Chapter 16

1. Let y_1 = 2.0 mm (corresponding to time t_1) and y_2 = –2.0 mm (corresponding to time t_2). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0)\ .$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2$ = 0.011 s (or 1.1 ms).

2. (a) The speed of the wave is the distance divided by the required time. Thus,

$$v = \frac{853\ \text{seats}}{39\ \text{s}} = 21.87\ \text{seats/s} \approx 22\ \text{seats/s}\ .$$

(b) The width w is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit. Thus,

$$w = vt = (21.87\ \text{seats/s})(1.8\ \text{s}) \approx 39\ \text{seats}\ .$$

3. (a) The angular wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \dfrac{\lambda\omega}{2\pi} = \dfrac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

4. The distance d between the beetle and the scorpion is related to the transverse speed v_t and longitudinal speed v_ℓ as

$$d = v_t t_t = v_\ell t_\ell$$

where t_t and t_ℓ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150 \text{ m/s}}{50 \text{ m/s}} = 3.0 .$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0 t_\ell - t_\ell = 2.0 t_\ell = 4.0\times10^{-3} \text{ s} \quad \Rightarrow \quad t_\ell = 2.0\times10^{-3} \text{ s} ,$$

then $d = v_\ell t_\ell = (150 \text{ m/s})(2.0\times10^{-3} \text{ s}) = 0.30 \text{ m} = 30 \text{ cm}.$

5. (a) The motion from maximum displacement to zero is one-fourth of a cycle. One-fourth of a period is 0.170 s, so the period is T = 4(0.170 s) = 0.680 s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680 \text{ s}} = 1.47 \text{ Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40 \text{ m}}{0.680 \text{ s}} = 2.06 \text{ m/s}.$$

6. The slope that they are plotting is the physical slope of the sinusoidal waveshape (not to be confused with the more abstract "slope" of its time development; the physical slope is an x-derivative, whereas the more abstract "slope" would be the t-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d}{dx}\left[y_m \sin(kx - \omega t) \right] = y_m k \cos(kx - \omega t) .$$

The problem additionally gives t = 0, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40 \text{ m}} = 15.7 \text{ rad/m} .$$

Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude y_m. We find

$$y_m = \frac{0.20}{15.7 \text{ rad/m}} = 0.0127 \text{ m} \approx 1.3 \text{ cm} .$$

7. (a) From the simple harmonic motion relation $u_m = y_m \omega$, we have

$$\omega = \frac{16 \text{ m/s}}{0.040 \text{ m}} = 400 \text{ rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = (80 \text{ m/s})/(64 \text{ Hz}) = 1.26 \text{ m} \approx 1.3 \text{ m}$.

(c) The amplitude of the transverse displacement is $y_m = 4.0 \text{ cm} = 4.0 \times 10^{-2} \text{ m}$.

(d) The wave number is $k = 2\pi/\lambda = 5.0$ rad/m.

(e) As shown in (a), the angular frequency is $\omega = (16 \text{ m/s})/(0.040 \text{ m}) = 4.0 \times 10^2$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040 \sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.040$ at $x = t = 0$. Therefore, $\sin \phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040 \sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

8. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives

$$u = -\omega y_m \cos(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$, or

$$\frac{du}{dt} = \frac{d}{dt}\left[-\omega y_m \cos(-\omega t + \phi)\right] = -y_m \omega^2 \sin(-\omega t + \phi) > 0$$

at $t = 0$. This implies that $-\sin\phi > 0$ and consequently that ϕ is in either the third or fourth quadrant. The graph shows (at $t = 0$) $u = -4$ m/s, and (at some later t) $u_{\max} = 5$ m/s. We note that $u_{\max} = y_m \omega$. Therefore,

$$u = -u_{\max} \cos(-\omega t + \phi)\Big|_{t=0} \;\Rightarrow\; \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

9. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, that is, $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = k\,v = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction, see Section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16\,x - 2.4\times10^2\, t) .$$

10. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

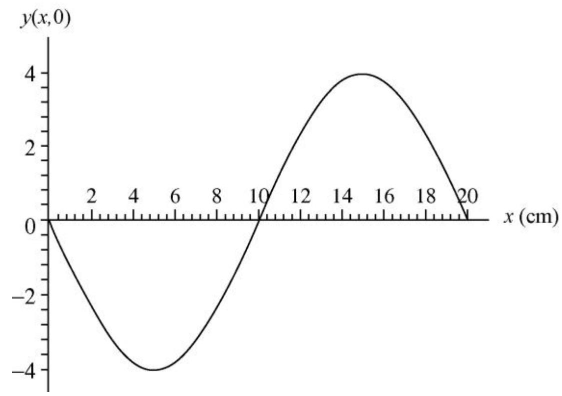
$$u_{\text{max}} = 2\pi\, f y_m = \left(4.0\,\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

11. From Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi) .$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, that is, $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At $t = 0$, we then have



$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a negative sine function. A plot of $y(x, 0)$ is depicted on the right.

(b) From the figure we see that the amplitude is y_m = 4.0 cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of y with respect to t, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0, 5.0) = -2.5$ cm/s.

12. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = (225\pi) \sin(\pi x - 15\pi t) .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2}.$$

Therefore, where $y = 12$, u must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

13. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700 \text{ m} = 700 \text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6$ = (700 mm)/6 = 117 mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

14. (a) Comparing with Eq. 16-2, we see that k = 20/m and ω = 600 rad/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017 \text{ kg/m} = 17 \text{ g/m}.$$

15. **THINK** Numerous physical properties of a traveling wave can be deduced from its wave function.

EXPRESS We first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where y_m is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and ϕ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string.

ANALYZE (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wavelength is $\lambda = v/f = \sqrt{\tau/\mu}\,/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction.

LEARN In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

16. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = (120\,\text{N})\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2 = 135\text{N}.$$

17. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\text{ m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\text{ rad/s})/(2.0\text{ m}^{-1}) = 15\text{ m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\text{ kg/m}\right)(15\,\text{m/s})^2 = 0.036\,\text{N}.$$

18. The volume of a cylinder of height ℓ is $V = \pi r^2\ell = \pi d^2\ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho \pi d^2 \,\ell/4}{\ell} = \frac{\pi \rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

19. **THINK** The speed of a transverse wave in a rope is related to the tension in the rope and the linear mass density of the rope.

EXPRESS The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the rope's linear mass density, which is defined as the mass per unit length of rope $\mu = m/L$.

ANALYZE With a linear mass density of

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m},$$

we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

LEARN Since $v \sim 1/\sqrt{\mu}$, the thicker the rope (larger μ), the slower the speed of the rope under the same tension τ.

20. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

21. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30$ ms. If this time is denoted by t_0, then the coordinate of this wave at time t is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v=\sqrt{\frac{\tau L}{m}}=\sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}}=158\,\text{m/s}.$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x=\frac{10.0\,\text{m}+(158\,\text{m/s})(30.0\times10^{-3}\text{s})}{2}=7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L - x$ = (10.0 m – 7.37 m) = 2.63 m.

22. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at x = 10 cm, becomes $y(x = 10 \text{ cm}, t) = y_m \sin[k(10 \text{ cm} - \omega t)]$. Comparing this with the expression given, we find ω = 4.0 rad/s, or $f = \omega/2\pi$ = 0.64 Hz.

(b) Since k(10 cm) = 1.0, the wave number is k = 0.10/cm. Consequently, the wavelength is $\lambda = 2\pi/k$ = 63 cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is k = 0.10/cm.

(e) The angular frequency is ω = 4.0 rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of k and ω into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t)=5.0\sin(0.10x-4.0t).$$

(g) Since $v=\omega/k=\sqrt{\tau/\mu}$, the tension is

$$\tau=\frac{\omega^2\mu}{k^2}=\frac{(4.0\,\text{g/cm})(4.0\,\text{s}^{-1})^2}{(0.10\,\text{cm}^{-1})^2}=6400\,\text{g}\cdot\text{cm/s}^2=0.064\,\text{N}.$$

23. **THINK** Various properties of the sinusoidal wave can be deduced from the plot of its displacement as a function of position.

EXPRESS In analyzing the properties of the wave, we first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where y_m is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and ϕ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string.

ANALYZE (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})\,(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9\times10^2$ rad/s .

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin \phi$. We wish to select ϕ so that

$$(5.0 \times 10^{-2} \text{ m})\sin \phi = (4.0 \times 10^{-2} \text{ m}).$$

The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction.

LEARN Summarizing the results obtained above, the wave function of the traveling wave can be written as

$$y(x,t) = \left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

24. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})\,(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})\,(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1\,\text{m/s}.$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1 = \frac{M}{1+\mu_2/\mu_1} = \frac{500\,\text{g}}{1+5.00/3.00} = 187.5\,\text{g} \approx 188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313\text{ g}$.

25. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by μgy, so the tension is $\tau = \mu gy$. The wave speed is $v = \sqrt{\mu gy/\mu} = \sqrt{gy}.$

(b) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $dt = dy/v = dy\big/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\,\Bigg|_0^L = 2\sqrt{\frac{L}{g}}\,.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f=\frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}}=\frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}}=198\text{ Hz.}$$

27. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}} = 500$ Hz, and that the wavelength $\lambda = 0.20$ m. We also note from the graph that the maximum value of *dK/dt* is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\tfrac{1}{2}\mu v\,\omega^2 y_m{}^2 = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m=\sqrt{\frac{10}{2\pi^2\mu\lambda f^3}}=0.0032\text{ m}.$$

28. Comparing

$$y(x,t)=(3.00\text{ mm})\sin[(4.00\text{ m}^{-1})x-(7.00\text{ s}^{-1})t]$$

to the general expression $y(x,t)=y_m\sin(kx-\omega t)$, we see that $k=4.00\text{ m}^{-1}$ and $\omega=7.00$ rad/s. The speed of the wave is

$$v=\omega/k=(7.00\text{ rad/s})/(4.00\text{ m}^{-1})=1.75\text{ m/s.}$$

29. The wave

$$y(x,t)=(2.00\text{ mm})[(20\text{ m}^{-1})x-(4.0\text{ s}^{-1})t]^{1/2}$$

is of the form $h(kx-\omega t)$ with angular wave number $k=20\text{ m}^{-1}$ and angular frequency $\omega=4.0$ rad/s. Thus, the speed of the wave is

$$v=\omega/k=(4.0\text{ rad/s})/(20\text{ m}^{-1})=0.20\text{ m/s.}$$

30. The wave $y(x,t)=(4.00\text{ mm})\,h[(30\text{ m}^{-1})x+(6.0\text{ s}^{-1})t]$ is of the form $h(kx-\omega t)$ with angular wave number $k=30\text{ m}^{-1}$ and angular frequency $\omega=6.0$ rad/s. Thus, the speed of the wave is

$$v=\omega/k=(6.0\text{ rad/s})/(30\text{ m}^{-1})=0.20\text{ m/s.}$$

31. **THINK** By superposition principle, the resultant wave is the algebraic sum of the two interfering waves.

EXPRESS The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \;= 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where we have used

$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta).$$

ANALYZE The two waves are out of phase by $\phi = \pi/2$, so the amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41y_m .$$

LEARN The interference between two waves can be constructive or destructive, depending on their phase difference.

32. (a) Let the phase difference be ϕ. Then from Eq. 16-52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to 1.45 rad/2π = 0.230 wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00\text{ mm}$, as stated in the problem.

(b) The figure indicates that λ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude y_m. The amplitude of their resultant wave, as shown in the figure, is y'_m = 4.00 mm. Equation 16-52 applies:

$$y'_m = 2y_m \cos(\tfrac{1}{2}\phi_2) \quad\Rightarrow\quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it was shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, ϕ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16\,x + 1100\,t) \quad \text{and} \quad y_2 = (0.00900)\sin(16\,x + 1100\,t + 2.7\,)\,.$$

34. (a) We use Eq. 16-26 and Eq. 16-33 with $\mu = 0.00200$ kg/m and $y_m = 0.00300$ m. These give $v = \sqrt{\tau/\mu} = 775\text{ m/s}$ and

$$P_{\text{avg}} = \tfrac{1}{2}\,\mu v\,\omega^2 y_m^{\,2} = 10\text{ W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); $P = 20$ W.

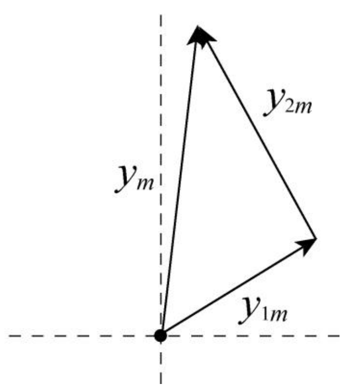
(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-13(a)) then the amplitude y_m is doubled, which means its square $y_m^{\,2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); $P = 40$ W.

(d) Equation 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi) = 1.618$ times the original amplitude y_m. Squared, this results in an increase in the power by a factor of 2.618. Thus, $P = 26$ W in this case.

(e) Now the situation depicted in Fig. 16-13(b) applies, so $P = 0$.

35. **THINK** We use phasors to add the two waves and calculate the amplitude of the resultant wave.

EXPRESS The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle.



ANALYZE The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus, the amplitude of the resultant wave is y_m = 5.0 cm.

LEARN When adding two waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. The same result, however, could also be obtained as follows: Writing the two waves as $y_1 = 3\sin(kx-\omega t)$ and $y_2 = 4\sin(kx-\omega t+\pi/2) = 4\cos(kx-\omega t)$, we have, after a little algebra,

$$\begin{aligned} y &= y_1 + y_2 = 3\sin(kx-\omega t) + 4\cos(kx-\omega t) = 5\left[\frac{3}{5}\sin(kx-\omega t) + \frac{4}{5}\cos(kx-\omega t)\right] \\ &= 5\sin(kx-\omega t+\phi) \end{aligned}$$

where $\phi = \tan^{-1}(4/3)$. In deducing the phase ϕ, we set $\cos\phi = 3/5$ and $\sin\phi = 4/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta+\phi)$.

