Chapter 29

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With r = 20 ft = 6.10 m, we have

$$B = \frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(100\ \mathrm{A})}{2\pi(6.10\ \mathrm{m})} = 3.3\times 10^{-6}\ \mathrm{T} = 3.3\ \mu\mathrm{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. Equation 29-1 is maximized (with respect to angle) by setting θ = 90º (= π/2 rad). Its value in this case is

$$dB_{\max} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}.$$

From Fig. 29-35(b), we have $B_{\max} = 60\times 10^{-12}$ T. We can relate this $B_{\max}$ to our $dB_{\max}$ by setting "ds" equal to 1×10^{-6} m and R = 0.025 m. This allows us to solve for the current: i = 0.375 A. Plugging this into Eq. 29-4 (for the infinite wire) gives B_∞ = 3.0 μT.

3. **THINK** The magnetic field produced by a current-carrying wire can be calculated using the Biot-Savart law.

EXPRESS The magnitude of the magnetic field at a distance r from a long straight wire carrying current i is, using the Biot-Savart law, $B = \mu_0 i/2\pi r$.

ANALYZE (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Therefore,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\ \mathrm{m})(39\times 10^{-6}\ \mathrm{T})}{4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}} = 16\ \mathrm{A}.$$

(b) The current must be from west to east to produce a field that is directed southward at points below it.

LEARN The direction of the current is given by the right-hand rule: grasp the element in your right hand with your thumb pointing in the direction of the current. The direction of the field due to the current-carrying element corresponds to the direction your fingers naturally curl.

4. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current"). Also, the fields from the two semicircular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

5. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 29-7) and a semicircular arc (Eq. 29-9 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 29-7(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{2(0.0050\,\mathrm{m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\ \mathrm{T}$$

upon substituting i = 10 A and R = 0.0050 m.

(b) The direction of this field is out of the page, as Fig. 29-7(c) makes clear.

(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 29-4,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{\pi(0.0050\,\mathrm{m})} = 8.0\times10^{-4}\,\mathrm{T}.$$

(d) This field, too, points out of the page.

6. With the "usual" x and y coordinates used in Fig. 29-38, then the vector $\vec{r}$ pointing from a current element to P is $\vec{r} = -s\,\hat{\mathrm{i}} + R\,\hat{\mathrm{j}}$. Since $d\vec{s} = ds\,\hat{\mathrm{i}}$, then $|d\vec{s}\times\vec{r}| = Rds.$ Therefore, with $r = \sqrt{s^2+R^2}$, Eq. 29-3 gives

$$dB = \frac{\mu_0}{4\pi}\frac{iR\,ds}{(s^2+R^2)^{3/2}}\,.$$

(a) Clearly, considered as a function of s (but thinking of "ds" as some finite-sized constant value), the above expression is maximum for s = 0. Its value in this case is $dB_{\max} = \mu_0 i\,ds/4\pi R^2$.

(b) We want to find the s value such that $dB = dB_{\max}/10$. This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is $s = \sqrt{10^{2/3}-1}\,R$. If we set $R = 2.00$ cm, then we obtain s = 3.82 cm.

7. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with P do not contribute to the field at that point. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$B=\frac{\mu_0 i\theta}{4}\left(\frac{1}{b}-\frac{1}{a}\right)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.411\text{A})(74°\cdot\pi/180°)}{4\pi}\left(\frac{1}{0.107\text{m}}-\frac{1}{0.135\text{m}}\right)$$
$$=1.02\times10^{-7}\,\text{T}.$$

(b) The direction is out of the page.

8. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in segments AH and JD do not contribute to the field at point C. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc $H\,J$ contributes $\mu_0 i/4R_1$ (into the page) to the field at C. Also, arc $D\,A$ contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at C is

$$B=\frac{\mu_0 i}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.281\text{A})}{4}\left(\frac{1}{0.0315\text{m}}-\frac{1}{0.0780\text{m}}\right)=1.67\times10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

9. **THINK** The net magnetic field at a point half way between the two long straight wires is the vector sum of the magnetic fields due to the currents in the two wires.

EXPRESS Since the magnitude of the magnetic field at a distance r from a long straight wire carrying current i is given by $B=\mu_0 i/2\pi r$, at a point half way between the two sires, the magnetic field is $\vec{B}=\vec{B}_1+\vec{B}_2$, where $B_1=B_2=\mu_0 i/2\pi r$ (assuming the two wires to be $2r$ apart). The directions of $\vec{B}_1$ and $\vec{B}_2$ are determined by the right-hand rule.

ANALYZE (a) The currents must be opposite or anti-parallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) The total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi\left(0.040\text{ m}\right)\left(300\times10^{-6}\text{T}\right)}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 30\,\text{A}.$$

LEARN For two parallel wires carrying currents in the opposite directions, a point that is a distance d from one wire and $2r - d$ from the other, the magnitude of the magnetic field is

$$B = B_1 + B_2 = \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{2\pi(2r-d)} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d} + \frac{1}{2r-d}\right).$$

10. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with C do not contribute to the field at that point.

Equation 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. Thus, the magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.0348\text{A})}{4(0.0926\text{m})} = 1.18\times10^{-7}\,\text{T}.$$

(b) The right-hand rule shows that this field is into the page.

11. (a) $B_{P_1} = \mu_0 i_1 / 2\pi r_1$ where i_1 = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2} = \mu_0 i_2 / 2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = \left(6.5\,\text{A}\right)\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right) = 4.3\,\text{A}.$$

(b) Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page.

12. (a) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current 3.00i, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad\Rightarrow\quad r = \frac{d}{4} = \frac{16.0\text{ cm}}{4} = 4.0\text{ cm}.$$

(b) Doubling the currents does not change the location where the magnetic field is zero.

13. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of

the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\,\theta$ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from x = –L/2 to x = L/2. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$
$$= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(0.0582\ \mathrm{A}\right)}{2\pi\left(0.131\,\mathrm{m}\right)}\frac{0.180\mathrm{m}}{\sqrt{(0.180\mathrm{m})^2+4(0.131\mathrm{m})^2}} = 5.03\times10^{-8}\ \mathrm{T}.$$

14. We consider Eq. 29-6 but with a finite upper limit (L/2 instead of ∞). This leads to

$$B = \frac{\mu_0 i}{2\pi R}\frac{L/2}{\sqrt{(L/2)^2+R^2}}\,.$$

In terms of this expression, the problem asks us to see how large L must be (compared with R) such that the infinite wire expression B_∞ (Eq. 29-4) can be used with no more than a 1% error. Thus we must solve

$$\frac{B_\infty - B}{B} = 0.01.$$

This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is

$$L = \frac{200R}{\sqrt{201}} \approx 14.1R \quad \Rightarrow \quad \frac{L}{R} \approx 14.1\,.$$

15. (a) As discussed in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," the radial segments do not contribute to $\vec{B}_P$ and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0\left(0.40\,\mathrm{A}\right)\left(\pi\,\mathrm{rad}\right)}{4\pi\left(0.050\,\mathrm{m}\right)}\hat{\mathrm{k}} - \frac{\mu_0\left(0.80\,\mathrm{A}\right)\left(2\pi/3\,\mathrm{rad}\right)}{4\pi\left(0.040\,\mathrm{m}\right)}\hat{\mathrm{k}} = -(1.7\times10^{-6}\ \mathrm{T})\hat{\mathrm{k}}$$

or $|\vec{B}| = 1.7\times10^{-6}\,\mathrm{T}$.

(b) The direction is $-\hat{\mathrm{k}}$, or into the page.

(c) If the direction of i_1 is reversed, we then have

$$\vec{B} = -\frac{\mu_0 (0.40\,\text{A})(\pi\,\text{rad})}{4\pi (0.050\,\text{m})}\hat{\text{k}} - \frac{\mu_0 (0.80\,\text{A})(2\pi/3\,\text{rad})}{4\pi (0.040\,\text{m})}\hat{\text{k}} = -(6.7\times 10^{-6}\ \text{T})\hat{\text{k}}$$

or $|\vec{B}| = 6.7\times 10^{-6}\,\text{T}$.

(d) The direction is $-\hat{\text{k}}$, or into the page.

