

Chapter 28

1. **THINK** The magnetic force on a charged particle is given by $\vec{F}_B = q\vec{v} \times \vec{B}$, where \vec{v} is the velocity of the charged particle and \vec{B} is the magnetic field.

EXPRESS The magnitude of the magnetic force on the proton (of charge $+e$) is $F_B = evB \sin \phi$, where ϕ is the angle between \vec{v} and \vec{B} .

ANALYZE (a) The speed of the proton is

$$v = \frac{F_B}{eB \sin \phi} = \frac{6.50 \times 10^{-17} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(2.60 \times 10^{-3} \text{ T}) \sin 23.0^\circ} = 4.00 \times 10^5 \text{ m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(4.00 \times 10^5 \text{ m/s})^2 = 1.34 \times 10^{-16} \text{ J},$$

which is equivalent to

$$K = (1.34 \times 10^{-16} \text{ J}) / (1.60 \times 10^{-19} \text{ J/eV}) = 835 \text{ eV}.$$

LEARN from the definition of \vec{B} given by the expression $\vec{F}_B = q\vec{v} \times \vec{B}$, we see that the magnetic force \vec{F}_B is always perpendicular to \vec{v} and \vec{B} .

2. The force associated with the magnetic field must point in the \hat{j} direction in order to cancel the force of gravity in the $-\hat{j}$ direction. By the right-hand rule, \vec{B} points in the $-\hat{k}$ direction (since $\hat{i} \times (-\hat{k}) = \hat{j}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90^\circ$). Therefore, with $m = 1.0 \times 10^{-2} \text{ kg}$, $v = 2.0 \times 10^4 \text{ m/s}$, and $q = 8.0 \times 10^{-5} \text{ C}$, we find

$$\vec{B} = B_z \hat{k} = -\left(\frac{mg}{qv}\right) \hat{k} = (-0.061 \text{ T}) \hat{k}.$$

3. (a) The force on the electron is

$$\begin{aligned}
\vec{F}_B &= q\vec{v} \times \vec{B} = q(v_x \hat{i} + v_y \hat{j}) \times (B_x \hat{i} + B_y \hat{j}) = q(v_x B_y - v_y B_x) \hat{k} \\
&= (-1.6 \times 10^{-19} \text{ C}) [(2.0 \times 10^6 \text{ m/s})(-0.15 \text{ T}) - (3.0 \times 10^6 \text{ m/s})(0.030 \text{ T})] \\
&= (6.2 \times 10^{-14} \text{ N}) \hat{k}.
\end{aligned}$$

Thus, the magnitude of \vec{F}_B is $6.2 \times 10^{-14} \text{ N}$, and \vec{F}_B points in the positive z direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, \vec{F}_B has the same magnitude but points in the negative z direction, namely, $\vec{F}_B = -(6.2 \times 10^{-14} \text{ N}) \hat{k}$.

4. (a) We use Eq. 28-3:

$$F_B = |q| v B \sin \phi = (+ 3.2 \times 10^{-19} \text{ C}) (550 \text{ m/s}) (0.045 \text{ T}) (\sin 52^\circ) = 6.2 \times 10^{-18} \text{ N}.$$

(b) The acceleration is

$$a = F_B/m = (6.2 \times 10^{-18} \text{ N}) / (6.6 \times 10^{-27} \text{ kg}) = 9.5 \times 10^8 \text{ m/s}^2.$$

(c) Since it is perpendicular to \vec{v} , \vec{F}_B does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q(v_x B_y - v_y B_x) \hat{k} = q(v_x (3B_x) - v_y B_x) \hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z \hat{k}$ where $F_z = 6.4 \times 10^{-19} \text{ N}$, then we are led to the condition

$$q(3v_x - v_y) B_x = F_z \Rightarrow B_x = \frac{F_z}{q(3v_x - v_y)}.$$

Substituting $v_x = 2.0 \text{ m/s}$, $v_y = 4.0 \text{ m/s}$, and $q = -1.6 \times 10^{-19} \text{ C}$, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4 \times 10^{-19} \text{ N}}{(-1.6 \times 10^{-19} \text{ C})[3(2.0 \text{ m/s}) - 4.0 \text{ m/s}]} = -2.0 \text{ T}.$$

6. The magnetic force on the proton is given by $\vec{F} = q\vec{v} \times \vec{B}$, where $q = +e$. Using Eq. 3-30 this becomes

$$(4 \times 10^{-17})\hat{\mathbf{i}} + (2 \times 10^{-17})\hat{\mathbf{j}} = e[(0.03v_y + 40)\hat{\mathbf{i}} + (20 - 0.03v_x)\hat{\mathbf{j}} - (0.02v_x + 0.01v_y)\hat{\mathbf{k}}]$$

with SI units understood. Equating corresponding components, we find

(a) $v_x = -3.5 \times 10^3 \text{ m/s}$, and

(b) $v_y = 7.0 \times 10^3 \text{ m/s}$.

7. We apply $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = m_e \vec{a}$ to solve for \vec{E} :

$$\begin{aligned}\vec{E} &= \frac{m_e \vec{a}}{q} + \vec{B} \times \vec{v} \\ &= \frac{(9.11 \times 10^{-31} \text{ kg})(2.00 \times 10^{12} \text{ m/s}^2)\hat{\mathbf{i}}}{-1.60 \times 10^{-19} \text{ C}} + (400 \mu\text{T})\hat{\mathbf{i}} \times (12.0 \text{ km/s})\hat{\mathbf{j}} + (15.0 \text{ km/s})\hat{\mathbf{k}} \\ &= (-11.4\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} + 4.80\hat{\mathbf{k}}) \text{ V/m}.\end{aligned}$$

8. Letting $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$, we get $vB \sin \phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the $\sin \phi$ factor is at its maximum value (which is 1, corresponding to $\phi = 90^\circ$). So

$$v_{\min} = \frac{E}{B} = \frac{1.50 \times 10^3 \text{ V/m}}{0.400 \text{ T}} = 3.75 \times 10^3 \text{ m/s}.$$

9. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Note that $\vec{v} \perp \vec{B}$ so $|\vec{v} \times \vec{B}| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100 \text{ V}/(20 \times 10^{-3} \text{ m})}{\sqrt{2(1.0 \times 10^3 \text{ V})(1.60 \times 10^{-19} \text{ C})/(9.11 \times 10^{-31} \text{ kg})}} = 2.67 \times 10^{-4} \text{ T}.$$

In unit-vector notation, $\vec{B} = -(2.67 \times 10^{-4} \text{ T})\hat{\mathbf{k}}$.

10. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} = (1.60 \times 10^{-19} \text{ C})[(4.00 \text{ V/m})\hat{\mathbf{k}} + (2000 \text{ m/s})\hat{\mathbf{j}} \times (-2.50 \times 10^{-3} \text{ T})\hat{\mathbf{i}}] \\ &= (1.44 \times 10^{-18} \text{ N})\hat{\mathbf{k}}.\end{aligned}$$

(b) In this case, we have

$$\begin{aligned}
\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \\
&= (1.60 \times 10^{-19} \text{ C}) \left[(-4.00 \text{ V/m}) \hat{k} + (2000 \text{ m/s}) \hat{j} \times (-2.50 \text{ mT}) \hat{i} \right] \\
&= (1.60 \times 10^{-19} \text{ N}) \hat{k}.
\end{aligned}$$

(c) In the final case, we have

$$\begin{aligned}
\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B} \\
&= (1.60 \times 10^{-19} \text{ C}) \left[(4.00 \text{ V/m}) \hat{i} + (2000 \text{ m/s}) \hat{j} \times (-2.50 \text{ mT}) \hat{i} \right] \\
&= (6.41 \times 10^{-19} \text{ N}) \hat{i} + (8.01 \times 10^{-19} \text{ N}) \hat{k}.
\end{aligned}$$