36. We see that y_1 and y_3 cancel (they are 180°) out of phase, and y_2 cancels with y_4 because their phase difference is also equal to π rad (180°). There is no resultant wave in this case.

37. (a) Using the phasor technique, we think of these as two “vectors” (the first of “length” 4.6 mm and the second of “length” 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144°). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8° (or 1.55 rad).

(c) Clearly, it should in “in phase” with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8° (or 1.55 rad).

38. (a) As shown in Figure 16-13(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

(c) As shown in Figure 16-13(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

(e) Using phasor terminology, the angle “between them” in this case is π/2 rad (90°), so the Pythagorean theorem applies:

$$\sqrt{(8.0 \text{ mm})^2 + (5.0 \text{ mm})^2} = 9.4 \text{ mm} .$$

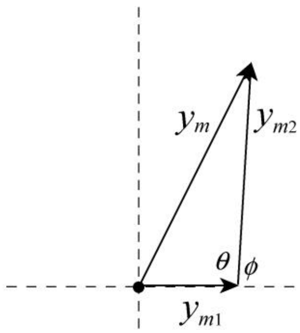
39. The phasor diagram is shown to the right. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$

We solve for cos ϕ:

$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.



40. The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude T = 2(0.50 s) = 1.0 s. Thus, $f = 1/T$ = 1.0 Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10 \text{ cm}.$$

41. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. The possible wavelengths of a standing wave are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer.

ANALYZE (a) The wave speed is

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0 \text{ N})\,(8.40 \text{ m})}{0.120 \text{ kg}}} = 82.0 \text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda_1/2$ (n = 1), so $\lambda_1 = 2L$ = 2(8.40 m) = 16.8 m.

(c) The corresponding frequency is $f_1 = v/\lambda_1$ = (82.0 m/s)/(16.8 m) = 4.88 Hz.

LEARN The resonant frequencies are given by

$$f_n = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_1,$$

where $f_1 = v/\lambda_1 = v/2L$. The oscillation mode with $n = 1$ is called the fundamental mode or the first harmonic.

42. Use Eq. 16-66 (for the resonant frequencies) and Eq. 16-26 $(v=\sqrt{\tau/\mu})$ to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2\,f_3.$$

(b) And we get the new wavelength $\lambda'_3 = \dfrac{v'}{f_3'} = \dfrac{2L}{3} = \lambda_3.$

43. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS Possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. The wave speed is given by $v=\sqrt{\tau/\mu}=\sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus,

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\text{ N}}{(10.0\text{ m})\,(0.100\text{ kg})}} = n\ (7.91\text{ Hz}).$$

ANALYZE (a) The lowest frequency is $f_1 = 7.91\text{ Hz}.$

(b) The second lowest frequency is $f_2 = 2(7.91\text{ Hz}) = 15.8\text{ Hz}.$

(c) The third lowest frequency is $f_3 = 3(7.91\text{ Hz}) = 23.7\text{ Hz}.$

LEARN The frequencies are integer multiples of the fundamental frequency f_1. This means that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency f_1.

44. (a) The wave speed is given by $v=\sqrt{\dfrac{\tau}{\mu}}=\sqrt{\dfrac{7.00\,\text{N}}{2.00\times10^{-3}\,\text{kg}/1.25\,\text{m}}}=66.1\,\text{m/s}.$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where L = 125 cm. Thus,

$$f_1=\frac{v}{\lambda_1}=\frac{66.1\text{ m/s}}{2(1.25\text{ m})}=26.4\text{ Hz}.$$

45. **THINK** The difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency.

EXPRESS The resonant wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are

$$f_n = v/\lambda = nv/2L = nf_1,$$

where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with n + 1. The frequency difference between successive modes is

$$\Delta f = f_{n+1}-f_n=\frac{v}{2L}=f_1.$$

ANALYZE (a) The lowest possible resonant frequency is

$$f_1=\Delta f = f_{n+1}-f_n=420\text{ Hz}-315\text{ Hz}=105\text{ Hz}.$$

(b) The longest possible wavelength is $\lambda_1 = 2L$. If f_1 is the lowest possible frequency then

$$v = \lambda_1 f_1 = (2L)f_1 = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

LEARN Since 315 Hz = 3(105 Hz) and 420 Hz = 4(105 Hz), the two frequencies correspond to n = 3 and n = 4, respectively.

46. The nth resonant frequency of string A is

$$f_{n,A}=\frac{v_A}{2l_A}n=\frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string B it is

$$f_{n,B}=\frac{v_B}{2l_B}n=\frac{n}{8L}\sqrt{\frac{\tau}{\mu}}=\frac{1}{4}f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of *B* matches the frequency of *A*'s first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of *B* would match $f_{3,A} = \dfrac{3v_A}{2l_A} = \dfrac{3}{2L}\sqrt{\dfrac{\tau}{\mu}}$.

47. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus,

$$f_1 = (390 \text{ Hz} - 325 \text{ Hz}) = 65 \text{ Hz}.$$

This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

48. Using Eq. 16-26, we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{65.2\times10^{6}\text{ N}}{3.35\text{ kg/ m}}} = 4412\text{ m/s}.$$

The corresponding resonant frequencies are

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The wavelength of the wave with the lowest (fundamental) resonant frequency f_1 is $\lambda_1 = 2L$, where L = 347 m. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{4412\text{ m/s}}{2(347\text{ m})} = 6.36\text{ Hz}.$$

(b) The frequency difference between successive modes is

$$\Delta f = f_n - f_{n-1} = \frac{v}{2L} = \frac{4412\text{ m/s}}{2(347\text{ m})} = 6.36\text{ Hz}.$$

49. (a) Equation 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\text{ N}}{7.20\times10^{-3}\text{ kg/m}}} = 144.34\text{ m/s} \approx 1.44\times10^{2}\text{ m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0 \text{ cm}) = 60.0 \text{ cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^2 \text{ m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

50. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form

$$y(x,t) = -(0.04)\cos(kx)\sin(\omega t),$$

where $\omega = 2\pi/T = \pi$ rad/s, with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040\,\text{m} .$$

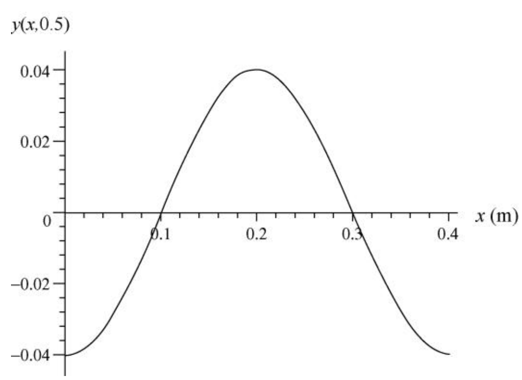
(b) The above equation yields $y(0.30 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$.

