M/M_{\text{max}} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4$ K.

(b) Now $B/T = 2.0$, so $T = 2/2.0 = 1$ K.

65. Let the area of each circular plate be *A* and that of the central circular section be *a*. Then

$$\frac{A}{a}=\frac{\pi R^2}{\pi\left(R/2\right)^2}=4\ .$$

Thus, from Eqs. 32-14 and 32-15 the total discharge current is given by $i = i_d = 4(2.0\text{ A}) = 8.0\text{ A}$.

66. Ignoring points where the determination of the slope is problematic, we find the interval of largest $|\Delta\vec{E}|/\Delta t$ is 6 μs < t < 7 μs. During that time, we have, from Eq. 32-14,

$$i_d=\varepsilon_0 A\frac{|\Delta\vec{E}|}{\Delta t}=(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m}^2)(2.0\times10^6\ \text{V/m})=3.5\times10^{-5}\,\text{A}.$$

67. (a) Using Eq. 32-13 but noting that the capacitor is being *discharged*, we have

$$\frac{d\,|\vec{E}\,|}{dt}=-\frac{i}{\varepsilon_0 A}=-\frac{5.0\text{ A}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0080\text{ m})^2}=-8.8\times10^{15}\ \text{V/m}\cdot\text{s}\ .$$

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in Section 32-4), we follow part (a) of Sample Problem 32.02 — "Treating a changing electric field as a displacement current" and relate the (absolute value of the) line integral to the portion of displacement current enclosed:

$$\left|\oint \vec{B}\cdot d\vec{s}\right| = \mu_0 i_{d,\text{enc}} = \mu_0\left(\frac{WH}{L^2}i\right) = 5.9\times10^{-7}\,\text{Wb/m}.$$

68. (a) Using Eq. 32-31, we find

$$\mu_{\text{orb},z} = -3\mu_B = -2.78 \times 10^{-23}\ \text{J/T}.$$

That these are acceptable units for magnetic moment is seen from Eq. 32-32 or Eq. 32-27; they are equivalent to A·m^2.

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z}$ = 3.71 × 10^{-23} J/T.

69. (a) Since the field lines of a bar magnet point toward its South pole, then the $\vec{B}$ arrows in one's sketch should point generally toward the left and also towards the central axis.

(b) The sign of $\vec{B}\cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

(c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \vec{B}\cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

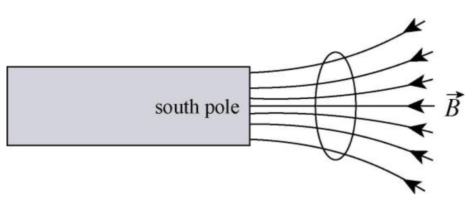
70. (a) From Eq. 21-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(5.2\times10^{-11}\ \text{m}\right)^2} = 5.3\times10^{11}\ \text{N/C}\ .$$

(b) We use Eq. 29-28: $B = \dfrac{\mu_0}{2\pi}\dfrac{\mu_p}{r^3} = \dfrac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.4\times10^{-26}\ \text{J/T}\right)}{2\pi\left(5.2\times10^{-11}\ \text{m}\right)^3} = 2.0\times10^{-2}\ \text{T}$.

(c) From Eq. 32-30, $\dfrac{\mu_{\text{orb}}}{\mu_p} = \dfrac{eh/4\pi m_e}{\mu_p} = \dfrac{\mu_B}{\mu_p} = \dfrac{9.27\times10^{-24}\ \text{J/T}}{1.4\times10^{-26}\ \text{J/T}} = 6.6\times10^2$.

71. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) For paramagnetic materials, the dipole moment $\vec{\mu}$ is in the same direction as $\vec{B}$. From the above figure, $\vec{\mu}$ points in the $-x$ direction.

(c) Form the right-hand rule, since $\vec{\mu}$ points in the $-x$ direction, the current flows counterclockwise, from the perspective of the bar magnet.