11. Since the total force given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ vanishes, the electric field \vec{E} must be perpendicular to both the particle velocity \vec{v} and the magnetic field \vec{B} . The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V , $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B \sqrt{\frac{2eV}{m}} = (1.2 \text{ T}) \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(10 \times 10^3 \text{ V})}{(9.99 \times 10^{-27} \text{ kg})}} = 6.8 \times 10^5 \text{ V/m}.$$

12. (a) The force due to the electric field ($\vec{F} = q\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F} = q\vec{v} \times \vec{B}$) in that the latter vanishes when the speed is zero and the former is independent of speed. The graph shows that the force (y-component) is negative at $v = 0$ (specifically, its value is $-2.0 \times 10^{-19} \text{ N}$ there), which (because $q = -e$) implies that the electric field points in the $+y$ direction. Its magnitude is

$$E = \frac{F_{\text{net},y}}{|q|} = \frac{2.0 \times 10^{-19} \text{ N}}{1.6 \times 10^{-19} \text{ C}} = 1.25 \text{ N/C} = 1.25 \text{ V/m}.$$

(b) We are told that the x and z components of the force remain zero throughout the motion, implying that the electron continues to move along the x axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in Section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v = 2.50 \times 10^{-2} \text{ T}$.

For $\vec{F} = q\vec{v} \times \vec{B}$ to be in the opposite direction of $\vec{F} = q\vec{E}$ we must have $\vec{v} \times \vec{B}$ in the opposite direction from \vec{E} , which points in the $+y$ direction, as discussed in part (a). Since the velocity is in the $+x$ direction, then (using the right-hand rule) we conclude that

the magnetic field must point in the $+z$ direction ($\hat{i} \times \hat{k} = -\hat{j}$). In unit-vector notation, we have $\vec{B} = (2.50 \times 10^{-2} \text{ T})\hat{k}$.

13. We use Eq. 28-12 to solve for V :

$$V = \frac{iB}{nle} = \frac{(23\text{ A})(0.65 \text{ T})}{(8.47 \times 10^{28}/\text{m}^3)(150\mu\text{m})(1.6 \times 10^{-19}\text{ C})} = 7.4 \times 10^{-6} \text{ V}.$$

14. For a free charge q inside the metal strip with velocity \vec{v} we have $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90 \times 10^{-9} \text{ V})}{(1.20 \times 10^{-3} \text{ T})(0.850 \times 10^{-2} \text{ m})} = 0.382 \text{ m/s}.$$

15. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec{E}| = v|\vec{B}| = (20.0 \text{ m/s})(0.030 \text{ T}) = 0.600 \text{ V/m}.$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec{v} \times \vec{B}$. In summary,

$$\vec{E} = -(0.600 \text{ V/m})\hat{k}$$

which insures that $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes.

(b) Equation 28-9 yields $V = Ed = (0.600 \text{ V/m})(2.00 \text{ m}) = 1.20 \text{ V}$.

16. We note that \vec{B} must be along the x axis because when the velocity is along that axis there is no induced voltage. Combining Eq. 28-7 and Eq. 28-9 leads to

$$d = \frac{V}{E} = \frac{V}{vB}$$

where one must interpret the symbols carefully to ensure that \vec{d} , \vec{v} , and \vec{B} are mutually perpendicular. Thus, when the velocity is parallel to the y axis the absolute value of the voltage (which is considered in the same “direction” as \vec{d}) is 0.012 V, and

$$d = d_z = \frac{0.012 \text{ V}}{(3.0 \text{ m/s})(0.020 \text{ T})} = 0.20 \text{ m}.$$

On the other hand, when the velocity is parallel to the z axis the absolute value of the appropriate voltage is 0.018 V, and

$$d = d_y = \frac{0.018 \text{ V}}{(3.0 \text{ m/s})(0.020 \text{ T})} = 0.30 \text{ m}.$$

Thus, our answers are

(a) $d_x = 25 \text{ cm}$ (which we arrive at “by elimination,” since we already have figured out d_y and d_z),

(b) $d_y = 30 \text{ cm}$, and

(c) $d_z = 20 \text{ cm}$.

17. (a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\text{u}} = \frac{2(4.50 \times 10^{-2} \text{ m})(1.60 \times 10^{-19} \text{ C})(1.20 \text{ T})}{(4.00\text{u})(1.66 \times 10^{-27} \text{ kg/u})} = 2.60 \times 10^6 \text{ m/s}.$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2} \text{ m})/(2.60 \times 10^6 \text{ m/s}) = 1.09 \times 10^{-7} \text{ s}$.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2} m_\alpha v^2 = \frac{(4.00\text{u})(1.66 \times 10^{-27} \text{ kg/u})(2.60 \times 10^6 \text{ m/s})^2}{2(1.60 \times 10^{-19} \text{ J/eV})} = 1.40 \times 10^5 \text{ eV}.$$

(d) $\Delta V = K/q = 1.40 \times 10^5 \text{ eV}/2e = 7.00 \times 10^4 \text{ V}$.

18. With the \vec{B} pointing “out of the page,” we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle’s path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99 \times 10^6 \text{ m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find $r = 0.00710 \text{ m}$.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9} \text{ s}$.

19. Let ξ stand for the ratio $(m/|q|)$ we wish to solve for. Then Eq. 28-17 can be written as $T = 2\pi\xi/B$. Noting that the horizontal axis of the graph (Fig. 28-37) is inverse-field $(1/B)$ then we conclude (from our previous expression) that the slope of the line in the graph must be equal to $2\pi\xi$. We estimate that slope is $7.5 \times 10^{-9} \text{ T}\cdot\text{s}$, which implies

$$\xi = m/|q| = 1.2 \times 10^{-9} \text{ kg/C}.$$

20. Combining Eq. 28-16 with energy conservation ($eV = \frac{1}{2} m_e v^2$ in this particular application) leads to the expression

$$r = \frac{m_e}{eB} \sqrt{\frac{2eV}{m_e}}$$

which suggests that the slope of the r versus \sqrt{V} graph should be $\sqrt{2m_e/eB^2}$. From Fig. 28-38, we estimate the slope to be 5×10^{-5} in SI units. Setting this equal to $\sqrt{2m_e/eB^2}$ and solving, we find $B = 6.7 \times 10^{-2} \text{ T}$.

21. **THINK** The electron is in circular motion because the magnetic force acting on it points toward the center of the circle.

EXPRESS The kinetic energy of the electron is given by $K = \frac{1}{2} m_e v^2$, where m_e is the mass of electron and v is its speed. The magnitude of the magnetic force on the electron is $F_B = evB$ which is equal to the centripetal force:

$$evB = \frac{m_e v^2}{r}.$$

ANALYZE (a) From $K = \frac{1}{2} m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ eV/J})}{9.11 \times 10^{-31} \text{ kg}}} = 2.05 \times 10^7 \text{ m/s}.$$

(b) Since $evB = m_e v^2 / r$, we find the magnitude of the magnetic field to be

$$B = \frac{m_e v}{er} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.05 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(25.0 \times 10^{-2} \text{ m})} = 4.67 \times 10^{-4} \text{ T}.$$

(c) The “orbital” frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07 \times 10^7 \text{ m/s}}{2\pi(25.0 \times 10^{-2} \text{ m})} = 1.31 \times 10^7 \text{ Hz.}$$

(d) The period is simply equal to the reciprocal of frequency:

$$T = 1/f = (1.31 \times 10^7 \text{ Hz})^{-1} = 7.63 \times 10^{-8} \text{ s.}$$

LEARN The period of the electron's circular motion can be written as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|e|B} = \frac{2\pi m}{|e|B}.$$

The period is inversely proportional to B .

22. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $K = (rqB)^2/2m \propto q^2 m^{-1}$.

$$(a) K_\alpha = (q_\alpha/q_p)^2 (m_p/m_\alpha) K_p = (2)^2 (1/4) K_p = K_p = 1.0 \text{ MeV};$$

$$(b) K_d = (q_d/q_p)^2 (m_p/m_d) K_p = (1)^2 (1/2) K_p = 1.0 \text{ MeV}/2 = 0.50 \text{ MeV}.$$

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.30 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ m})} = 2.11 \times 10^{-5} \text{ T.}$$

24. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2} m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(350 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 1.11 \times 10^7 \text{ m/s.}$$

(b) Equation 28-16 gives

$$r = \frac{m_e v}{eB} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.11 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(200 \times 10^{-3} \text{ T})} = 3.16 \times 10^{-4} \text{ m}.$$

25. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0 \times 10^{-6} \text{ T})(1.60 \times 10^{-19} \text{ C})}{2\pi(9.11 \times 10^{-31} \text{ kg})} = 9.78 \times 10^5 \text{ Hz}.$$

(b) Using Eq. 28-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(100 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}}{(1.60 \times 10^{-19} \text{ C})(35.0 \times 10^{-6} \text{ T})} = 0.964 \text{ m}.$$

26. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v} \times \vec{B}$ points leftward, which indeed seems to be the direction it is “pushed”; therefore, $q > 0$ (it is a proton).

(a) Equation 28-17 becomes $T = 2\pi m_p / e|\vec{B}|$, or

$$2(130 \times 10^{-9}) = \frac{2\pi(1.67 \times 10^{-27})}{(1.60 \times 10^{-19})|\vec{B}|}$$

which yields $|\vec{B}| = 0.252 \text{ T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130 \text{ ns}$.

27. (a) We solve for B from $m = B^2 q x^2 / 8V$ (see Sample Problem 28.04 — “Uniform circular motion of a charged particle in a magnetic field”):

$$B = \sqrt{\frac{8Vm}{qx^2}}.$$

We evaluate this expression using $x = 2.00 \text{ m}$:

$$B = \sqrt{\frac{8(100 \times 10^3 \text{ V})(3.92 \times 10^{-25} \text{ kg})}{(3.20 \times 10^{-19} \text{ C})(2.00 \text{ m})^2}} = 0.495 \text{ T}.$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100 \times 10^{-6} \text{ kg}}{3600 \text{ s}} = 2.78 \times 10^{-8} \text{ kg/s} .$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{(3.20 \times 10^{-19} \text{ C})(2.78 \times 10^{-8} \text{ kg/s})}{3.92 \times 10^{-25} \text{ kg}} = 2.27 \times 10^{-2} \text{ A} .$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV \Delta t = \frac{iqV}{q} \Delta t = iV \Delta t .$$

For $\Delta t = 1.0 \text{ h}$,

$$E = (2.27 \times 10^{-2} \text{ A})(100 \times 10^3 \text{ V})(3600 \text{ s}) = 8.17 \times 10^6 \text{ J} .$$

To obtain the second expression, i/q is substituted for N .

28. Using $F = mv^2 / r$ (for the centripetal force) and $K = mv^2 / 2$, we can easily derive the relation

$$K = \frac{1}{2} Fr .$$

With the values given in the problem, we thus obtain $K = 2.09 \times 10^{-22} \text{ J}$.

29. Reference to Fig. 28-11 is very useful for interpreting this problem. The distance traveled parallel to \vec{B} is $d_{\parallel} = v_{\parallel} T = v_{\parallel}(2\pi m_e / |q|B)$ using Eq. 28-17. Thus,

$$v_{\parallel} = \frac{d_{\parallel} eB}{2\pi m_e} = 50.3 \text{ km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_{\perp}$, then we find $v_{\perp} = 41.7 \text{ km/s}$. The speed is therefore $v = \sqrt{v_{\perp}^2 + v_{\parallel}^2} = 65.3 \text{ km/s}$.

30. Eq. 28-17 gives $T = 2\pi m_e / eB$. Thus, the total time is

$$\left(\frac{T}{2}\right)_1 + t_{\text{gap}} + \left(\frac{T}{2}\right)_2 = \frac{\pi m_e}{e} \left(\frac{1}{B_1} + \frac{1}{B_2}\right) + t_{\text{gap}} .$$

The time spent in the gap (which is where the electron is accelerating in accordance with Eq. 2-15) requires a few steps to figure out: letting $t = t_{\text{gap}}$ then we want to solve

$$d = v_0 t + \frac{1}{2} a t^2 \Rightarrow 0.25 \text{ m} = \sqrt{\frac{2K_0}{m_e}} t + \frac{1}{2} \left(\frac{e\Delta V}{m_e d} \right) t^2$$

for t . We find in this way that the time spent in the gap is $t \approx 6 \text{ ns}$. Thus, the total time is 8.7 ns .

31. Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. Therefore, using Eq. 28-17, the time is given by

$$t = \frac{T}{2} = \frac{\pi m}{Bq} = \frac{\pi (9.11 \times 10^{-31} \text{ kg})}{(3.53 \times 10^{-3} \text{ T})(1.60 \times 10^{-19} \text{ C})} = 5.07 \times 10^{-9} \text{ s}.$$

32. Let $v_{\parallel} = v \cos \theta$. The electron will proceed with a uniform speed v_{\parallel} in the direction of \vec{B} while undergoing uniform circular motion with frequency f in the direction perpendicular to B : $f = eB/2\pi m_e$. The distance d is then

$$d = v_{\parallel} T = \frac{v_{\parallel}}{f} = \frac{(v \cos \theta) 2\pi m_e}{eB} = \frac{2\pi (1.5 \times 10^7 \text{ m/s}) (9.11 \times 10^{-31} \text{ kg}) (\cos 10^\circ)}{(1.60 \times 10^{-19} \text{ C})(1.0 \times 10^{-3} \text{ T})} = 0.53 \text{ m}.$$

33. **THINK** The path of the positron is helical because its velocity \vec{v} has components parallel and perpendicular to the magnetic field \vec{B} .

EXPRESS If v is the speed of the positron then $v \sin \phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here $\phi = 89^\circ$ is the angle between the velocity and the field. Newton's second law yields $eBv \sin \phi = m_e(v \sin \phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v / eB) \sin \phi$. The period is given by

$$T = \frac{2\pi r}{v \sin \phi} = \frac{2\pi m_e}{eB}.$$

The equation for r is substituted to obtain the second expression for T . For part (b), the pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT \cos \phi$.

ANALYZE (a) Substituting the values given, we find the period to be

$$T = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.100 \text{ T})} = 3.58 \times 10^{-10} \text{ s}.$$

(b) We use the kinetic energy, $K = \frac{1}{2} m_e v^2$, to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.00 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 2.65 \times 10^7 \text{ m/s}.$$

Thus, the pitch is $p = (2.65 \times 10^7 \text{ m/s})(3.58 \times 10^{-10} \text{ s}) \cos 89^\circ = 1.66 \times 10^{-4} \text{ m}$.

(c) The orbit radius is

$$R = \frac{m_e v \sin \phi}{eB} = \frac{(9.11 \times 10^{-31} \text{ kg})(2.65 \times 10^7 \text{ m/s}) \sin 89^\circ}{(1.60 \times 10^{-19} \text{ C})(0.100 \text{ T})} = 1.51 \times 10^{-3} \text{ m}.$$

LEARN The parallel component of the velocity, $v_{\parallel} = v \cos \phi$, is what determines the pitch of the helix. On the other hand, the perpendicular component, $v_{\perp} = v \sin \phi$, determines the radius of the helix.

34. (a) Equation 3-20 gives $\phi = \cos^{-1}(2/19) = 84^\circ$.

(b) No, the magnetic field can only change the direction of motion of a free (unconstrained) particle, not its speed or its kinetic energy.

(c) No, as reference to Fig. 28-11 should make clear.

(d) We find $v_{\perp} = v \sin \phi = 61.3 \text{ m/s}$, so $r = mv_{\perp}/eB = 5.7 \text{ nm}$.