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega\,\cos(kx)\cos(\omega t) = 0\,.$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown next:



51. **THINK** In this problem, in order to produce the standing wave pattern, the two waves must have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions.

EXPRESS We take the two waves to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The superposition principle gives

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = \left[2y_m \sin kx\right]\cos \omega t\ .$$

ANALYZE (a) The amplitude y_m is half the maximum displacement of the standing wave, or (0.01 m)/2 = 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With L = 3.0m, λ = 2.0 m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0\text{ m}) = 3.1\text{ m}^{-1}.$$

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\text{ m/s})}{2(3.0\text{ m})} = 50\text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50\text{ Hz}) = 314\text{ rad/s}.$$

(d) If one of the waves has the form $y_2(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y_1(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

LEARN Using the results above, the two waves can be written as

$$y_1 = \left(5.0 \times 10^{-3}\text{ m}\right)\sin\left[\left(3.14\text{ m}^{-1}\right)x - \left(314\text{ s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0 \times 10^{-3}\text{ m}\right)\sin\left[\left(3.14\text{ m}^{-1}\right)x + \left(314\text{ s}^{-1}\right)t\right].$$

52. Since the rope is fixed at both ends, then the phrase “second-harmonic standing wave pattern” describes the oscillation shown in Figure 16-20(b), where (see Eq. 16-65)

$$\lambda = L, \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$, then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \quad \Rightarrow \quad \lambda = 4.0\,\text{m} \quad \Rightarrow \quad L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$, then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \quad \Rightarrow \quad v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16-26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\,\text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

the period is $T = 1/f = 0.11$ s.

53. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^{2}\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by

$$u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t).$$

For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\ (0.50\,\text{cm})\ \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\ \left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$

54. Reference to point A as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Using Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90$ cm = 9.0 mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so T = 12 ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t]$$

and

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] \, .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in y_1, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in y_2. Therefore, the sign in front of ω is minus.

55. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(k\,x \pm \omega\,t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point, which occurs during a half-period. Since the period T is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\pi} = 0.500 \text{ s}.$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \omega / k = 1.00$ m/s. Therefore, Δx = (1.00 m/s)(0.250 s) = 0.250 m.

56. The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of x that corresponds to a node is $x = 0$.

(b) The second smallest value of x that corresponds to a node is x = 0.20 m.

(c) The third smallest value of x that corresponds to a node is x = 0.40 m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16-58 through Eq. 16-60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s = 0.025 s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s = 0.050 s.

57. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is

$$f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00 \text{ Hz}.$$

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi \text{ m}^{-1}$, so the wavelength is

$$\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00 \text{ m}.$$

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using cos α = sin (α + π/2), then apply

$$\cos\alpha+\cos\beta=\sin\left(\alpha+\frac{\pi}{2}\right)+\sin\left(\beta+\frac{\pi}{2}\right)=2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right).$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m\cos(kx+\omega t)+y_m\cos(kx-\omega t)=2y_m\cos(kx)\cos(\omega t).$$

Nodes occur where cos(kx) = 0 or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi\ \text{m}^{-1}$, this means $x=\left(n+\frac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of x that corresponds to a node is $x = 0.500$ m ($n = 0$).

(e) The second smallest value of x that corresponds to a node is $x = 1.50$ m ($n = 1$).

(f) The third smallest value of x that corresponds to a node is $x = 2.50$ m ($n = 2$).

(g) The displacement is a maximum where cos(kx) = ±1. This means $kx = n\pi$, where n is an integer. Thus, $x = n(1.00\text{ m})$. The smallest value of x that corresponds to an anti-node (maximum) is $x = 0$ ($n = 0$).

(h) The second smallest value of x that corresponds to an anti-node (maximum) is $x=1.00$ m ($n = 1$).

(i) The third smallest value of x that corresponds to an anti-node (maximum) is $x=2.00$ m ($n = 2$).

58. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f=\frac{nv}{2L}=\frac{n}{2L}\sqrt{\frac{\tau}{\mu}}=\frac{n}{2L}\sqrt{\frac{mg}{\mu}},\qquad n=1,2,3,\ldots$$

(a) The mass that allows the oscillator to set up the 4th harmonic ($n=4$) on the string is

$$m=\left.\frac{4L^2f^2\mu}{n^2g}\right|_{n=4}=\frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{(4)^2(9.80\text{ m/s}^2)}=0.846\text{ kg}$$

(b) If the mass of the block is $m=1.00$ kg, the corresponding n is

$$n = \sqrt{\frac{4L^2 f^2 \mu}{g}} = \sqrt{\frac{4(1.20\text{ m})^2 (120\text{ Hz})^2 (0.00160\text{ kg/m})}{9.80\text{ m/s}^2}} = 3.68$$

which is not an integer. Therefore, the mass cannot set up a standing wave on the string.

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1 v_1/2f,$$

where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1v_1/2L_1$. A similar expression holds for the steel section: $f = n_2v_2/2L_2$. Since the frequency is the same for the two sections, $n_1v_1/L_1 = n_2v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$$

and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times10^3\text{ kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\text{ kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is

$$f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}.$$

The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus,

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\text{kg/m}^3)(1.00\times10^{-6}\text{m}^2)}} = 324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

60. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

The mass that allows the oscillator to set up the nth harmonic on the string is

$$m = \frac{4L^2 f^2 \mu}{n^2 g}.$$

Thus, we see that the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number n, then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n+1}{n^2}.$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer ($2n + 1$) divided by a squared integer (n^2). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc.) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging in $m = 0.447$ kg, $n = 4$, and the other values given in the problem, we find

$$\mu = 0.000845 \text{ kg/m} = 0.845 \text{ g/m}.$$

61. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049 \text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27 \text{ m/s})^2(0.049 \text{ kg/m}) = 36 \text{ N}.$$

62. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is $y_m = 2.0$ cm $= 0.020$ m, as given in the problem.

(b) The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10\text{ m}) = 63\text{ m}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400\text{ Hz}) = 2510\text{ rad/s} = 2.5\times10^3\text{ rad/s}$.

(d) A minus sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left(\left(62.8\,\text{m}^{-1}\right)x - \left(2510\,\text{s}^{-1}\right)t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of y:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m = (2510\text{ rad/s})(0.020\text{ m}) = 50\text{ m/s}$.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

63. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75\text{ Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

64. (a) At $x = 2.3$ m and $t = 0.16$ s the displacement is

$$y(x,t) = 0.15\sin[(0.79)(2.3) - 13(0.16)]\,\text{m} = -0.039\,\text{m}.$$

(b) We choose $y_m = 0.15$ m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79\text{ m}^{-1}$,

(d) and $\omega = 13\text{ rad/s}$.

(e) The wave must be traveling in the $-x$ direction, implying a plus sign in front of ω.

Thus, its general form is $y'(x,t) = (0.15\text{ m})\sin(0.79x + 13t)$.

(f) The displacement of the standing wave at $x = 2.3$ m and $t = 0.16$ s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

65. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed u.