16. Using the law of cosines and the requirement that B = 100 nT, we have

$$\theta = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1B_2}\right) = 144°,$$

where Eq. 29-10 has been used to determine B_1 (168 nT) and B_2 (151 nT).

17. **THINK** We apply the Biot-Savart law to calculate the magnetic field at point P_2. An integral is required since the length of the wire is finite.

EXPRESS We take the x axis to be along the wire with the origin at the right endpoint. The current is in the $+x$ direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L$ to $x = 0$.

ANALYZE The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$
$$= \frac{(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A})(0.693\,\text{A})}{4\pi(0.251\,\text{m})}\frac{0.136\,\text{m}}{\sqrt{(0.136\,\text{m})^2+(0.251\,\text{m})^2}} = 1.32\times 10^{-7}\,\text{T}.$$

LEARN In calculating B at P_2, we could have chosen the origin to be at the left endpoint. This only changes the integration limit, but the result remains the same:

$$B = \frac{\mu_0 iR}{4\pi}\int_0^L \frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_0^L = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}} .$$

18. In the one case we have $B_{\text{small}} + B_{\text{big}} = 47.25\ \mu\text{T}$, and the other case gives $B_{\text{small}} - B_{\text{big}} = 15.75\ \mu\text{T}$ (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{\text{small}})+(1/r_{\text{big}})}{(1/r_{\text{small}})-(1/r_{\text{big}})} = \frac{1+(r_{\text{small}}/r_{\text{big}})}{1-(r_{\text{small}}/r_{\text{big}})} = 3 .$$

The solution of this is straightforward: $r_{\text{small}} = r_{\text{big}}/2$. Using the given fact that the $r_{\text{big}} = 4.00$ cm, then we conclude that the small radius is $r_{\text{small}} = 2.00$ cm.

19. The contribution to $\vec{B}_{\text{net}}$ from the first wire is (using Eq. 29-4)

$$\vec{B}_1 = \frac{\mu_0 i_1}{2\pi r_1}\hat{\text{k}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(30\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{k}} = (3.0\times10^{-6}\ \text{T})\hat{\text{k}}.$$

The distance from the second wire to the point where we are evaluating $\vec{B}_{\text{net}}$ is r_2 = 4 m – 2 m = 2 m. Thus,

$$\vec{B}_2 = \frac{\mu_0 i_2}{2\pi r_2}\hat{\text{i}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(40\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{i}} = (4.0\times10^{-6}\ \text{T})\hat{\text{i}}.$$

and consequently is perpendicular to $\vec{B}_1$. The magnitude of $\vec{B}_{\text{net}}$ is therefore

$$|\vec{B}_{\text{net}}| = \sqrt{(3.0\times10^{-6}\ \text{T})^2 + (4.0\times10^{-6}\ \text{T})^2} = 5.0\times10^{-6}\ \text{T} .$$

20. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2} = \dfrac{\mu_0 i}{2R}$. Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right) = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\ \text{A}\right)}{2\left(0.0189\ \text{m}\right)}\left(1+\frac{1}{\pi}\right) = 2.53\times10^{-7}\text{T}.$$

$\vec{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\text{T})\hat{\text{k}}$

(b) Now, $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\,\text{A}\right)}{2\left(0.0189\,\text{m}\right)}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\text{T}.$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\text{T}(\cos17.66°\,\hat{\text{i}} + \sin17.66°\,\hat{\text{k}}) = (1.92\times10^{-7}\,\text{T})\hat{\text{i}} + (6.12\times10^{-8}\,\text{T})\hat{\text{k}}.$$

21. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the y axis (passing through P), consisting of two equal magnetic field y-components. Using Eq. 29-17,

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where i = 4.00 A, r = $r = \sqrt{d_2^2 + d_1^2/4} = 5.00\text{ m},$ and

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{4.00\text{ m}}{6.00\text{ m}/2}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°.$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.00\text{ A})}{\pi(5.00\text{ m})}\sin53.1° = 2.56\times10^{-7}\text{ T}.$$

22. The fact that $B_y = 0$ at x = 10 cm implies the currents are in opposite directions. Thus,

$$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

using Eq. 29-4 and the fact that $i_1 = 4i_2$. To get the maximum, we take the derivative with respect to x and set equal to zero. This leads to $3x^2 - 2Lx - L^2 = 0$, which factors and becomes $(3x + L)(x - L) = 0$, which has the physically acceptable solution: $x = L$. This produces the maximum B_y: $\mu_0 i_2/2\pi L$. To proceed further, we must determine L.

Examination of the datum at x = 10 cm in Fig. 29-50(b) leads (using our expression above for B_y and setting that to zero) to L = 30 cm.

(a) The maximum value of B_y occurs at $x = L$ = 30 cm.

(b) With i_2 = 0.003 A we find $\mu_0 i_2 /2\pi L$ = 2.0 nT.

(c) and (d) Figure 29-50(b) shows that as we get <u>very</u> close to wire 2 (where its field strongly dominates over that of the more distant wire 1) B_y points along the $-y$ direction. The right-hand rule leads us to conclude that wire 2's current is consequently is *into the page*. We previously observed that the currents were in opposite directions, so wire 1's current is *out of the page*.

23. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge q is $\vec{B} = (\mu_0 i / 2\pi d)\,\hat{\mathrm{k}}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 iq}{2\pi d}\left(\vec{v} \times \hat{\mathrm{k}}\right).$$

In this situation, $\vec{v} = v\left(-\hat{\mathrm{j}}\right)$ (where v is the speed and is a positive value), and $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left(\left(-\hat{\mathrm{j}}\right) \times \hat{\mathrm{k}}\right) = -\frac{\mu_0 iqv}{2\pi d}\,\hat{\mathrm{i}} = -\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.350\mathrm{A})(1.60\times10^{-19}\,\mathrm{C})(200\mathrm{m/s})}{2\pi(0.0289\text{ m})}\,\hat{\mathrm{i}}$$
$$= (-7.75\times10^{-23}\,\mathrm{N})\hat{\mathrm{i}}.$$

24. Initially, we have $B_{\text{net},y}$ = 0 and $B_{\text{net},x} = B_2 + B_4 = 2(\mu_0 i /2\pi d)$ using Eq. 29-4, where $d = 0.15$ m. To obtain the 30º condition described in the problem, we must have

$$B_{\text{net},y} = B_{\text{net},x}\tan(30°) \quad \Rightarrow \; B_1' - B_3 = 2\left(\frac{\mu_0 i}{2\pi d}\right)\tan(30°)$$

where $B_3 = \mu_0 i /2\pi d$ and $B_1' = \mu_0 i / 2\pi d'$. Since tan(30º) = $1/\sqrt{3}$, this leads to

$$d' = \frac{\sqrt{3}}{\sqrt{3}+2}\,d = 0.464d\ .$$

(a) With d = 15.0 cm, this gives d' = 7.0 cm. Being very careful about the geometry of the situation, then we conclude that we must move wire 1 to x = –7.0 cm.

(b) To restore the initial symmetry, we would have to move wire 3 to x = +7.0 cm.

25. **THINK** The magnetic field at the center of the circle is the vector sum of the fields of the two straight wires and the arc.

EXPRESS Each of the semi-infinite straight wires contributes $B_{\text{straight}} = \mu_0 i/4\pi R$ (Eq. 29-7) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 29-9: $B_{\text{arc}} = \dfrac{\mu_0 i\phi}{4\pi R}$, pointing into the page.

ANALYZE The total magnetic field is

$$B = 2B_{\text{straight}} - B_{\text{arc}} = 2\left(\frac{\mu_0 i}{4\pi R}\right) - \frac{\mu_0 i\phi}{4\pi R} = \frac{\mu_0 i}{4\pi R}(2-\phi).$$

Therefore, $\phi = 2.00$ rad would produce zero total field at the center of the circle.

LEARN The total contribution of the two semi-infinite wires is the same as that of an infinite wire. Note that the angle ϕ is in radians rather than degrees.