35. (a) By conservation of energy (using qV for the potential energy, which is converted into kinetic form) the kinetic energy gained in each pass is 200 eV.

(b) Multiplying the part (a) result by $n = 100$ gives $\Delta K = n(200 \text{ eV}) = 20.0 \text{ keV}$.

(c) Combining Eq. 28-16 with the kinetic energy relation ($n(200 \text{ eV}) = m_p v^2/2$ in this particular application) leads to the expression

$$r = \frac{m_p}{eB} \sqrt{\frac{2n(200 \text{ eV})}{m_p}}$$

which shows that r is proportional to \sqrt{n} . Thus, the percent increase defined in the problem in going from $n = 100$ to $n = 101$ is $\sqrt{101/100} - 1 = 0.00499$ or 0.499%.

36. (a) The magnitude of the field required to achieve resonance is

$$B = \frac{2\pi f m_p}{q} = \frac{2\pi(12.0 \times 10^6 \text{ Hz})(1.67 \times 10^{-27} \text{ kg})}{1.60 \times 10^{-19} \text{ C}} = 0.787 \text{ T}.$$

(b) The kinetic energy is given by

$$\begin{aligned} K = \frac{1}{2} m v^2 &= \frac{1}{2} m (2\pi R f)^2 = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg}) 4\pi^2 (0.530 \text{ m})^2 (12.0 \times 10^6 \text{ Hz})^2 \\ &= 1.33 \times 10^{-12} \text{ J} = 8.34 \times 10^6 \text{ eV}. \end{aligned}$$

(c) The required frequency is

$$f = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(1.57 \text{ T})}{2\pi (1.67 \times 10^{-27} \text{ kg})} = 2.39 \times 10^7 \text{ Hz}.$$

(d) The kinetic energy is given by

$$\begin{aligned} K = \frac{1}{2} m v^2 &= \frac{1}{2} m (2\pi R f)^2 = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg}) 4\pi^2 (0.530 \text{ m})^2 (2.39 \times 10^7 \text{ Hz})^2 \\ &= 5.3069 \times 10^{-12} \text{ J} = 3.32 \times 10^7 \text{ eV}. \end{aligned}$$

37. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3 \text{ eV}$. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6 \text{ eV}}{2(80 \times 10^3 \text{ eV})} = 104.$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB} \sqrt{\frac{2K}{m}} = \frac{1}{qB} \sqrt{2Km}.$$

For the average energy

$$r = \frac{\sqrt{2(8.3 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})(3.34 \times 10^{-27} \text{ kg})}}{(1.60 \times 10^{-19} \text{ C})(1.57 \text{ T})} = 0.375 \text{ m}.$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2 \text{ m.}$$

38. (a) Using Eq. 28-23 and Eq. 28-18, we find

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(1.20 \text{ T})}{2\pi(1.67 \times 10^{-27} \text{ kg})} = 1.83 \times 10^7 \text{ Hz.}$$

(b) From $r = m_p v / qB = \sqrt{2m_p k} / qB$ we have

$$K = \frac{(rqB)^2}{2m_p} = \frac{[(0.500 \text{ m})(1.60 \times 10^{-19} \text{ C})(1.20 \text{ T})]^2}{2(1.67 \times 10^{-27} \text{ kg})(1.60 \times 10^{-19} \text{ J/eV})} = 1.72 \times 10^7 \text{ eV.}$$

39. **THINK** The magnetic force on a wire that carries a current i is given by $\vec{F}_B = i\vec{L} \times \vec{B}$, where \vec{L} is the length vector of the wire and \vec{B} is the magnetic field.

EXPRESS The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin \phi$, where ϕ is the angle between the current and the field.

ANALYZE (a) With $\phi = 70^\circ$, we have

$$F_B = (5000 \text{ A})(100 \text{ m})(60.0 \times 10^{-6} \text{ T}) \sin 70^\circ = 28.2 \text{ N.}$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

LEARN From the expression $\vec{F}_B = i\vec{L} \times \vec{B}$, we see that the magnetic force acting on a current-carrying wire is a maximum when \vec{L} is perpendicular to \vec{B} ($\phi = 90^\circ$), and is zero when \vec{L} is parallel to \vec{B} ($\phi = 0^\circ$).

40. The magnetic force on the (straight) wire is

$$F_B = iBL \sin \theta = (13.0 \text{ A})(1.50 \text{ T})(1.80 \text{ m})(\sin 35.0^\circ) = 20.1 \text{ N.}$$

41. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \Rightarrow i = \frac{mg}{LB} = \frac{(0.0130 \text{ kg})(9.8 \text{ m/s}^2)}{(0.620 \text{ m})(0.440 \text{ T})} = 0.467 \text{ A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

42. (a) From symmetry, we conclude that any x -component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the \hat{k} direction produces on each part of the bent wire a y -component of force pointing in the $-\hat{j}$ direction; each of these components has magnitude

$$|F_y| = i\ell |\vec{B}| \sin 30^\circ = (2.0 \text{ A})(2.0 \text{ m})(4.0 \text{ T}) \sin 30^\circ = 8 \text{ N}.$$

Therefore, the force on the wire shown in the figure is $(-16\hat{j}) \text{ N}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

43. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50 \text{ cm}$ side runs along the $+y$ axis, while the $\ell_x = 120 \text{ cm}$ side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is

$$\theta = \tan^{-1}(50/120) = 22.6^\circ,$$

relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is $180^\circ - 22.6^\circ = +157^\circ$. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis). We take \vec{B} to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750 \text{ T}$,

$$B_x = -B \cos \theta = -0.0692 \text{ T}, \quad B_y = B \sin \theta = 0.0288 \text{ T}.$$

(a) Equation 28-26 produces zero force when $\vec{L} \parallel \vec{B}$ so there is no force exerted on the hypotenuse of length 130 cm .

(b) On the 50 cm side, the B_x component produces a force $i\ell_y B_x \hat{k}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00 \text{ A})(0.500 \text{ m})(0.0692 \text{ T}) = 0.138 \text{ N}.$$

(c) On the 120 cm side, the B_y component produces a force $i\ell_x B_y \hat{k}$, and there is no contribution from the B_x component. The magnitude of the force on the ℓ_x side is also

$$(4.00 \text{ A})(1.20 \text{ m})(0.0288 \text{ T}) = 0.138 \text{ N}.$$

(d) The net force is

$$i\ell_y B_x \hat{k} + i\ell_x B_y \hat{k} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed \vec{B} went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$, but $B_y < 0$ and a zero net force would still be the result.

44. Consider an infinitesimal segment of the loop, of length ds . The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB ds$. The horizontal component of the force has magnitude

$$dF_h = (iB \cos \theta) ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_v = (iB \sin \theta) ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$\begin{aligned} F_v &= iB \sin \theta \int ds = 2\pi a i B \sin \theta = 2\pi(0.018 \text{ m})(4.6 \times 10^{-3} \text{ A})(3.4 \times 10^{-3} \text{ T}) \sin 20^\circ \\ &= 6.0 \times 10^{-7} \text{ N}. \end{aligned}$$

We note that i , B , and θ have the same value for every segment and so can be factored from the integral.

45. The magnetic force on the wire is

$$\begin{aligned} \vec{F}_B &= i\vec{L} \times \vec{B} = iL\hat{i} \times (B_y\hat{j} + B_z\hat{k}) = iL(-B_z\hat{j} + B_y\hat{k}) \\ &= (0.500 \text{ A})(0.500 \text{ m}) \left[-(0.0100 \text{ T})\hat{j} + (0.00300 \text{ T})\hat{k} \right] \\ &= (-2.50 \times 10^{-3} \hat{j} + 0.750 \times 10^{-3} \hat{k}) \text{ N}. \end{aligned}$$

46. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13 \times 10^{-3} \text{ A})(2.56 \times 10^{-2} \text{ m})(5.63 \times 10^{-2} \text{ T})(0.0611 \text{ s})}{2.41 \times 10^{-5} \text{ kg}} \\ = 3.34 \times 10^{-2} \text{ m/s}.$$

(b) The direction is to the left (away from the generator).