(d) The effect of $\vec{F}$ is to move the material toward regions of larger $|\vec{B}|$ values. Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the left, or $-x$.

72. (a) Inside the gap of the capacitor, $B_1 = \mu_0 i_d\, r_1/2\pi R^2$ (Eq. 32-16); outside the gap the magnetic field is $B_2 = \mu_0 i_d/2\pi r_2$ (Eq. 32-17). Consequently, $B_2 = B_1 R^2/r_1 r_2 = 16.7$ nT.

(b) The displacement current is $i_d = 2\pi B_1 R^2/\mu_0 r_1 = 5.00$ mA.

73. **THINK** The z component of the orbital angular momentum is give by $L_{\text{orb},z} = m_\ell h/2\pi$, where h is the Planck constant and m_ℓ is the orbital magnetic quantum number.

EXPRESS The "limit" for m_ℓ is 3. This means that the allowed values of m_ℓ are: $0, \pm 1, \pm 2,$ and ± 3.

ANALYZE (a) The number of different m_ℓ's is 2(3) + 1 = 7. Since $L_{\text{orb},z} \propto m_\ell$, there are a total of seven different values of $L_{\text{orb},z}$.

(b) Similarly, since $\mu_{\text{orb},z} \propto m_\ell$, there are also a total of seven different values of $\mu_{\text{orb},z}$.

(c) The greatest allowed value of $L_{\text{orb},z}$ is given by $|m_\ell|_{\max} h/2\pi = 3h/2\pi$.

(d) Similar to part (c), since $\mu_{\text{orb},z} = -m_\ell \mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_\ell|_{\max}\mu_B = 3eh/4\pi m_e$.

(e) From Eqs. 32-23 and 32-29 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_\ell h}{2\pi} + \frac{m_s h}{2\pi}.$$

For the maximum value of $L_{\text{net},z}$ let $m_\ell = [m_\ell]_{\max} = 3$ and $m_s = \frac{1}{2}$. Thus

$$L_{\text{net},z\ \max} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}.$$

(f) Since the maximum value of $L_{net,z}$ is given by $[m_J]_{max}h/2\pi$ with $[m_J]_{max} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{net,z}$ is given by $2[m_J]_{max} + 1 = 2(3.5) + 1 = 8$.

LEARN As we shall see in Chapter 40, the allowed values of m_ℓ range from $-\ell$ to $+\ell$, where ℓ is called the orbital quantum number.

74. The definition of displacement current is Eq. 32-10, and the formula of greatest convenience here is Eq. 32-17:

$$i_d = \frac{2\pi r\ B}{\mu_0} = \frac{2\pi\left(0.0300\,\text{m}\right)\left(2.00\times10^{-6}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 0.300\ \text{A}\ .$$

75. (a) The complete set of values are

$$\{-4, -3, -2, -1, 0, +1, +2, +3, +4\} \quad\Rightarrow\quad \text{nine values in all.}$$

(b) The maximum value is $4\mu_B = 3.71 \times 10^{-23}$ J/T.

(c) Multiplying our result for part (b) by 0.250 T gives $U = +9.27 \times 10^{-24}$ J.

(d) Similarly, for the lower limit, $U = -9.27 \times 10^{-24}$ J.

76. (a) The z component of the orbital magnetic dipole moment is

$$\mu_{\text{orb},z} = -m_\ell \mu_B$$

where $\mu_B = eh/4\pi m = 9.27\times10^{-24}\,\text{J/T}$ is the Bohr magneton. For $m_l = 3,$ we have

$$\mu_{\text{orb,z}} = -m_\ell \mu_B - (3)(9.27\times10^{-24}\ \text{J/T}) = -2.78\times10^{-23}\ \text{J/T}.$$

(b) Similarly, for $m_l = -4,$ the result is

$$\mu_{\text{orb},z} = -m_\ell \mu_B - (-4)(9.27\times10^{-24}\ \text{J/T}) = 3.71\times10^{-23}\ \text{J/T}.$$