26. Using the Pythagorean theorem, we have

$$B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i_2}{2\pi R}\right)^2$$

which, when thought of as the equation for a line in a B^2 versus i_2^2 graph, allows us to identify the first term as the "y-intercept" (1×10^{-10}) and the part of the second term that multiplies i_2^2 as the "slope" (5×10^{-10}). The latter observation leads to

$$5.00\times10^{-10}\ \text{T}^2/\text{A}^2 = \left(\frac{\mu_0}{2\pi R}\right)^2 = \left(\frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi R}\right)^2$$

or

$$R^2 = \frac{4.00\times10^{-14}\ \text{T}^2\cdot\text{m}^2/\text{A}^2}{5.00\times10^{-10}\ \text{T}^2/\text{A}^2} = 8.00\times10^{-5}\,\text{m}^2 \ \Rightarrow\ R = 8.94\times10^{-3}\ \text{m} \approx 8.9\ \text{mm}.$$

The other observation about the "y-intercept" determines the angle subtended by the arc:

$$1.00\times10^{-10}\ \text{T}^2 = \left(\frac{\mu_0 i_1\phi}{4\pi R}\right)^2 = \left(\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.50\ \text{A})}{4\pi(8.94\times10^{-3}\text{m})}\right)^2 \phi^2 = (3.13\times10^{-11}\phi^2)\ \text{T}^2$$

or

$$\phi^2 = \frac{1.00\times10^{-10}\ \text{T}^2}{3.13\times10^{-11}\ \text{T}^2} = 3.19 \ \Rightarrow\ \phi = 1.79\ \text{rad} \approx 1.8\ \text{rad}.$$

27. We use Eq. 29-4 to relate the magnitudes of the magnetic fields B_1 and B_2 to the currents (i_1 and i_2, respectively) in the two long wires. The angle of their net field is

$$\theta = \tan^{-1}(B_2/B_1) = \tan^{-1}(i_2/i_1) = 53.13°.$$

The accomplish the net field rotation described in the problem, we must achieve a final angle $\theta' = 53.13° - 20° = 33.13°$. Thus, the final value for the current i_1 must be $i_2/\tan\theta'$ = 61.3 mA.

28. Letting "out of the page" in Fig. 29-56(a) be the positive direction, the net field is

$$B = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi(R/2)}$$

from Eqs. 29-9 and 29-4. Referring to Fig. 29-56, we see that $B = 0$ when $i_2 = 0.5$ A, so (solving the above expression with B set equal to zero) we must have

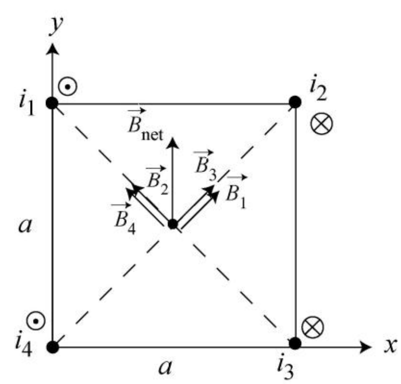
$$\phi = 4(i_2/i_1) = 4(0.5/2) = 1.00 \text{ rad (or } 57.3°).$$

29. **THINK** Our system consists of four long straight wires whose cross section form a square of length a. The magnetic field at the center of the square is the vector sum of the fields of the four wires.

EXPRESS Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left (wire 1) and lower right (wire 3) corners both point toward the upper right corner of the square. The fields due to the wires at the upper right (wire 2) and lower left (wire 4) corners both point toward the upper left corner.

ANALYZE The horizontal components of the fields cancel and the vertical components sum to

$$B_{\text{net}} = 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a} = \frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(20\ \text{A})}{\pi(0.20\ \text{m})}$$
$$= 8.0\times10^{-5}\ \text{T}.$$



In the calculation, cos 45° was replaced with $1/\sqrt{2}$. The total field points upward, or in the $+y$ direction. Thus, $\vec{B}_{\text{net}} = (8.0\times10^{-5}\ \text{T})\hat{\text{j}}$.

LEARN In the figure to the right, we show the contributions from the individual wires. The directions of the fields are deduced using the right-hand rule.

30. We note that when there is no *y*-component of magnetic field from wire 1 (which, by the right-hand rule, relates to when wire 1 is at 90º = $\pi/2$ rad), the <u>total</u> *y*-component of magnetic field is zero (see Fig. 29-58(c)). This means wire #2 is either at $+\pi/2$ rad or $-\pi/2$ rad.

(a) We now make the assumption that wire #2 must be at $-\pi/2$ rad (–90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the θ_1 = 90º datum in Fig. 29-58(b)), where there is a *maximum* in $B_{\text{net}\,x}$ (equal to +6 μT), we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(4.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 4.0\text{ A}\,.$$

(c) The fact that Fig. 29-58(b) increases as θ_1 progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net}\,y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-58(c)).

(d) Referring now to Fig. 29-58(b) we note that there is no *x*-component of magnetic field from wire 1 when θ_1 = 0, so that plot tells us that B_{2x} = +2.0 μT. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 2.0\text{ A}\,.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

31. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *P* do not contribute to the field at that point. We use the result of Problem 29-21 to evaluate the contributions to the field at *P*, noting that the nearest wire segments (each of length *a*) produce magnetism into the page at *P* and the further wire segments (each of length 2*a*) produce magnetism pointing out of the page at *P*. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} = \frac{\sqrt{2}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(13\text{ A})}{8\pi\left(0.047\text{ m}\right)}$$
$$= 1.96\times10^{-5}\,\text{T} \approx 2.0\times10^{-5}\,\text{T}.$$

(b) The direction of the field is into the page.

32. Initially we have

$$B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$$

using Eq. 29-9. In the final situation we use Pythagorean theorem and write

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2.$$

If we square B_i and divide by B_f^2, we obtain

$$\left(\frac{B_i}{B_f}\right)^2 = \frac{[(1/R)+(1/r)]^2}{(1/R)^2 + (1/r)^2} .$$

From the graph (see Fig. 29-60(c), note the maximum and minimum values) we estimate $B_i/B_f = 12/10 = 1.2$, and this allows us to solve for r in terms of R:

$$r = R\,\frac{1 \pm 1.2\sqrt{2 - 1.2^2}}{1.2^2 - 1} = 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}.$$

Since we require $r < R$, then the acceptable answer is $r = 2.3$ cm.

33. **THINK** The magnetic field at point P produced by the current-carrying ribbon (shown in Fig. 29-61) can be calculated using the Biot-Savart law.

EXPRESS Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}.$$

ANALYZE Integrating over the length of the ribbon, we obtain

$$B_P = \int dB_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.61\times10^{-6}\ \text{A})}{2\pi\left(0.0491\ \text{m}\right)}\ln\left(1+\frac{0.0491}{0.0216}\right)$$
$$= 2.23\times10^{-11}\text{T}.$$

and $\vec{B}_P$ points upward. In unit-vector notation, $\vec{B}_P = (2.23\times10^{-11}\text{T})\hat{\text{j}}$.

LEARN In the limit where $d \gg w$, using

$$\ln(1+x) = x - x^2/2 + \cdots,$$

the magnetic field becomes

$$B_P = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) \approx \frac{\mu_0 i}{2\pi w}\cdot\frac{w}{d} = \frac{\mu_0 i}{2\pi d}$$

which is the same as that due to a thin wire.

34. By the right-hand rule (which is "built-into" Eq. 29-3) the field caused by wire 1's current, evaluated at the coordinate origin, is along the +y axis. Its magnitude B_1 is given by Eq. 29-4. The field caused by wire 2's current will generally have both an x and a y component, which are related to its magnitude B_2 (given by Eq. 29-4), and sines and cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle θ_2 (shown in Fig. 29-62) then its components are

$$B_{2x} = B_2 \sin\theta_2, \quad B_{2y} = -B_2\cos\theta_2 .$$

The magnitude-squared of their net field is then (by Pythagoras' theorem) the sum of the square of their net x-component and the square of their net y-component:

$$B^2 = (B_2\sin\theta_2)^2 + (B_1 - B_2\cos\theta_2)^2 = B_1^2 + B_2^2 - 2B_1B_2\cos\theta_2 .$$

(since $\sin^2\theta + \cos^2\theta = 1$), which we could also have gotten directly by using the law of cosines. We have

$$B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}.$$

With the requirement that the net field have magnitude B = 80 nT, we find

$$\theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1B_2}\right) = \cos^{-1}(-1/4) = 104°,$$

where the positive value has been chosen.