47. (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: \vec{F} , the force of the magnetic field; mg , the magnitude of the (downward) force of gravity; \vec{F}_N , the normal force exerted by the stationary rails upward on the rod; and \vec{f} , the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that \vec{f} points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, \vec{F} has an eastward component F_x and an upward component F_y , which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component (B_d) of \vec{B} will produce the eastward F_x , and a westward component (B_w) will produce the upward F_y . Specifically,

$$F_x = iLB_d, \quad F_y = iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s (mg - iLB_w).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \Rightarrow iLB_d = \mu_s (mg - iLB_w).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B \sin \theta$ and $B_d = B \cos \theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB \cos \theta = \mu_s (mg - iLB \sin \theta) \Rightarrow B = \frac{\mu_s mg}{iL(\cos \theta + \mu_s \sin \theta)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31^\circ.$$

Consequently,

$$B_{\min} = \frac{0.60(1.0 \text{ kg})(9.8 \text{ m/s}^2)}{(50 \text{ A})(1.0 \text{ m})(\cos 31^\circ + 0.60 \sin 31^\circ)} = 0.10 \text{ T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31^\circ$.

48. We use $d\vec{F}_B = id\vec{L} \times \vec{B}$, where $d\vec{L} = dx\hat{i}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$. Thus,

$$\begin{aligned}\vec{F}_B &= \int id\vec{L} \times \vec{B} = \int_{x_i}^{x_f} idx\hat{i} \times (B_x\hat{i} + B_y\hat{j}) = i \int_{x_i}^{x_f} B_y dx \hat{k} \\ &= (-5.0 \text{ A}) \left(\int_{1.0}^{3.0} (8.0x^2 dx) (\text{m} \cdot \text{mT}) \right) \hat{k} = (-0.35 \text{ N}) \hat{k}.\end{aligned}$$

49. **THINK** Magnetic forces on the loop produce a torque that rotates it about the hinge line. Our applied field has two components: $B_x > 0$ and $B_z > 0$.

EXPRESS Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of \vec{B} which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050 \text{ m}$, which has length $L = 0.10 \text{ m}$ and is shown in Fig. 28-45 carrying current in the $-y$ direction.

Now, the B_z component will produce a force on this straight segment which points in the $-x$ direction (back toward the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B \cos \theta$ where $B = 0.50 \text{ T}$ and $\theta = 30^\circ$) produces a force equal to $F = NiLB_x$ which points (by the right-hand rule) in the $+z$ direction.

ANALYZE Since the action of the force F is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\begin{aligned}\tau &= (NiLB_x)(x) = NiLxB \cos \theta = (20)(0.10 \text{ A})(0.10 \text{ m})(0.050 \text{ m})(0.50 \text{ T}) \cos 30^\circ \\ &= 0.0043 \text{ N} \cdot \text{m}.\end{aligned}$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. In unit-vector notation, the torque is $\vec{\tau} = (-4.3 \times 10^{-3} \text{ N} \cdot \text{m}) \hat{j}$

LEARN An alternative way to do this problem is through the use of Eq. 28-37:

$\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic moment vector is

$$\vec{\mu} = -(NiA) \hat{k} = -(20)(0.10 \text{ A})(0.0050 \text{ m}^2) \hat{k} = -(0.01 \text{ A} \cdot \text{m}^2) \hat{k}.$$

The torque on the loop is

$$\begin{aligned} \vec{\tau} &= \vec{\mu} \times \vec{B} = (-\mu \hat{k}) \times (B \cos \theta \hat{i} + B \sin \theta \hat{k}) = -(\mu B \cos \theta) \hat{j} \\ &= -(0.01 \text{ A} \cdot \text{m}^2)(0.50 \text{ T}) \cos 30^\circ \hat{j} \\ &= (-4.3 \times 10^{-3} \text{ N} \cdot \text{m}) \hat{j}. \end{aligned}$$

50. We use $\tau_{\max} = |\vec{\mu} \times \vec{B}|_{\max} = \mu B = i \pi r^2 B$, and note that $i = qf = qv/2\pi r$. So

$$\begin{aligned} \tau_{\max} &= \left(\frac{qv}{2\pi r} \right) \pi r^2 B = \frac{1}{2} qvrB = \frac{1}{2} (1.60 \times 10^{-19} \text{ C})(2.19 \times 10^6 \text{ m/s})(5.29 \times 10^{-11} \text{ m})(7.10 \times 10^{-3} \text{ T}) \\ &= 6.58 \times 10^{-26} \text{ N} \cdot \text{m}. \end{aligned}$$

51. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and \vec{B} is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg , acting downward from the center of mass, the normal force of the incline F_N , acting perpendicularly to the incline through the center of mass, and the force of friction f , acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg \sin \theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B \sin \theta$, and the force of friction produces a torque with magnitude fr , where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B \sin \theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is

rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250 \text{ kg})(9.8 \text{ m/s}^2)}{2(10.0)(0.100 \text{ m})(0.500 \text{ T})} = 2.45 \text{ A}.$$

52. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is $x = 4 \text{ cm}$ (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of x must be $\frac{1}{4}$ of 8 cm ; that is, $x = 2 \text{ cm}$ when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020 \text{ m})^2 = 0.00040 \text{ m}^2$. Since $N = 1$ and the torque in this case is given as $4.8 \times 10^{-4} \text{ N}\cdot\text{m}$, then the aforementioned equations lead immediately to $i = 0.0030 \text{ A}$.

53. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area that was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by \vec{B} on the n th rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB \sin\theta \Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB \sum_n \Delta A_n = NiAB \sin\theta.$$

54. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B \cos\theta - (-\mu B \cos 0^\circ).$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77^\circ.$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta = 77^\circ$ on the other side of the alignment axis.

55. **THINK** Our system consists of two concentric current-carrying loops. The net magnetic dipole moment is the vector sum of the individual contributions.

EXPRESS The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. Each of the loops is a circle, so the area is $A = \pi r^2$, where r is the radius of the loop.

ANALYZE (a) Since the currents are in the same direction, the magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi (7.00 \text{ A}) \left[(0.200 \text{ m})^2 + (0.300 \text{ m})^2 \right] = 2.86 \text{ A} \cdot \text{m}^2.$$

(b) Now, the two currents flow in the opposite directions, so the magnitude of the magnetic moment vector is

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi (7.00 \text{ A}) \left[(0.300 \text{ m})^2 - (0.200 \text{ m})^2 \right] = 1.10 \text{ A} \cdot \text{m}^2.$$

LEARN In both cases, the directions of the dipole moments are into the page. The direction of $\vec{\mu}$ is that of the normal vector \vec{n} to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19(b).

$$56. (a) \mu = N i A = \pi r^2 i = \pi (0.150 \text{ m})^2 (2.60 \text{ A}) = 0.184 \text{ A} \cdot \text{m}^2.$$

(b) The torque is

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B \sin \theta = (0.184 \text{ A} \cdot \text{m}^2)(12.0 \text{ T}) \sin 41.0^\circ = 1.45 \text{ N} \cdot \text{m}.$$

57. **THINK** Magnetic forces on a current-carrying loop produce a torque that tends to align the magnetic dipole moment with the magnetic field.

EXPRESS The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn.