(a) The amplitude is $y_m = 2.0$ mm.

(b) Since $\omega = 600$ rad/s, the frequency is found to be $f = 600/2\pi \approx 95$ Hz.

(c) Since $k = 20$ rad/m, the velocity of the wave is $v = \omega/k = 600/20 = 30$ m/s in the $+x$ direction.

(d) The wavelength is $\lambda = 2\pi/k \approx 0.31$ m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is $u_m = (600)(2.0) = 1200$ mm/s, or 1.2 m/s.

66. Setting $x = 0$ in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its t-derivative) at $t = 0$, or

$$\frac{dy}{dt} = \frac{d}{dt}\left[y_m \sin(-\omega t + \phi)\right] = -y_m \omega \cos(-\omega t + \phi) > 0$$

at $t = 0$. This implies that $-\cos\phi > 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $y = 2.00$ mm, and (at some later t) $y_m = 6.00$ mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad\Rightarrow\quad \phi = \sin^{-1}\left(\tfrac{1}{3}\right) = 0.34\text{ rad} \quad\text{or}\quad 2.8\text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.8$ rad because this is about 161° and is in second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.8 - 2\pi = -3.48$ rad is also an acceptable result.

67. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m\cos\left(\frac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\frac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

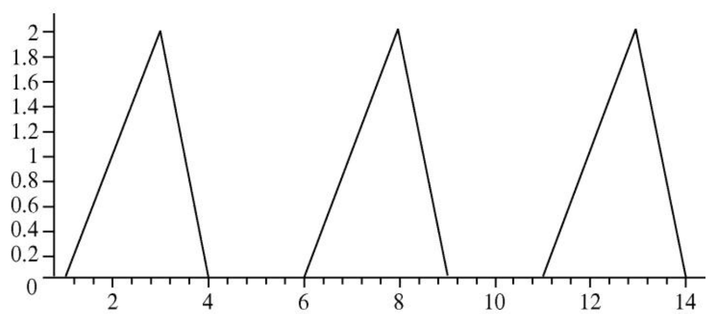
(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

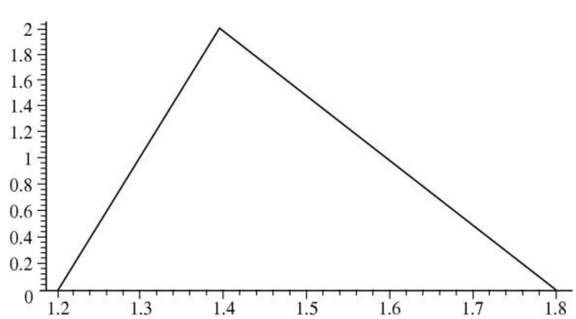
68. (a) Recalling the discussion in Section 16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where x is in centimeters and t is in seconds) must be $5.0\ \text{cm/s}$.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16-44 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.

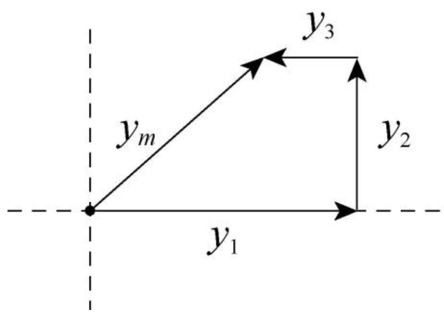


(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4.0)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3.0)/5 = 1.4$ s. Finally, the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1.0)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



69. **THINK** We use phasors to add the three waves and calculate the amplitude of the resultant wave.

EXPRESS The phasor diagram is shown here: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave.



The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$.

ANALYZE (a) The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

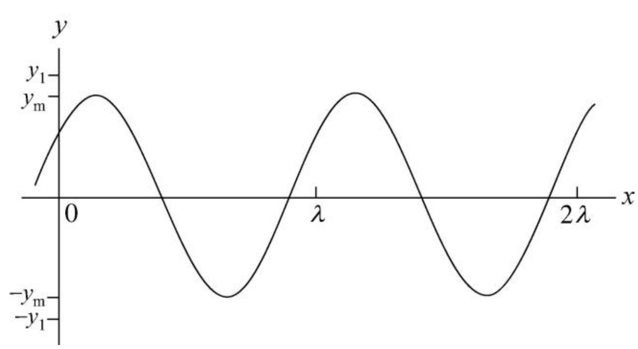
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



LEARN In adding the three sinusoidal waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. However, adding the three terms explicitly gives, after a little algebra,

$$\begin{aligned} y_1 + y_2 + y_3 &= y_1 \sin(kx - \omega t) + \frac{1}{2} y_1 \sin(kx - \omega t + \pi/2) + \frac{1}{3} y_1 \sin(kx - \omega t + \pi) \\ &= y_1 \sin(kx - \omega t) + \frac{1}{2} y_1 \cos(kx - \omega t) - \frac{1}{3} y_1 \sin(kx - \omega t) \\ &= \frac{2}{3} y_1 \sin(kx - \omega t) + \frac{1}{2} y_1 \cos(kx - \omega t) \\ &= \frac{5}{6} y_1 \left[\frac{4}{5} \sin(kx - \omega t) + \frac{3}{5} \cos(kx - \omega t) \right] \\ &= \frac{5}{6} y_1 \sin(kx - \omega t + \phi) \end{aligned}$$

where $\phi = \tan^{-1}(3/4) = 0.644\text{ rad}$. In deducing the phase ϕ, we set $\cos\phi = 4/5$ and $\sin\phi = 3/5$, and use the relation $\cos\phi \sin\theta + \sin\phi \cos\theta = \sin(\theta + \phi)$. The result indeed agrees with that obtained in (c).

70. Setting $x = 0$ in $a_y = -\omega^2 y$, where $y = y_m \sin(kx - \omega t + \phi)$ gives

$$a_y = -\omega^2 y_m \sin(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a negative "slope" (referring to its t-derivative) at $t = 0$, or

$$\frac{da_y}{dt} = \frac{d}{dt}\left[-\omega^2 y_m \sin(-\omega t + \phi)\right] = \omega^3 y_m \cos(-\omega t + \phi) < 0$$

at $t = 0$. This implies that $\cos\phi < 0$ and consequently that ϕ is in either the second or third quadrant. The graph shows (at $t = 0$) $a_y = -100\text{ m/s}^2$, and (at another t) $a_{\max} = 400\text{ m/s}^2$. Therefore,

$$a_y = -a_{\max} \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}(\tfrac{1}{4}) = 0.25\text{ rad} \quad \text{or} \quad 2.9\text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

71. (a) Let the displacement of the string be of the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega\, y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is f = 120 Hz and the angular frequency is $\omega = 2\pi f = 2\pi$ (120 Hz) = 754 rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754\ \text{rad/s})\,(5.00 \times 10^{-3}\ \text{m}) = 3.77\ \text{m/s}.$$

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = ky_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = ky_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus,

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\text{m}) = 0.138$$

and θ = 7.83°. The maximum value of the transverse component of the tension in the string is

$$\tau_{\text{trans}} = (90.0\ \text{N}) \sin 7.83° = 12.3\ \text{N}.$$

We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is

$$u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}}u.$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above, $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

72. We use Eq. 16-52 in interpreting the figure.

(a) Since $y' = 6.0$ mm when $\phi = 0$, then Eq. 16-52 can be used to determine $y_m = 3.0$ mm.