35. **THINK** The magnitude of the force of wire 1 on wire 2 is given by $F_{21} = \mu_0 i_1 i_2 L / 2\pi r,$ where i_1 is the current in wire 1, i_2 is the current in wire 2, and r is the distance between the wires.

EXPRESS The distance between the wires is $r = \sqrt{d_1^2 + d_2^2}$. The x component of the force is $F_{21,x} = F_{21}\cos\phi,$ where $\cos\phi = d_2/\sqrt{d_1^2 + d_2^2}$.

ANALYZE Substituting the values given, the x component of the force per unit length is

$$\frac{F_{21,x}}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2 + d_2^2)} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(4.00\times10^{-3}\ \mathrm{A})(6.80\times10^{-3}\ \mathrm{A})(0.050\ \mathrm{m})}{2\pi[(0.0240\ \mathrm{m})^2 + (0.050\ \mathrm{m})^2]}$$
$$= 8.84\times10^{-11}\ \mathrm{N/m}.$$

LEARN Since the two currents flow in the opposite directions, the force between the wires is repulsive. Thus, the direction of $\vec{F}_{21}$ is along the line that joins the wire and is away from wire 1.

36. We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\hat{\mathrm{j}} = \frac{25\mu_0 i^2 l}{24\pi d}\,\hat{\mathrm{j}} = \frac{25\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(3.00\mathrm{A}\right)^2(10.0\mathrm{m})}{24\pi\left(8.00\times10^{-2}\,\mathrm{m}\right)}\,\hat{\mathrm{j}}$$
$$= (4.69\times10^{-4}\,\mathrm{N})\,\hat{\mathrm{j}}.$$

(b) Similarly, for wire 2, we have

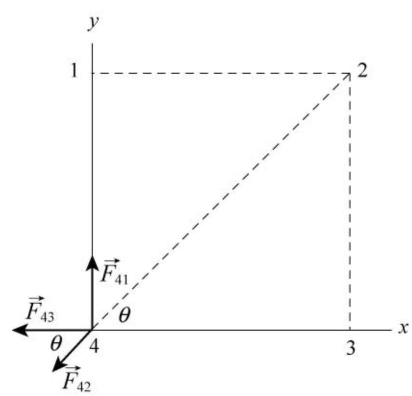
$$\vec{F}_2 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\mathrm{j}} = \frac{5\mu_0 i^2 l}{12\pi d}\,\hat{\mathrm{j}} = (1.88\times10^{-4}\mathrm{N})\,\hat{\mathrm{j}}.$$

(c) $F_3 = 0$ (because of symmetry).

(d) $\vec{F}_4 = -\vec{F}_2 = (-1.88\times10^{-4}\mathrm{N})\hat{\mathrm{j}}$, and

(e) $\vec{F}_5 = -\vec{F}_1 = -(4.69\times10^{-4}\,\mathrm{N})\hat{\mathrm{j}}$.

37. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.



The components of $\vec{F}_4$ are given by

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

and

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$

Thus,

$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(7.50\mathrm{A}\right)^2}{4\pi\left(0.135\mathrm{m}\right)}$$
$$= 1.32\times10^{-4}\ \mathrm{N/m}$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°.$$

In unit-vector notation, we have

$$\vec{F}_1 = (1.32\times10^{-4}\ \mathrm{N/m})[\cos 162°\hat{\mathrm{i}} + \sin 162°\hat{\mathrm{j}}] = (-1.25\times10^{-4}\ \mathrm{N/m})\hat{\mathrm{i}} + (4.17\times10^{-5}\ \mathrm{N/m})\hat{\mathrm{j}}$$

38. (a) The fact that the curve in Fig. 29-65(b) passes through zero implies that the currents in wires 1 and 3 exert forces in opposite directions on wire 2. Thus, current i_1 points *out of the page*. When wire 3 is a great distance from wire 2, the only field that affects wire 2 is that caused by the current in wire 1; in this case the force is negative according to Fig. 29-65(b). This means wire 2 is attracted to wire 1, which implies (by the discussion in Section 29-2) that wire 2's current is in the same direction as wire 1's current: *out of the page*. With wire 3 infinitely far away, the force per unit length is given (in magnitude) as 6.27×10^{-7} N/m. We set this equal to $F_{12} = \mu_0 i_1 i_2 / 2\pi d$. When wire 3 is at x = 0.04 m the curve passes through the zero point previously mentioned, so the force between 2 and 3 must equal F_{12} there. This allows us to solve for the distance between wire 1 and wire 2:

$$d = (0.04\ \mathrm{m})(0.750\ \mathrm{A})/(0.250\ \mathrm{A}) = 0.12\ \mathrm{m}.$$

Then we solve 6.27×10^{-7} N/m= $\mu_0 i_1 i_2 / 2\pi d$ and obtain i_2 = 0.50 A.

(b) The direction of i_2 is out of the page.

39. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting d = 0.500 m, we find the net force (per meter length) using Eq. 29-13, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.00\times10^{-7}\ \mathrm{N/m}$.

40. Using Eq. 29-13, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing toward wire 3, which is at the lower right). Only the forces

(or their components) along the diagonal direction contribute. With $\theta = 45°$, we find the force per unit meter on wire 1 to be

$$F_1 = |\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}| = 2F_{12}\cos\theta + F_{13} = 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} = \frac{3}{2\sqrt{2}\pi}\left(\frac{\mu_0 i^2}{a}\right)$$
$$= \frac{3}{2\sqrt{2}\pi}\frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(15.0\,\mathrm{A}\right)^2}{\left(8.50\times10^{-2}\,\mathrm{m}\right)} = 1.12\times10^{-3}\,\mathrm{N/m}.$$

The direction of $\vec{F}_1$ is along $\hat{r} = (\hat{\mathrm{i}} - \hat{\mathrm{j}})/\sqrt{2}$. In unit-vector notation, we have

$$\vec{F}_1 = \frac{(1.12\times10^{-3}\,\mathrm{N/m})}{\sqrt{2}}(\hat{\mathrm{i}} - \hat{\mathrm{j}}) = (7.94\times10^{-4}\,\mathrm{N/m})\hat{\mathrm{i}} + (-7.94\times10^{-4}\,\mathrm{N/m})\hat{\mathrm{j}}$$

41. The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with i_1 = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\ \mathrm{sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\,dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$
$$= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(30.0\,\mathrm{A}\right)\left(20.0\,\mathrm{A}\right)\left(8.00\,\mathrm{cm}\right)\left(300\times10^{-2}\,\mathrm{m}\right)}{2\pi\left(1.00\,\mathrm{cm} + 8.00\,\mathrm{cm}\right)} = 3.20\times10^{-3}\,\mathrm{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\mathrm{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\mathrm{N})\hat{\mathrm{j}}$ in unit-vector notation.

42. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(15\,\mathrm{A/m^2}\right)\left(6\right)\left(0.20\,\mathrm{m}\right)^2 = 4.5\times10^{-6}\,\mathrm{T\cdot m}.$$

43. We use Eq. 29-20 $B = \mu_0 ir/2\pi a^2$ for the B-field inside the wire ($r < a$) and Eq. 29-17 $B = \mu_0 i/2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

44. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A} + 3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-2.0\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A} - 5.0\text{A} - 3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-13.0\text{A}\right) = -1.6\times10^{-5}\,\text{T}\cdot\text{m}.$$

45. **THINK** The value of the line integral $\oint \vec{B}\cdot d\vec{s}$ is proportional to the net current enclosed.

EXPRESS By Ampere's law, we have $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path.

ANALYZE (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "Amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B}\cdot d\vec{s} = -\mu_0 i = -(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(2.0\,\text{A}\right) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

LEARN The value of $\oint \vec{B}\cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the Amperian loop.

46. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(7i-6i+3i+i\right)=5\mu_0 i = 5\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(4.50\times10^{-3}\,\text{A}\right)=2.83\times10^{-8}\,\text{T}\cdot\text{m}.$$

47. For $r \le a$,

$$B(r)=\frac{\mu_0 i_{\text{enc}}}{2\pi r}=\frac{\mu_0}{2\pi r}\int_0^r J(r)\,2\pi r dr=\frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r dr=\frac{\mu_0 J_0 r^2}{3a}.$$

(a) At $r=0,\ B=0$.

(b) At $r=a/2$, we have

$$B(r)=\frac{\mu_0 J_0 r^2}{3a}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m}/2)^2}{3(3.1\times10^{-3}\,\text{m})}=1.0\times10^{-7}\,\text{T}.$$

(c) At $r=a$,

$$B(r=a)=\frac{\mu_0 J_0 a}{3}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m})}{3}=4.0\times10^{-7}\,\text{T}.$$

48. (a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C=\frac{\mu_0 i_{\text{wire}}}{2\pi(3R)}=\frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R}-\frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\text{A})/8 = 3.00\times10^{-3}\,\text{A}$.

(b) The direction is into the page.

49. (a) We use Eq. 29-24. The inner radius is r = 15.0 cm, so the field there is

$$B=\frac{\mu_0 iN}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(0.800\,\text{A}\right)\left(500\right)}{2\pi\left(0.150\,\text{m}\right)}=5.33\times10^{-4}\,\text{T}.$$

(b) The outer radius is r = 20.0 cm. The field there is

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.200\text{m})} = 4.00\times10^{-4}\ \text{T}.$$

50. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 3.60 A, $\ell = 0.950$ m, and N = 1200. This yields B = 0.00571 T.

51. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B = \mu_0 in = \mu_0 i\left(\frac{N}{\ell}\right)$$

where i = 0.30 A, $\ell = 0.25$ m, and N = 200. This yields $B = 3.0\times10^{-4}$ T.

52. We find N, the number of turns of the solenoid, from the magnetic field $B = \mu_0 in = \mu_o iN/\ell$: $N = B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN = \frac{2\pi rB\ell}{\mu_0 i} = \frac{2\pi(2.60\times10^{-2}\ \text{m})(23.0\times10^{-3}\ \text{T})(1.30\,\text{m})}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(18.0\,\text{A})} = 108\ \text{m}.$$

53. The orbital radius for the electron is

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$i = \frac{mv}{e\mu_0 nr} = \frac{(9.11\times10^{-31}\,\text{kg})(0.0460)(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\,\text{C})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(100/0.0100\text{m})(2.30\times10^{-2}\,\text{m})}$$
$$= 0.272\,\text{A}.$$

54. As the problem states near the end, some idealizations are being made here to keep the calculation straightforward (but are slightly unrealistic). For circular motion (with speed, $v_\perp$, which represents the magnitude of the component of the velocity perpendicular to the magnetic field [the field is shown in Fig. 29-20]), the period is (see Eq. 28-17)

$$T = 2\pi r/v_\perp = 2\pi m/eB.$$

Now, the time to travel the length of the solenoid is $t = L/v_\parallel$ where $v_\parallel$ is the component of the velocity in the direction of the field (along the coil axis) and is equal to $v\cos\theta$ where θ = 30º. Using Eq. 29-23 ($B = \mu_0 in$) with $n = N/L$, we find the number of revolutions made is $t/T = 1.6 \times 10^6$.

55. **THINK** The net field at a point inside the solenoid is the vector sum of the fields of the solenoid and that of the long straight wire along the central axis of the solenoid.

EXPRESS The magnetic field at a point P is given by $\vec{B} = \vec{B}_s + \vec{B}_w$, where $\vec{B}_s$ and $\vec{B}_w$ are the fields due to the solenoid and the wire, respectively. The direction of $\vec{B}_s$ is along the axis of the solenoid, and the direction of $\vec{B}_w$ is perpendicular to it, so the two fields are perpendicular to each other, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis, we must have $B_s = B_w$.

ANALYZE (a) Thus,

$$B_s = B_w \Rightarrow \quad \mu_0 i_s n = \frac{\mu_0 i_w}{2\pi d},$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi\left(20.0\times10^{-3}\,\text{A}\right)\left(10\,\text{turns/cm}\right)} = 4.77\,\text{cm}.$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\ \text{turns}/0.0100\ \text{m}\right) = 3.55\times10^{-5}\,\text{T}.$$

LEARN In general, the angle $\vec{B}$ makes with the solenoid axis is give by

$$\phi = \tan^{-1}\left(\frac{B_w}{B_s}\right) = \tan^{-1}\left(\frac{\mu_0 i_w/2\pi d}{\mu_0 i_s n}\right) = \tan^{-1}\left(\frac{i_w}{2\pi d\ n i_s}\right).$$

56. We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 i R^2 N}{2\left[R^2 + (R/2)^2\right]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R} = \frac{8\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(200)\left(0.0122\,\mathrm{A}\right)}{5\sqrt{5}\left(0.25\mathrm{m}\right)} = 8.78\times10^{-6}\ \mathrm{T}.$$

$\vec{B}_P$ is in the positive x direction.

57. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area.

EXPRESS The cross-sectional area is a circle, so $A = \pi R^2$, where R is the radius. The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27: $B = \dfrac{\mu_0}{2\pi}\dfrac{\mu}{z^3}$.

ANALYZE (a) Substituting the values given, we find the magnitude of the dipole moment to be

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\,\mathrm{A\cdot m^2}.$$

(b) Solving for z, we obtain

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(2.36\,\mathrm{A\cdot m^2}\right)}{2\pi\left(5.0\times10^{-6}\ \mathrm{T}\right)}\right)^{1/3} = 46\,\mathrm{cm}.$$

LEARN Note the similarity between $B = \dfrac{\mu_0}{2\pi}\dfrac{\mu}{z^3}$, the magnetic field of a magnetic dipole μ and $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{p}{z^3}$, the electric field of an electric dipole p (see Eq. 22-9).

58. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0\,.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50.$$

59. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area.

EXPRESS The cross-sectional area is a circle, so $A = \pi R^2$, where R is the radius.

ANALYZE With N = 200, i = 0.30 A, and R = 0.050 m, the magnitude of the dipole moment is

$$\mu = (200)(0.30\,\text{A})\pi(0.050\,\text{m})^2 = 0.47\,\text{A}\cdot\text{m}^2.$$

LEARN The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19.

60. Using Eq. 29-26, we find that the net y-component field is

$$B_y = \frac{\mu_0 i_1 R^2}{2(R^2+z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2(R^2+z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ (see Fig. 29-74(a)) and $z_2^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-74(b) corresponding to B_y = 0 would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1:

$$\begin{aligned} i_1 &= \frac{2(R^2+z_1^2)^{3/2}B_y}{\mu_0 R^2} = \frac{2R[1+(L/R)^2]^{3/2}B_y}{\mu_0} \\ &= \frac{2(0.040\text{ m})[1+(0.030\text{ m}/0.040\text{ m})^2]^{3/2}(7.2\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 0.895\text{ A} \approx 0.90\text{ A}. \end{aligned}$$

(b) With loop 2 at y = 0.06 m (see Fig. 29-74(b)) we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2+L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2+y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A $\approx$ 2.7 A.

61. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(15\,\text{A})}{2(0.12\,\text{m})} = 7.9\times10^{-5}\,\text{T}.$$

(b) The torque has magnitude equal to

$$\tau = |\vec{\mu}_2 \times \vec{B}_1| = \mu_2 B_1 \sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 = \pi\left(50\right)\left(1.3\text{A}\right)\left(0.82\times10^{-2}\,\text{m}\right)^2\left(7.9\times10^{-5}\text{T}\right)$$
$$= 1.1\times10^{-6}\,\text{N}\cdot\text{m}.$$

62. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(0.0562\,\text{A}\right)}{4}\left(\frac{1}{0.0572\text{m}}+\frac{1}{0.0936\text{m}}\right)$$
$$= 4.97\times10^{-7}\,\text{T}.$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-7(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2+b^2) = \frac{\pi(0.0562\text{A})}{2}[(0.0572\text{m})^2 + (0.0936\text{m})^2] = 1.06\times10^{-3}\,\text{A}\cdot\text{m}^2.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

63. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\vec{\mu} = \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = \left(ia^2\right)\left(\hat{\text{j}} - \hat{\text{i}} + \hat{\text{i}}\right) = ia^2\hat{\text{j}}$$
$$= \left(6.0\,\text{A}\right)\left(0.10\,\text{m}\right)^2\hat{\text{j}} = (6.0\times10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\text{j}}\,.$$

(b) Since both points are far from the cube we can use the dipole approximation. For $(x, y, z) = (0, 5.0\text{ m}, 0)$,

$$\vec{B}(0,\,5.0\text{ m},\,0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\text{ T}\cdot\text{m/A})(6.0\times10^{-2}\text{m}^2\cdot\text{A})\,\hat{\text{j}}}{2\pi(5.0\text{ m})^3} = (9.6\times10^{-11}\text{ T})\,\hat{\text{j}}\,.$$

64. (a) The radial segments do not contribute to $\vec{B}_P$, and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}} - \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}$$

where i = 0.200 A. This yields $\vec{B} = -2.75 \times 10^{-8}\,\hat{\text{k}}$ T, or $|\vec{B}| = 2.75 \times 10^{-8}$ T.

(b) The direction is $-\hat{\text{k}}$, or into the page.

65. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r\,,$$

we find that r = 0.00128 m gives the desired field value.

66. (a) We designate the wire along $y = r_A$ = 0.100 m wire A and the wire along $y = r_B$ = 0.050 m wire B. Using Eq. 29-4, we have

$$\vec{B}_{\text{net}} = \vec{B}_A + \vec{B}_B = -\frac{\mu_0 i_A}{2\pi r_A}\hat{\text{k}} - \frac{\mu_0 i_B}{2\pi r_B}\hat{\text{k}} = (-52.0\times 10^{-6}\text{ T})\hat{\text{k}}.$$

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0 i_A}{2\pi(r_A - y)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 13/160 \approx 0.0813$ m.

(c) We eliminate the $y < r_B$ possibility due to wire B carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0 i_A}{2\pi(y - r_A)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 7/40 \approx 0.0175$ m.

67. Let the length of each side of the square be a. The center of a square is a distance $a/2$ from the nearest side. There are four sides contributing to the field at the center. The result is

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2 + 4(a/2)^2}}\right) = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

On the other hand, the magnetic field at the center of a circular wire of radius R is $\mu_0 i/2R$ (e.g., Eq. 29-10). Thus, the problem is equivalent to showing that

$$\frac{2\sqrt{2}\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad \Rightarrow \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R} \ .$$

To do this we must relate the parameters a and R. If both wires have the same length L then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \quad \Rightarrow \quad a = \frac{\pi R}{2}.$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R},$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

68. We take the current (i = 50 A) to flow in the $+x$ direction, and the electron to be at a point P, which is r = 0.050 m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 29-4 with Eq. 28-2, we obtain

$$\vec{F}_e = \left(-e\mu_0 i/2\pi r\right)\left(\vec{v}\times\hat{\mathrm{k}}\right).$$

(a) The electron is moving down: $\vec{v} = -v\hat{\mathrm{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\left(-\hat{\mathrm{i}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{i}}\ ,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N .

(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{\mathrm{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\left(-\hat{\mathrm{j}}\right) = (3.2\times10^{-16}\ \mathrm{N})\,\hat{\mathrm{j}},$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N .

(c) Now, $\vec{v} = \pm v\hat{\mathrm{k}}$ so $\vec{F}_e \propto \hat{\mathrm{k}}\times\hat{\mathrm{k}} = 0.$

69. (a) By the right-hand rule, the magnetic field $\vec{B}_1$ (evaluated at a) produced by wire 1 (the wire at bottom left) is at ϕ = 150° (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at ϕ = 210°. By symmetry $\left(\vec{B}_1 = \vec{B}_2\right)$ we observe that only the x-components survive, yielding

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \left(2\frac{\mu_0 i}{2\pi\ell}\cos 150°\right)\hat{\mathrm{i}} = (-3.46\times10^{-5}\,\mathrm{T})\hat{\mathrm{i}}$$

where i = 10 A, ℓ = 0.10 m, and Eq. 29-4 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\mathrm{k}}$ direction) of value

$$i_b = B\frac{2\pi r}{\mu_0} = \left(3.46\times10^{-5}\ \mathrm{T}\right)\frac{2\pi(0.087\ \mathrm{m})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 15\,\mathrm{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and Eq. 29-4 has again been used.

(b) As stated above, to cancel this, wire b must carry current into the page (that is, the $-z$ direction).

70. The radial segments do not contribute to $\vec{B}$ (at the center), and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0 i(\pi\ \mathrm{rad})}{4\pi(4.00\ \mathrm{m})}\hat{\mathrm{k}} + \frac{\mu_0 i(\pi/2\ \mathrm{rad})}{4\pi(2.00\ \mathrm{m})}\hat{\mathrm{k}} - \frac{\mu_0 i(\pi/2\ \mathrm{rad})}{4\pi(4.00\ \mathrm{m})}\hat{\mathrm{k}}$$

where i = 2.00 A. This yields $\vec{B}$ = (1.57 × 10^{-7} T) $\hat{\mathrm{k}}$, or $|\vec{B}| = 1.57\times10^{-7}\,\mathrm{T}$.

71. Since the radius is R = 0.0013 m, then the i = 50 A produces

$$B = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(50\ \mathrm{A})}{2\pi(0.0013\ \mathrm{m})} = 7.7\times10^{-3}\,\mathrm{T}$$

at the edge of the wire. The three equations, Eq. 29-4, Eq. 29-17, and Eq. 29-20, agree at this point.

72. (a) With cylindrical symmetry, we have, external to the conductors,

$$\left|\vec{B}\right| = \frac{\mu_0\, i_{\mathrm{enc}}}{2\pi r}$$

which produces i_{enc} = 25 mA from the given information. Therefore, the thin wire must carry 5.0 mA.

(b) The direction is downward, opposite to the 30 mA carried by the thin conducting surface.

73. (a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and

has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi\left(a^2 - b^2\right)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(5.25\text{A}\right)(0.0200\text{m})}{2\pi[(0.0400\text{m})^2 - (0.0150\text{m})^2]} = 1.53\times10^{-5}\,\text{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes $B = \dfrac{\mu_0 id}{2\pi a^2}$. This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

Note: One may apply Ampere's law to show that the magnetic field in the hole is uniform. Consider a rectangular path with two long sides (side 1 and 2, each with length L) and two short sides (each of length less than b). If side 1 is directly along the axis of the hole, then side 2 would also be parallel to it and in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make L *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\oint_{\text{rectangle}} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$\int_{side1} \vec{B}\cdot d\vec{s} + \int_{\text{side}\,2} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{in hole}}$$

$$\left(B_{\text{side}1} - B_{\text{side}2}\right)L = 0$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

74. Equation 29-4 gives

$$i = \frac{2\pi RB}{\mu_0} = \frac{2\pi(0.880\,\text{m})(7.30\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 32.1\,\text{A}.$$

75. **THINK** In this problem, we apply the Biot-Savart law to calculate the magnetic field due to a current-carrying segment at various locations.

EXPRESS The Biot-Savart law can be written as

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\hat{\text{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\vec{r}}{r^3}.$$

With $\Delta\vec{s} = \Delta s\hat{\text{j}}$ and $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, their cross product is

$$\Delta\vec{s}\times\vec{r} = (\Delta s\hat{\text{j}})\times(x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}) = \Delta s(z\hat{\text{i}} - x\hat{\text{k}})$$

where we have used $\hat{\text{j}}\times\hat{\text{i}}=-\hat{\text{k}}$, $\hat{\text{j}}\times\hat{\text{j}}=0$, and $\hat{\text{j}}\times\hat{\text{k}}=\hat{\text{i}}$. Thus, the Biot-Savart equation becomes

$$\vec{B}(x,y,z)=\frac{\mu_0 i\,\Delta s\left(z\hat{\text{i}}-x\hat{\text{k}}\right)}{4\pi\left(x^2+y^2+z^2\right)^{3/2}}.$$

ANALYZE (a) The field on the z axis (at x = 0, y = 0, and z = 5.0 m) is

$$\vec{B}(0,\,0,\,5.0\text{ m})=\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(2.0\,\text{A})\left(3.0\times10^{-2}\,\text{m}\right)(5.0\text{ m})\hat{\text{i}}}{4\pi\left(0^2+0^2+(5.0\text{ m})^2\right)^{3/2}}=(2.4\times10^{-10}\,\text{T})\hat{\text{i}}.$$

(b) Similarly, $\vec{B}$ (0, 6.0 m, 0) = 0, since x = z = 0.