ANALYZE (a) Thus, the current is

$$i = \frac{\mu}{N \pi r^2} = \frac{2.30 \text{ A} \cdot \text{m}^2}{(160)(\pi)(0.0190 \text{ m})^2} = 12.7 \text{ A}.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{\max} = \mu B = (2.30 \text{ A} \cdot \text{m}^2)(35.0 \times 10^{-3} \text{ T}) = 8.05 \times 10^{-2} \text{ N} \cdot \text{m}.$$

LEARN The torque on the coil can be written as $\vec{\tau} = \vec{\mu} \times \vec{B}$, with $\tau = |\vec{\tau}| = \mu B \sin \theta$, where θ is the angle between $\vec{\mu}$ and \vec{B} . Thus, τ is a maximum when $\theta = 90^\circ$, and zero when $\theta = 0^\circ$.

58. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22} \text{ J/T}}{\pi (3500 \times 10^3 \text{ m})^2} = 2.08 \times 10^9 \text{ A}.$$

59. (a) The area of the loop is $A = \frac{1}{2}(30\text{ cm})(40\text{ cm}) = 6.0 \times 10^2 \text{ cm}^2$, so

$$\mu = iA = (5.0 \text{ A})(6.0 \times 10^{-2} \text{ m}^2) = 0.30 \text{ A} \cdot \text{m}^2.$$

(b) The torque on the loop is

$$\tau = \mu B \sin \theta = (0.30 \text{ A} \cdot \text{m}^2)(80 \times 10^3 \text{ T}) \sin 90^\circ = 2.4 \times 10^{-2} \text{ N} \cdot \text{m}.$$

60. Let $a = 30.0 \text{ cm}$, $b = 20.0 \text{ cm}$, and $c = 10.0 \text{ cm}$. From the given hint, we write

$$\begin{aligned} \vec{\mu} &= \vec{\mu}_1 + \vec{\mu}_2 = iab(-\hat{k}) + iac(\hat{j}) = ia(c\hat{j} - b\hat{k}) = (5.00 \text{ A})(0.300 \text{ m})[(0.100 \text{ m})\hat{j} - (0.200 \text{ m})\hat{k}] \\ &= (0.150\hat{j} - 0.300\hat{k}) \text{ A} \cdot \text{m}^2. \end{aligned}$$

61. **THINK** Magnetic forces on a current-carrying coil produce a torque that tends to align the magnetic dipole moment with the field. The magnetic energy of the dipole depends on its orientation relative to the field.

EXPRESS The magnetic potential energy of the dipole is given by $U = -\vec{\mu} \cdot \vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment of the coil and \vec{B} is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA$, where i is the current in the coil, N is the number of turns, A is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu} \times \vec{B}$.

ANALYZE (a) By using the right-hand rule, we see that $\vec{\mu}$ is in the $-y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{j}) = -(3)(2.00 \text{ A})(4.00 \times 10^{-3} \text{ m}^2)\hat{j} = -(0.0240 \text{ A} \cdot \text{m}^2)\hat{j}.$$

The corresponding magnetic energy is

$$U = -\vec{\mu} \cdot \vec{B} = -\mu_y B_y = -(-0.0240 \text{ A} \cdot \text{m}^2)(-3.00 \times 10^{-3} \text{ T}) = -7.20 \times 10^{-5} \text{ J}.$$

(b) Using the fact that $\hat{j} \cdot \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, and $\hat{j} \times \hat{k} = \hat{i}$, the torque on the coil is

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = \mu_y B_z \hat{i} - \mu_y B_x \hat{k} \\ &= (-0.0240 \text{ A} \cdot \text{m}^2)(-4.00 \times 10^{-3} \text{ T})\hat{i} - (-0.0240 \text{ A} \cdot \text{m}^2)(2.00 \times 10^{-3} \text{ T})\hat{k} \\ &= (9.60 \times 10^{-5} \text{ N} \cdot \text{m})\hat{i} + (4.80 \times 10^{-5} \text{ N} \cdot \text{m})\hat{k}.\end{aligned}$$

LEARN The magnetic energy is highest when $\vec{\mu}$ is in the opposite direction of \vec{B} , and lowest when $\vec{\mu}$ lines up with \vec{B} .

62. Looking at the point in the graph (Fig. 28-51(b)) corresponding to $i_2 = 0$ (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0 \times 10^{-5} \text{ A} \cdot \text{m}^2$. Looking at the point where the line crosses the axis (at $i_2 = 5.0 \text{ mA}$) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0 \times 10^{-5} \text{ A} \cdot \text{m}^2$ when $i_2 = 0.0050 \text{ A}$, which means (Eq. 28-35)

$$N_2 A_2 = \frac{\mu_2}{i_2} = \frac{2.0 \times 10^{-5} \text{ A} \cdot \text{m}^2}{0.0050 \text{ A}} = 4.0 \times 10^{-3} \text{ m}^2.$$

Now the problem has us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions, from coil 1 and coil 2, specifically for the case that $i_2 = 0.007 \text{ A}$. We find that total moment is

$$\mu = (2.0 \times 10^{-5} \text{ A} \cdot \text{m}^2) + (N_2 A_2 i_2) = 4.8 \times 10^{-5} \text{ A} \cdot \text{m}^2.$$

63. The magnetic dipole moment is $\vec{\mu} = \mu(0.60\hat{i} - 0.80\hat{j})$, where

$$\mu = NiA = Ni\pi r^2 = 1(0.20 \text{ A})\pi(0.080 \text{ m})^2 = 4.02 \times 10^{-4} \text{ A} \cdot \text{m}^2.$$

Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = \mu(0.60\hat{i} - 0.80\hat{j}) \times (0.25\hat{i} + 0.30\hat{k}) \\ &= \mu(0.60)(0.30)(\hat{i} \times \hat{k}) - (0.80)(0.25)(\hat{j} \times \hat{i}) - (0.80)(0.30)(\hat{j} \times \hat{k}) \\ &= \mu(-0.18\hat{j} + 0.20\hat{k} - 0.24\hat{i}).\end{aligned}$$

Here $\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{i} = -\hat{k}$, and $\hat{j} \times \hat{k} = \hat{i}$ are used. We also use $\hat{i} \times \hat{i} = 0$. Now, we substitute the value for μ to obtain

$$\vec{\tau} = (-9.7 \times 10^{-4} \hat{i} - 7.2 \times 10^{-4} \hat{j} + 8.0 \times 10^{-4} \hat{k}) \text{ N} \cdot \text{m}.$$

(b) The orientation energy of the dipole is given by

$$U = -\vec{\mu} \cdot \vec{B} = -\mu(0.60\hat{i} - 0.80\hat{j}) \cdot (0.25\hat{i} + 0.30\hat{k}) = -\mu(0.60)(0.25) = -0.15\mu = -6.0 \times 10^{-4} \text{ J}.$$

Here $\hat{i} \cdot \hat{i} = 1$, $\hat{i} \cdot \hat{k} = 0$, $\hat{j} \cdot \hat{i} = 0$, and $\hat{j} \cdot \hat{k} = 0$ are used.

64. Eq. 28-39 gives $U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi$, so at $\phi = 0$ (corresponding to the lowest point on the graph in Fig. 28-52) the mechanical energy is

$$K + U = K_0 + (-\mu B) = 6.7 \times 10^{-4} \text{ J} + (-5 \times 10^{-4} \text{ J}) = 1.7 \times 10^{-4} \text{ J}.$$

The turning point occurs where $K = 0$, which implies $U_{\text{turn}} = 1.7 \times 10^{-4} \text{ J}$. So the angle where this takes place is given by

$$\phi = -\cos^{-1}\left(\frac{1.7 \times 10^{-4} \text{ J}}{\mu B}\right) = 110^\circ$$

where we have used the fact (see above) that $\mu B = 5 \times 10^{-4} \text{ J}$.

65. **THINK** The torque on a current-carrying coil is a maximum when its dipole moment is perpendicular to the magnetic field.