(b) We note that $y' = 0$ when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, $\lambda = 20$ cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since $f = 120$ Hz, $\omega = 2\pi f = 754\ \text{rad/s} \approx 7.5\times 10^2\ \text{rad/s}$.

(d) The sign in front of ω is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm}) \sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]]$ (this applies to each wave when they are in phase).

73. We note that

$$dy/dt = -\omega\cos(kx - \omega t + \phi),$$

which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2\ \text{rad}.$$

74. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau}\, f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\dfrac{\tau_2}{\tau_1}} = P_1\sqrt{\dfrac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1\left(\dfrac{f_2}{f_1}\right)^2 = P_1\left(\dfrac{f_1/2}{f_1}\right)^2 = \dfrac{1}{4}P_1$.

75. (a) Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^8\ \mathrm{N/m^2}}{7800\,\mathrm{kg/m^3}}} = 3.00\times10^2\ \mathrm{m/s}\,.$$

(b) The result does not depend on the diameter of the wire.

76. Repeating the steps of Eq. 16-47 → Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative x, the smallest value to produce cos $\pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x](-4\pi\sin 4\pi t).$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at $x = 0$ has zero velocity is $t = 0$.

(c) Using the result obtained in (b), the second time where the velocity at $x = 0$ vanishes would be $t = 0.25$ s,

(d) and the third time is $t = 0.50$ s.

77. **THINK** The speed of a transverse wave in the stretched rubber band is related to the tension in the band and the linear mass density of the band.

EXPRESS The wave speed v is given by $v = \sqrt{F/\mu}$, where F is the tension in the rubber band and μ is the band's linear mass density, which is defined as the mass per unit length $\mu = m/L$. The fact that the band obeys Hooke's law implies $F = k\Delta\ell$, where k is the spring constant and $\Delta\ell$ is the elongation. Thus, when a force F is applied, the rubber band has a length $L = \ell + \Delta\ell$, where ℓ is the unstretched length, resulting in a linear mass density $\mu = m/(\ell + \Delta\ell)$.

ANALYZE (a) The wave speed is $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{k\Delta\ell}{m/(\ell+\Delta\ell)}} = \sqrt{\dfrac{k\Delta\ell(\ell+\Delta\ell)}{m}}$.

(b) The time required for the pulse to travel the length of the rubber band is

$$t = \frac{2\pi(\ell+\Delta\ell)}{v} = \frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$. On the other hand, if $\ell/\Delta\ell \ll 1$, then we have $t \simeq 2\pi\sqrt{m/k} = \text{const}$.

LEARN When $\Delta\ell \ll \ell$, the applied force $F = k\Delta\ell$ is small while $\mu \approx m/\ell = \text{constant}$, leading to a small wave speed. On the other hand, when $\Delta\ell \gg \ell$, $\mu \approx m/\Delta\ell$ and $v = \sqrt{F/\mu} \propto \Delta\ell$, so that $t \simeq 2\pi\sqrt{m/k}$, which is a constant.

78. (a) For visible light

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz}.$$

(b) For radio waves

$$\lambda_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\,\text{m}.$$

(c) For X rays

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{5.0\times10^{-9}\ \text{m}} = 6.0\times10^{16}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-11}\ \text{m}} = 3.0\times10^{19}\ \text{Hz.}$$

79. **THINK** A wire held rigidly at both ends can be made to oscillate in standing wave patterns.

EXPRESS Possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$ where τ is the tension in the wire and μ is the linear mass density of the wire.

ANALYZE (a) The wave speed is $v = \sqrt{\dfrac{\tau}{\mu}} = \sqrt{\dfrac{120\ \text{N}}{8.70\times10^{-3}\ \text{kg}/1.50\ \text{m}}} = 144\ \text{m/s}.$

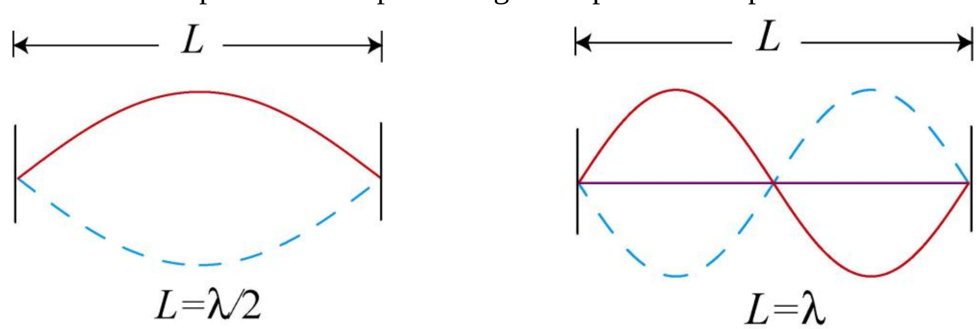
(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\ \text{m}) = 3.00\ \text{m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\ \text{m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\ \text{m/s})/(3.00\ \text{m}) = 48.0\ \text{Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\ \text{m/s})/(1.50\ \text{m}) = 96.0\ \text{Hz}$.

LEARN The one-loop and two-loop standing wave patterns are plotted below:



80. By Eq. 16–66, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880\ \text{Hz}$.

(b) The frequency of the third harmonic is $f_3 = 3(440) = 1320\ \text{Hz}$.

81. (a) The amplitude is y_m = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is f = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the $-x$ direction, the sign in front of ω is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$\begin{aligned} y(x,t) &= y_\mathrm{m} \sin(kx+\omega t) = y_\mathrm{m} \sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] \\ &= (0.010\,\mathrm{m})\sin\left[2\pi(550\,\mathrm{Hz})\left(\frac{x}{330\,\mathrm{m/s}}+t\right)\right] \\ &= (0.010\,\mathrm{m})\ \sin[(10.5 \text{ rad/s})\ x + (3.46\times10^3 \text{ rad/s})t]. \end{aligned}$$

82. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$\begin{aligned} (3.0\ \text{mm}) + (5.0\ \text{mm})\cos(70°) &= 4.71\text{mm along } x \text{ axis} \\ (5.0\ \text{mm})\sin(70°) &= 4.70 \text{ mm along } y \text{ axis.} \end{aligned}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71 \text{ mm})^2 + (4.70 \text{ mm})^2} = 6.7\,\text{mm}.$

(b) And the angle (phase constant) is $\tan^{-1}$ (4.70/4.71) = 45°.

83. **THINK** The speed of a point on the cord is given by $u(x, t) = \partial y/\partial t$, where $y\ (x, t)$ is displacement.