(c) The field in the xy plane, at (x, y, z) = (7 m, 7 m, 0), is

$$\vec{B}(7.0\text{ m},7.0\text{ m},0)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(-7.0\,\text{m})\hat{\text{k}}}{4\pi\left((7.0\text{ m})^2+(7.0\text{ m})^2+0^2\right)^{3/2}}=(-4.3\times10^{-11}\,\text{T})\hat{\text{k}}.$$

(d) The field in the xy plane, at (x, y, z) = (–3, –4, 0), is

$$\vec{B}(-3.0\,\text{m},\,-4.0\text{m},\,0)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(3.0\,\text{m})\hat{\text{k}}}{4\pi\left((-3.0\,\text{m})^2+(-4.0\,\text{m})^2+0^2\right)^{3/2}}=(1.4\times10^{-10}\,\text{T})\hat{\text{k}}.$$

LEARN Along the x and z axes, the expressions for $\vec{B}$ simplify to

$$\vec{B}(x,0,0)=-\frac{\mu_0}{4\pi}\frac{i\,\Delta s}{x^2}\hat{\text{k}},\quad \vec{B}(0,0,z)=\frac{\mu_0}{4\pi}\frac{i\,\Delta s}{z^2}\hat{\text{i}}.$$

The magnetic field at any point on the y axis vanishes because the current flows in the +y direction, so $d\vec{s}\times\hat{\text{r}}=0$.

76. We note that the distance from each wire to P is $r=d/\sqrt{2}=0.071\text{m}$. In both parts, the current is i = 100 A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at P) reveals that the vertical components cancel and the horizontal components add, yielding the result:

$$B=2\left(\frac{\mu_0 i}{2\pi r}\right)\cos 45.0°=4.00\times10^{-4}\,\text{T}$$

and directed in the –*x* direction. In unit-vector notation, we have $\vec{B} = (-4.00\times10^{-4}\ \text{T})\hat{\text{i}}$.

(b) Now, with the currents anti-parallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45.0° = 4.00\times10^{-4}\ \text{T}$$

and directed in the +*y* direction. In unit-vector notation, we have $\vec{B} = (4.00\times10^{-4}\ \text{T})\hat{\text{j}}$.

77. We refer to the center of the circle (where we are evaluating $\vec{B}$) as *C*. Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments that are collinear with *C* do not contribute to the field there. Eq. 29-9 (with $\phi = \pi/2$ rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at *C*. Thus, the nonzero contributions come from those straight segments that are not collinear with *C*. There are two of these "semi-infinite" segments, one a vertical distance *R* above *C* and the other a horizontal distance *R* to the left of *C*. Both contribute fields pointing out of the page (see Fig. 29-7(c)). Since the magnitudes of the two contributions (governed by Eq. 29-7) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 29-4). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 29-7(c)).

78. The points must be along a line parallel to the wire and a distance *r* from it, where *r* satisfies $B_{\text{wire}} = \dfrac{\mu_0 i}{2\pi r} = B_{\text{ext}}$, or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26\times10^{-6}\ \text{T}\cdot\text{m/A})(100\,\text{A})}{2\pi(5.0\times10^{-3}\ \text{T})} = 4.0\times10^{-3}\,\text{m}.$$

79. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 29-17,

$$\left|\vec{B}\right| = \frac{\mu_0\, i_w}{2\pi r} = 4.8\times10^{-3}\ \text{T}$$

where i_w = 24 A and r = 0.0010 m.

(b) Now the field consists of two contributions (which are anti-parallel) — from the wire (Eq. 29-17) and from a portion of the conductor (Eq. 29-20 modified for annular area):

$$|\vec{B}| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_0^2 - \pi R_i^2}\right)$$

where r = 0.0030 m, R_i = 0.0020 m, R_o = 0.0040 m, and i_c = 24 A. Thus, we find $|\vec{B}| = 9.3\times10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

80. Using Eq. 29-20 and Eq. 29-17, we have

$$|\vec{B}_1| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_1 \qquad |\vec{B}_2| = \frac{\mu_0 i}{2\pi r_2}$$

where $r_1 = 0.0040\,\text{m}$, $\left|\vec{B}_1\right| = 2.8\times10^{-4}$ T, $r_2 = 0.010\,\text{m}$, and $|\vec{B}_2| = 2.0\times10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find i = 10 A. Plugging this into the first equation yields $R = 5.3\times10^{-3}$ m.

81. **THINK** The objective of this problem is to calculate the magnetic field due to an infinite current sheet by applying Ampere's law.

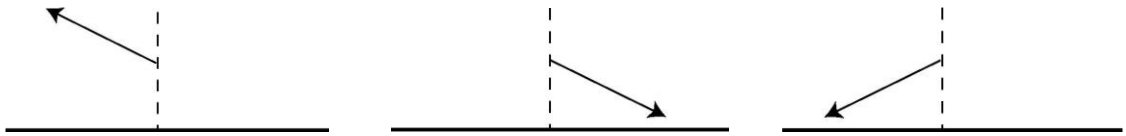
EXPRESS The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \int \vec{J}\cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it is clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to

$$\mu_0 JA = \mu_0 J\,\Delta y \Delta x = \mu_0 \lambda\,\Delta x.$$

ANALYZE (a) Figure 29-84 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P', so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure?



Our focus is on point P. Suppose the magnetic field is not parallel to the sheet, as shown in the upper left diagram. If we reverse the direction of the current, then the direction of the field will also be reversed (as shown in the upper middle diagram). Now, if we rotate the sheet by 180° about a line that is perpendicular to the sheet, the field will rotate and point in the direction shown in the diagram on the upper right. The current distribution now is exactly the same as the original; however, comparing the upper left and upper right diagrams, we see that the fields are not the same, unless the original field is parallel to the sheet and only has a horizontal component. That is, the field at P must be purely horizontal, as drawn in Fig. 29-84.

(b) The path used in evaluating $\oint \vec{B}\cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P', respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown next. Ampere's law yields

$$2B\Delta x = \mu_0 \lambda \Delta x \quad \Rightarrow \quad B = \frac{1}{2}\mu_0 \lambda.$$

LEARN In order to apply Ampere's law, the system must possess certain symmetry. In the case of an infinite current sheet, the symmetry is planar.

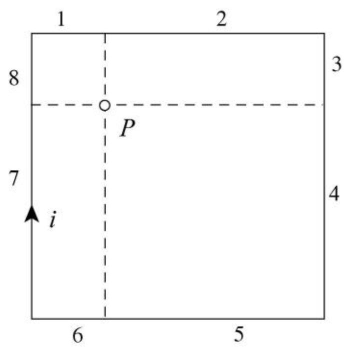
82. Equation 29-17 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

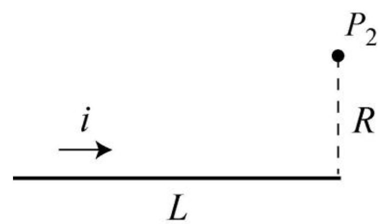
$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)} = 1.25\times 10^{-6}\ \text{T},$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement. Using the right-hand rule, we find both horizontal components point in the $+x$ direction. Thus, in unit-vector notation, we have $\vec{B} = (1.25\times 10^{-6}\ \text{T})\hat{\text{i}}$.

83. **THINK** The magnetic field at P is the vector sum of the fields of the individual wire segments.

EXPRESS The two small wire segments, each of length $a/4$, shown in Fig. 29-86 nearest to point P, are labeled 1 and 8 in the figure (below left). Let $-\hat{\mathrm{k}}$ be a unit vector pointing into the page.