EXPRESS The magnitude of the torque on the coil is given by $\tau = |\vec{\tau}| = \mu B \sin \theta$, where θ is the angle between $\vec{\mu}$ and \vec{B} . The magnitude of $\vec{\mu}$ is $\mu = NiA$, where i is the current in the coil, N is the number of turns, A is the area of the coil. Thus, if N closed loops are formed from the wire of length L , the circumference of each loop is L/N , the radius of each loop is $R = L/2\pi N$, and the area of each loop is

$$A = \pi R^2 = \pi(L/2\pi N)^2 = L^2/4\pi N^2.$$

ANALYZE (a) For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular (i.e., at a 90° angle) to the field.

(b) The magnitude of the torque is then

$$\tau = NiAB = (Ni)\left(\frac{L^2}{4\pi N^2}\right)B = \frac{iL^2 B}{4\pi N}.$$

To maximize the torque, we take the number of turns N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

(c) The magnitude of the maximum torque is

$$\tau = \frac{iL^2B}{4\pi} = \frac{(4.51 \times 10^{-3} \text{ A})(0.250 \text{ m})^2(5.71 \times 10^{-3} \text{ T})}{4\pi} = 1.28 \times 10^{-7} \text{ N} \cdot \text{m}.$$

LEARN The torque tends to align $\vec{\mu}$ with \vec{B} . The magnitude of the torque is a maximum when the angle between $\vec{\mu}$ and \vec{B} is $\theta = 90^\circ$, and is zero when $\theta = 0^\circ$.

66. The equation of motion for the proton is

$$\begin{aligned}\vec{F} &= q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \times B\hat{i} = qB(v_z\hat{j} - v_y\hat{k}) \\ &= m_p\vec{a} = m_p \left[\left(\frac{dv_x}{dt} \right) \hat{i} + \left(\frac{dv_y}{dt} \right) \hat{j} + \left(\frac{dv_z}{dt} \right) \hat{k} \right].\end{aligned}$$

Thus,

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = \omega v_z, \quad \frac{dv_z}{dt} = -\omega v_y,$$

where $\omega = eB/m$. The solution is $v_x = v_{0x}$, $v_y = v_{0y} \cos \omega t$, and $v_z = -v_{0y} \sin \omega t$. In summary, we have

$$\vec{v}(t) = v_{0x}\hat{i} + v_{0y} \cos(\omega t)\hat{j} - v_{0y}(\sin \omega t)\hat{k}.$$

67. (a) We use $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since \vec{B} points toward the one-hour (or “5-minute”) mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\begin{aligned}\tau &= |\vec{\mu} \times \vec{B}| = \mu B \sin 90^\circ = NiAB = \pi N i r^2 B = 6\pi (2.0 \text{ A})(0.15 \text{ m})^2 (70 \times 10^{-3} \text{ T}) \\ &= 5.9 \times 10^{-2} \text{ N} \cdot \text{m}.\end{aligned}$$

68. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{j}$. The (infinitesimal) force on this element is

$$d\vec{F} = i d\ell (-\hat{j}) \times (0.3y\hat{i} + 0.4y\hat{j})$$

with SI units (and 3 significant figures) understood. Since $\hat{j} \times \hat{i} = -\hat{k}$ and $\hat{j} \times \hat{j} = 0$, we obtain

$$d\vec{F} = 0.3iy \, d\ell \, \hat{k} = (6.00 \times 10^{-4} \text{ N/m}^2) y \, d\ell \, \hat{k}.$$

We integrate the force element found above, using the symbol ξ to stand for the coefficient $6.00 \times 10^{-4} \text{ N/m}^2$, and obtain

$$\vec{F} = \int d\vec{F} = \xi \hat{k} \int_0^{0.25} y \, dy = \xi \hat{k} \left(\frac{0.25^2}{2} \right) = (1.88 \times 10^{-5} \text{ N}) \hat{k}.$$

69. From $m = B^2 q x^2 / 8V$ we have $\Delta m = (B^2 q / 8V)(2x \Delta x)$. Here $x = \sqrt{8Vm / B^2 q}$, which we substitute into the expression for Δm to obtain

$$\Delta m = \left(\frac{B^2 q}{8V} \right) 2 \sqrt{\frac{8Vm}{B^2 q}} \Delta x = B \sqrt{\frac{mq}{2V}} \Delta x.$$

Thus, the distance between the spots made on the photographic plate is

$$\begin{aligned} \Delta x &= \frac{\Delta m}{B} \sqrt{\frac{2V}{mq}} = \frac{(37 \text{ u} - 35 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})}{0.50 \text{ T}} \sqrt{\frac{2(7.3 \times 10^3 \text{ V})}{(36 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})(1.60 \times 10^{-19} \text{ C})}} \\ &= 8.2 \times 10^{-3} \text{ m}. \end{aligned}$$

70. (a) Equating the magnitude of the electric force ($F_e = eE$) with that of the magnetic force (Eq. 28-3), we obtain $B = E / v \sin \phi$. The field is smallest when the $\sin \phi$ factor is at its largest value; that is, when $\phi = 90^\circ$. Now, we use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.5 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 2.96 \times 10^7 \text{ m/s}.$$

Thus,

$$B = \frac{E}{v} = \frac{10 \times 10^3 \text{ V/m}}{2.96 \times 10^7 \text{ m/s}} = 3.4 \times 10^{-4} \text{ T}.$$

The direction of the magnetic field must be perpendicular to both the electric field ($-\hat{j}$) and the velocity of the electron ($+\hat{i}$). Since the electric force $\vec{F}_e = (-e)\vec{E}$ points in the $+\hat{j}$ direction, the magnetic force $\vec{F}_b = (-e)\vec{v} \times \vec{B}$ points in the $-\hat{j}$ direction. Hence, the direction of the magnetic field is $-\hat{k}$. In unit-vector notation, $\vec{B} = (-3.4 \times 10^{-4} \text{ T})\hat{k}$.

71. The period of revolution for the iodine ion is

$$T = 2\pi r/v = 2\pi m/Bq,$$

which gives

$$m = \frac{BqT}{2\pi} = \frac{(45.0 \times 10^{-3} \text{ T})(1.60 \times 10^{-19} \text{ C})(1.29 \times 10^{-3} \text{ s})}{(7)(2\pi)(1.66 \times 10^{-27} \text{ kg/u})} = 127 \text{ u}.$$

72. (a) For the magnetic field to have an effect on the moving electrons, we need a non-negligible component of \vec{B} to be perpendicular to \vec{v} (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 28-16. The kinetic energy of the electron is $K = \frac{1}{2} m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB} \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2 B^2}}.$$

This must be less than d , so $\sqrt{\frac{2m_e K}{e^2 B^2}} \leq d$, or $B \geq \sqrt{\frac{2m_e K}{e^2 d^2}}$.

(b) If the electrons are to travel as shown in Fig. 28-53, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

73. **THINK** The electron moving in the Earth's magnetic field is being accelerated by the magnetic force acting on it.

EXPRESS Since the electron is moving in a line that is parallel to the horizontal component of the Earth's magnetic field, the magnetic force on the electron is due to the vertical component of the field only. The magnitude of the force acting on the electron is given by $F = evB$, where B represents the downward component of Earth's field. With $F = m_e a$, the acceleration of the electron is $a = evB/m_e$.