EXPRESS We take the form of the displacement to be

$$y\ (x, t) = y_m \sin(kx - \omega t).$$

The speed of a point on the cord is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t),$$

and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$.

(a) The ratio of the maximum particle speed to the wave speed is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on y_m/λ, the ratio of the amplitude to the wavelength.

LEARN Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

84. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx) \cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is

$$k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$$

and the angular frequency is

$$\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800 \text{ rad/s}.$$

With $y_m = 2.0$ mm, the displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos[(3800\,\text{s}^{-1})t].$$

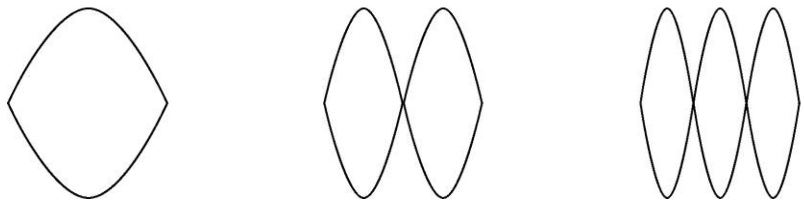
85. We make use of Eq. 16-65 with $L = 120$ cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown next:



86. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then

$$z(y,t) = z_m \sin(ky - \omega t),$$

where z_m = 3.0 mm, k = 60 cm^{-1}, and $\omega = 2\pi/T = 2\pi/0.20$ s = 10π s^{-1}. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

87. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when x = 6 and $t = \frac{1}{4}$, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when x = 6 and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin(-\pi/4)$, which yields a = 16.7 m/s^2.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s^2.

88. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = \left(2000\,\text{m/s}\right)\left(40\times10^{-6}\,\text{s}\right) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve *a* = 300/40 × 10^{-6}) we find the stopping distance *d*:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2 \left(40 \times 10^{-6}\right)}{2(300)}$$

which gives $d = 6.0\times10^{-3}$ m. This and the radius *r* form the legs of a right triangle (where *r* is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d \tan 60° = 1.0\times10^{-2}\,\text{m}.$$

89. Using Eq. 16-50, we have

$$y' = \left(0.60\cos\frac{\pi}{6}\right)\sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be $0.60\cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}$.

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12), $v = \dfrac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

90. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Using the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad \Rightarrow \quad y_m = 3.2\text{ cm}.$$

(c) As already noted, $f = 0.25$ Hz.

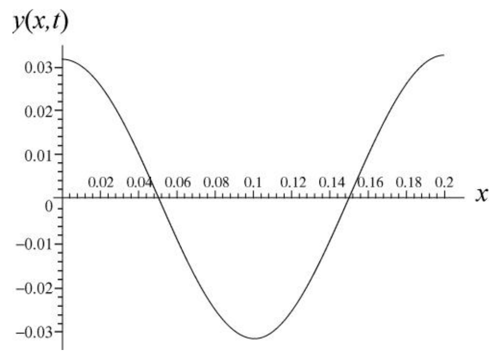
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the *t* and *x* terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050\,\sin\pi t/2$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050\sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at t = 2.0 s for $0 \le x \le 0.20$ m is shown below.



91. **THINK** The rope with both ends fixed and made to oscillate in fundamental mode has wavelength $\lambda = 2L$, where L is the length of the rope.

EXPRESS We first observe that the anti-node at x = 1.0 m having zero displacement at t = 0 suggests the use of sine instead of cosine for the simple harmonic motion factor. We take the form of the displacement to be

$$y(x, t) = y_m \sin(kx)\sin(\omega t).$$

A point on the rope undergoes simple harmonic motion with a speed

$$u(x, t) = \partial y/\partial t = \omega y_m \sin(kx)\cos(\omega t).$$

It has maximum speed $u_m = \omega y_m$ as it passes through its "middle" point. On the other hand, the wave speed is $v = \sqrt{\tau/\mu}$ where τ is the tension in the rope and μ is the linear mass density of the rope. For standing waves, possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the rope and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. For fundamental mode, we set n = 1.

ANALYZE (a) With f = 5.0 Hz, we find the angular frequency to be $\omega = 2\pi f = 10\pi$ rad/s. Thus, if the maximum speed of a point on the rope is u_m = 5.0 m/s, then its amplitude is

$$y_m = \frac{u_m}{\omega} = \frac{5.0\text{ m/s}}{10\pi\text{ rad/s}} = 0.16\text{ m}.$$

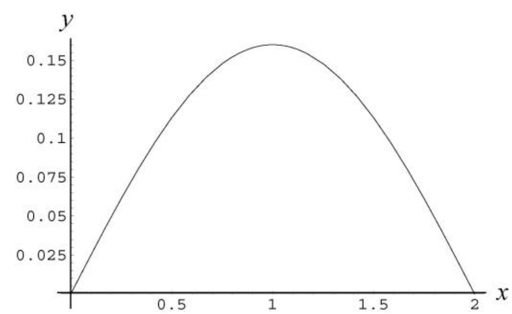
(b) Since the oscillation is in the *fundamental* mode, we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2\,\text{N}.$$

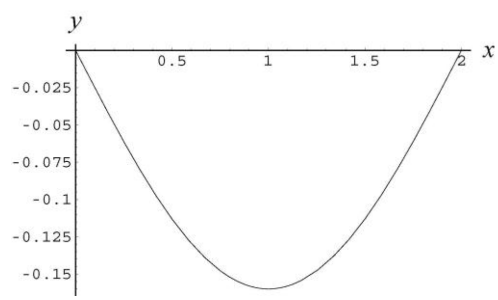
(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16\text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16\text{ m})\sin[(1.57\text{ m}^{-1})x]\sin[(31.4\text{ rad/s})t]$$

LEARN The period of oscillation is $T = 1/f = 0.20\,\text{s}$. The snapshots of the patterns at $t = T/4 = 0.05\,\text{s}$ and $t = 3T/4 = 0.15\,\text{s}$ are given below. At $t = T/2$ and T, the displacement is zero everywhere.

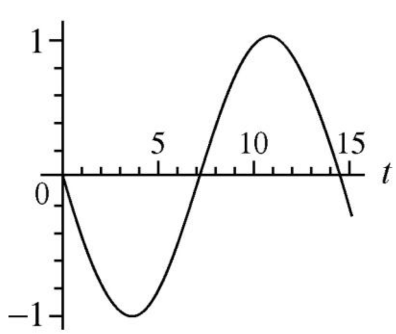


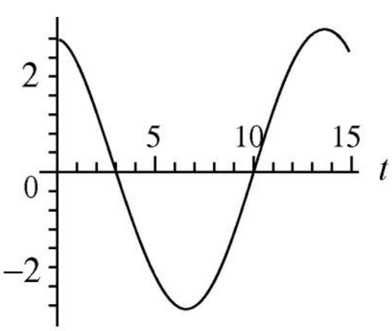
$t = T/4 = 0.05\,\text{s}$



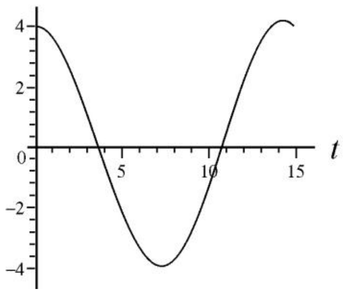
$t = 3T/4 = 0.15\,\text{s}$

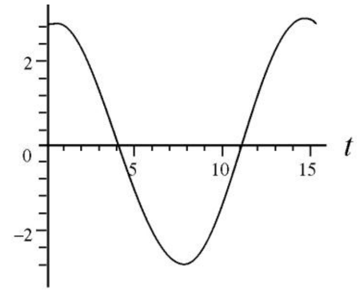
92. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250.3$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.