We use the result of Problem 29-17: namely, the magnetic field at P_2 (shown in Fig. 29-44 and upper right) is

$$B_{P_2}=\frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}\,.$$

Therefore, the magnetic fields due to the 8 segments are

$$B_{P1}=B_{P8}=\frac{\sqrt{2}\mu_0 i}{8\pi(a/4)}=\frac{\sqrt{2}\mu_0 i}{2\pi a},$$

$$B_{P4}=B_{P5}=\frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)}=\frac{\sqrt{2}\mu_0 i}{6\pi a},$$

$$B_{P2}=B_{P7}=\frac{\mu_0 i}{4\pi(a/4)}\cdot\frac{3a/4}{\left[(3a/4)^2+(a/4)^2\right]^{1/2}}=\frac{3\mu_0 i}{\sqrt{10}\pi a},$$

and

$$B_{P3}=B_{P6}=\frac{\mu_0 i}{4\pi(3a/4)}\cdot\frac{a/4}{\left[(a/4)^2+(3a/4)^2\right]^{1/2}}=\frac{\mu_0 i}{3\sqrt{10}\pi a}.$$

ANALYZE Adding up all the contributions, the total magnetic field at P is

$$\begin{aligned}\vec{B}_P&=\sum_{n=1}^{8}B_{Pn}(-\hat{\mathrm{k}})=2\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{\mathrm{k}})\\&=\frac{2\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)(10\mathrm{A})}{\pi\left(8.0\times10^{-2}\,\mathrm{m}\right)}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{\mathrm{k}})\\&=\left(2.0\times10^{-4}\,\mathrm{T}\right)(-\hat{\mathrm{k}}).\end{aligned}$$

LEARN If point P is located at the center of the square, then each segment would contribute

$$B_{P1} = B_{P2} = \cdots B_{P8} = \frac{\sqrt{2}\mu_0 i}{4\pi a},$$

making the total field

$$B_{\text{center}} = 8B_{P1} = \frac{8\sqrt{2}\mu_0 i}{4\pi a}.$$

84. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$\left|\vec{F}\right| = \frac{\mu_0\, L(5.0\,\text{A})(3.2\,\text{A})}{2\pi(0.10\,\text{m})} + \frac{\mu_0\, L(5.0\,\text{A})(5.0\,\text{A})}{2\pi(0.20\,\text{m})}$$

where L = 3.0 m. Thus, $\left|\vec{F}\right| = 1.7\times 10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\,\vec{F}\,| = \frac{\mu_0 L(5.0\text{A})(3.2\text{A})}{2\pi(0.10\text{m})} - \frac{\mu_0 L(5.0\text{A})(5.0\text{A})}{2\pi(0.20\text{m})} = 2.1\times 10^{-5}\,\text{N}\,.$$

85. **THINK** The hollow conductor has cylindrical symmetry, so Ampere's law can be applied to calculate the magnetic field due to the current distribution.

EXPRESS Ampere's law states that $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path, or Amperian loop. We choose the Amperian loop to be a circle of radius r and concentric with the cylindrical shell. Since the current is uniformly distributed throughout the cross section of the shell, the enclosed current is

$$i_{\text{enc}} = i\,\frac{\pi(r^2-b^2)}{\pi(a^2-b^2)} = i\left(\frac{r^2-b^2}{a^2-b^2}\right).$$

ANALYZE (a) Thus, in the region $b < r < a$, we have

$$\oint \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 i\left(\frac{r^2-b^2}{a^2-b^2}\right)$$

which gives $B = \dfrac{\mu_0 i}{2\pi(a^2-b^2)}\left(\dfrac{r^2-b^2}{r}\right).$

(b) At $r = a$, the magnetic field strength is

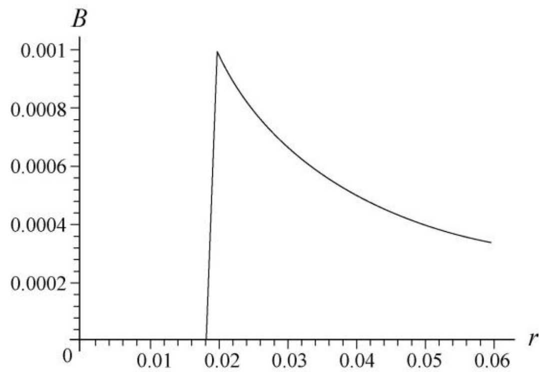
$$\frac{\mu_0 i}{2\pi\left(a^2-b^2\right)}\left(\frac{a^2-b^2}{a}\right)=\frac{\mu_0 i}{2\pi a}.$$

At $r=b, B\propto r^2-b^2=0$. Finally, for $b = 0$

$$B=\frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r}=\frac{\mu_0 ir}{2\pi a^2}$$

which agrees with Eq. 29-20.

(c) The field is zero for $r < b$ and is equal to Eq. 29-17 for $r > a$, so this along with the result of part (a) provides a determination of B over the full range of values. The graph (with SI units understood) is shown below.



LEARN For $r < b$, the field is zero, and for $r > a$, the field decreases as $1/r$. In the region $b < r < a$, the field increases with r as $r-b^2/r$.

86. We refer to the side of length L as the long side and that of length W as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of Problem 29-17 leads to

$$B=2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2+4(W/2)^2}}+2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2+4(L/2)^2}}.$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2+W^2}{\sqrt{W^2+L^2}}=\sqrt{W^2+L^2}.$$

87. (a) Equation 29-20 applies for $r < c$. Our sign choice is such that i is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2}, \; r \le c.$$

(b) Equation 29-17 applies in the region between the conductors:

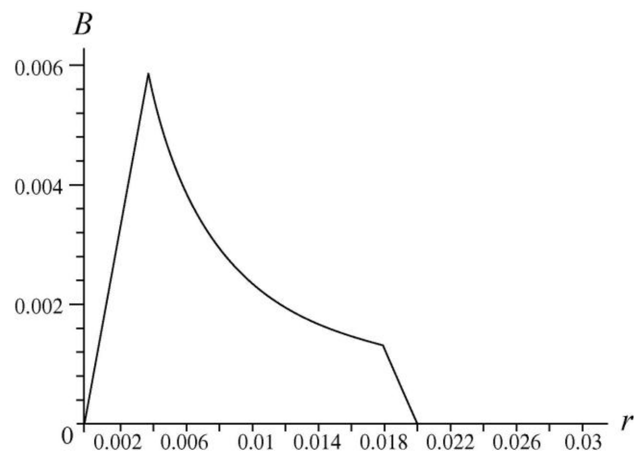
$$B = \frac{\mu_0 i}{2\pi r}, \; c \le r \le b.$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 29-17) plus the field due to the (negative) current in that part of the outer conductor at radius less than r. The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right), \;\; b < r \le a.$$

If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right).$$



(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \ge a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown to the right.

88. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 29-12 to find the magnetic force on the segment, and Eq. 29-7 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\mathrm{i}}$ direction, and the current in rail 2 is in the $-\hat{\mathrm{i}}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{\mathrm{k}}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\mathrm{j}}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = i dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_1 + dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_2 = i\left[B_1 + B_2\right]\hat{\mathrm{i}}\ dy$$
$$= i\left[\frac{\mu_0 i}{4\pi\left(2R+w-y\right)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\mathrm{i}}\ dy.$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R+w-y} + \frac{1}{y}\right)dy\hat{\mathrm{i}} = \frac{\mu_0 i^2}{2\pi}\ \ln\left(1+\frac{w}{R}\right)\hat{\mathrm{i}}.$$

(b) Using the work-energy theorem, we have

$$\Delta K = \tfrac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL.$$

Thus, the final speed of the projectile is

$$v_f = \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)L\right]^{1/2}$$
$$= \left[\frac{2\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(450\times10^3\ \mathrm{A}\right)^2\ \ln\left(1+1.2\ \mathrm{cm}/6.7\ \mathrm{cm}\right)\left(4.0\ \mathrm{m}\right)}{2\pi\left(10\times10^{-3}\ \mathrm{kg}\right)}\right]^{1/2}$$
$$= 2.3\times10^3\ \mathrm{m/s}.$$