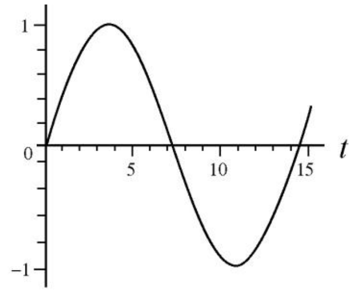




The following three graphs show the oscillation at $x = \lambda/4 = 62.6$ mm ≈ 63 mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.







(b) We can think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in Section 16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{max} = 4.0$ mm and occurs at $x = \lambda/4 = 62.6$ mm.

(e) The smallest amplitude of oscillation is $y_{min} = 1.0$ mm and occurs at $x = 0$ and at

$$x = \lambda/2 = 125 \text{ mm}.$$

(f) The largest amplitude can be related to the amplitudes of y_1 and y_2 in a simple way:

$$y_{max} = y_{1m} + y_{2m},$$

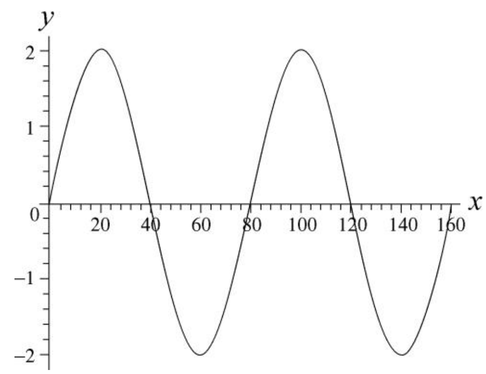
where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is

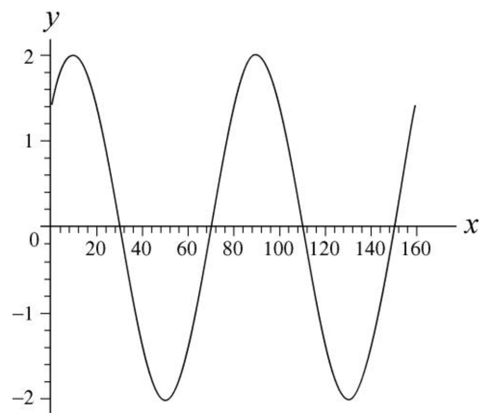
$$y_{min} = y_{1m} - y_{2m},$$

where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

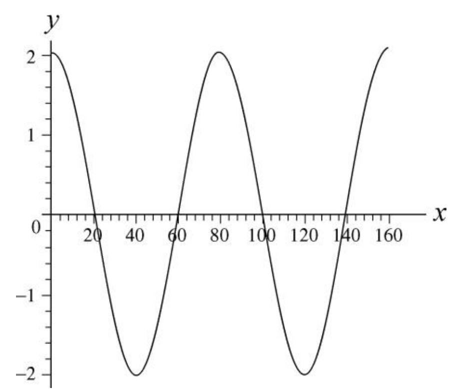
93. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at t = 0, and this next one is at t = 0.050 s.



And the final one, shown below, is at t = 0.010 s.



(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0\times 10^2$ cm/s $= 2.0$ m/s.

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the –x direction.

94. The speed of the transverse wave along the string is given by Eq. 16-26: $v = \sqrt{\tau / \mu}$, where τ is the tension and μ is the linear mass density of the string. Applying Newton's second law to a small segment of the string, the radial restoring force is (see Eq. 16-23)

$$F = 2(\tau \sin\theta) \approx \tau \frac{\Delta l}{R}$$

Since $F = (\Delta m)v_T^2 / R$, where v_T is the tangential speed of the segment of mass $\Delta m = \mu \Delta l$, and R is the radius of the circle, we have

$$\tau \frac{\Delta l}{R} = (\mu \Delta l)\frac{v_T^2}{R} \quad \Rightarrow \quad \tau = \mu v_T^2$$

On the other hand, the fact that $v = \sqrt{\tau / \mu}$ implies $\tau = \mu v^2$. Thus, we must have $v = v_T$, which in this case, is equal to 5.00 cm/s. Note that v is independent of the radius of the circular loop.

95. (a) With total reflection, $A = B$, and $\text{SWR} = \dfrac{A+B}{A-B} \rightarrow \infty$.

(b) With no reflection, $B = 0$, and $\text{SWR} = \dfrac{A+B}{A-B} = \dfrac{A}{A} = 1$.

(c) In terms of $R = (B/A)^2$, we can rewrite SWR as

$$\text{SWR} = \frac{A+B}{A-B} = \frac{1+(B/A)}{1-(B/A)} = \frac{1+\sqrt{R}}{1-\sqrt{R}} \quad \Rightarrow \quad R = \left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2$$

With SWR = 1.50, we obtain

$$R = \left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2 = \left(\frac{1.50-1}{1.50+1}\right)^2 = 0.040 = 4.0\%.$$

96. (a) The speed of each individual wave is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{40\ \text{N}}{(0.125\ \text{kg})/(2.25\ \text{m})}} = 26.83\ \text{m/s}.$$

The average rate at which energy is transmitted from one side is

$$P_{\text{avg},1} = \frac{1}{2}\mu v\omega^2 y_m^2 = \frac{1}{2}\left(\frac{0.125\text{ kg}}{2.25\text{ m}}\right)(26.83\text{ m/s})(2\pi\times 120\text{ Hz})^2(5.0\times 10^{-3}\text{ m})^2 = 10.6\text{ W}.$$

(b) From both sides, $P_{\text{avg}} = 2P_{\text{avg},1} = 2(10.6\text{ W}) = 21.2\text{ W}.$

(c) The rate of change of kinetic energy from one side is given by Eq. 16-30:

$$\frac{dK_1}{dt} = \frac{1}{2}\mu v\omega^2 y_m^2 \cos^2(kx-\omega t).$$

Integrating over one period for both sides, we obtain

$$K = \int\left(2\frac{dK_1}{dt}\right)dt = \mu v\omega^2 y_m^2\int_0^T \cos^2(kx-\omega t)dt = \frac{T}{2}\mu v\omega^2 y_m^2 = \frac{P_{\text{avg}}}{2f}$$
$$= \frac{21.2\text{ W}}{2(120\text{ Hz})} = 8.83\times 10^{-2}\text{ J}.$$