ANALYZE (a) The electron speed can be found from its kinetic energy $K = \frac{1}{2} m_e v^2$:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(12.0 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 6.49 \times 10^7 \text{ m/s}.$$

Therefore,

$$a = \frac{evB}{m_e} = \frac{(1.60 \times 10^{-19} \text{ C}) (6.49 \times 10^7 \text{ m/s}) (55.0 \times 10^{-6} \text{ T})}{9.11 \times 10^{-31} \text{ kg}}$$

$$= 6.27 \times 10^{14} \text{ m/s}^2 \approx 6.3 \times 10^{14} \text{ m/s}^2.$$

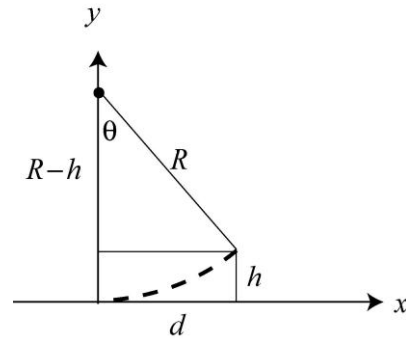
(b) We ignore any vertical deflection of the beam that might arise due to the horizontal component of Earth's field. Then, the path of the electron is a circular arc. The radius of the path is given by $a = v^2 / R$, or

$$R = \frac{v^2}{a} = \frac{(6.49 \times 10^7 \text{ m/s})^2}{6.27 \times 10^{14} \text{ m/s}^2} = 6.72 \text{ m}.$$

The dashed curve shown represents the path. Let the deflection be h after the electron has traveled a distance d along the x axis. With $d = R \sin \theta$, we have

$$h = R(1 - \cos \theta) = R(1 - \sqrt{1 - \sin^2 \theta})$$

$$= R(1 - \sqrt{1 - (d/R)^2}).$$



Substituting $R = 6.72 \text{ m}$ and $d = 0.20 \text{ m}$ into the expression, we obtain $h = 0.0030 \text{ m}$.

LEARN The deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$d = vt \Rightarrow t = \frac{d}{v} = \frac{0.200 \text{ m}}{6.49 \times 10^7 \text{ m/s}} = 3.08 \times 10^{-9} \text{ s}.$$

Then, with our y axis oriented eastward,

$$h = \frac{1}{2} at^2 = \frac{1}{2} (6.27 \times 10^{14}) (3.08 \times 10^{-9})^2 = 0.00298 \text{ m} \approx 0.0030 \text{ m}.$$

74. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 28-2 ($\vec{F} = q\vec{v} \times \vec{B}$) and Eq. 3-30, we obtain (with SI units understood)

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 2((4B_2 - 6B_1)\hat{i} + (6B_1 - 2B_2)\hat{j} + (2B_1 - 4B_1)\hat{k}).$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary,

$$\vec{B} = (-3.0\hat{i} - 3.0\hat{j} - 4.0\hat{k}) \text{ T}.$$

75. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $r \propto \sqrt{mK}/qB$.

$$(a) \frac{r_d}{r_p} = \sqrt{\frac{m_d K_d}{m_p K_p}} \frac{q_p}{q_d} = \sqrt{\frac{2.0\text{u}}{1.0\text{u}}} \frac{e}{e} = \sqrt{2} \approx 1.4, \text{ and}$$

$$(b) \frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}} \frac{q_p}{q_\alpha} = \sqrt{\frac{4.0\text{u}}{1.0\text{u}}} \frac{e}{2e} = 1.0.$$

76. Using Eq. 28-16, the charge-to-mass ratio is $\frac{q}{m} = \frac{v}{B'r}$. With the speed of the ion given by $v = E/B$ (using Eq. 28-7), the expression becomes

$$\frac{q}{m} = \frac{E/B}{B'r} = \frac{E}{BB'r}.$$

77. **THINK** Since both electric and magnetic fields are present, the net force on the electron is the vector sum of the electric force and the magnetic force.

EXPRESS The force on the electron is given by $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$, where \vec{E} is the electric field, \vec{B} is the magnetic field, and \vec{v} is the velocity of the electron. The fact that the fields are uniform with the feature that the charge moves in a straight line, implies that the speed is constant. Thus, the net force must vanish.

ANALYZE The condition $\vec{F} = 0$ implies that

$$E = vB = 500 \text{ V/m}.$$

Its direction (so that $\vec{F} = 0$) is downward, or $-\hat{j}$, in the “page” coordinates. In unit-vector notation, $\vec{E} = (-500 \text{ V/m})\hat{j}$

LEARN Electron moves in a straight line only when the condition $E = vB$ is met. In many experiments, a velocity selector can be set up so that only electrons with a speed given by $v = E/B$ can pass through.

78. (a) In Chapter 27, the electric field (called E_C in this problem) that “drives” the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho n e v_d.$$

Now, regarding the Hall effect, we use Eq. 28-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_C = B/ne\rho$.

(b) Using the value of copper’s resistivity given in Chapter 26, we obtain

$$\frac{E}{E_C} = \frac{B}{ne\rho} = \frac{0.65 \text{ T}}{(8.47 \times 10^{28} / \text{m}^3)(1.60 \times 10^{-19} \text{ C})(1.69 \times 10^{-8} \Omega \cdot \text{m})} = 2.84 \times 10^{-3}.$$

79. **THINK** We have charged particles that are accelerated through an electric potential difference, and then moved through a region of uniform magnetic field. Energy is conserved in the process.

EXPRESS The kinetic energy of a particle is given by $K = qV$, where q is the particle’s charge and V is the potential difference. With $K = mv^2/2$, the speed of the particle is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2qV}{m}}.$$

In the region with uniform magnetic field, the magnetic force on a particle of charge q is qvB , which according to Newton’s second law, is equal to mv^2/r , where r is the radius of the orbit. Thus, we have

$$r = \frac{mv}{qB} = \frac{m}{qB} \sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB}.$$

ANALYZE (a) Since $K = qV$ we have $K_p = \frac{1}{2} K_\alpha$ (as $q_\alpha = 2q_p$), or $K_p / K_\alpha = 0.50$.

(b) Similarly, $q_\alpha = 2q_p$, $K_d / K_\alpha = 0.50$.

(c) Since $r \propto \sqrt{mK}/q$, we have

$$r_d = \sqrt{\frac{m_d K_d}{m_p K_p}} \frac{q_p}{q_d} r_p = \sqrt{\frac{(2.00 \text{ u}) K_p}{(1.00 \text{ u}) K_p}} r_p = 10\sqrt{2} \text{ cm} = 14 \text{ cm}.$$

(d) Similarly, for the alpha particle, we have

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p} \frac{q_p}{q_\alpha}} r_p = \sqrt{\frac{(4.00\text{u}) K_\alpha}{(1.00\text{u}) (K_\alpha/2)}} \frac{e}{2e} r_p = 10\sqrt{2} \text{ cm} = 14 \text{ cm}.$$

LEARN The radius of the particle's path, given by $r = \sqrt{2mK} / qB$, depends on its mass, kinetic energy, and charge, in addition to the field strength.

80. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 28-3,

$$F_{B,\max} = |q| vB \sin(90^\circ) = evB = (1.60 \times 10^{-19} \text{ C})(7.20 \times 10^6 \text{ m/s})(83.0 \times 10^{-3} \text{ T}) \\ = 9.56 \times 10^{-14} \text{ N}.$$

(b) The smallest value occurs if they are parallel: $F_{B,\min} = |q| vB \sin(0) = 0$.

(c) By Newton's second law, $a = F_B/m_e = |q| vB \sin \theta / m_e$, so the angle θ between \vec{v} and \vec{B} is

$$\theta = \sin^{-1} \left(\frac{m_e a}{|q| vB} \right) = \sin^{-1} \left[\frac{(9.11 \times 10^{-31} \text{ kg})(4.90 \times 10^{14} \text{ m/s}^2)}{(1.60 \times 10^{-16} \text{ C})(7.20 \times 10^6 \text{ m/s})(83.0 \times 10^{-3} \text{ T})} \right] = 0.267^\circ.$$

81. The contribution to the force by the magnetic field ($\vec{B} = B_x \hat{i} = (-0.020 \text{ T}) \hat{i}$) is given by Eq. 28-2:

$$\vec{F}_B = q\vec{v} \times \vec{B} = q \left((17000\hat{i} \times B_x \hat{i}) + (-11000\hat{j} \times B_x \hat{i}) + (7000\hat{k} \times B_x \hat{i}) \right) \\ = q(-220\hat{k} - 140\hat{j})$$

in SI units. And the contribution to the force by the electric field ($\vec{E} = E_y \hat{j} = 300\hat{j} \text{ V/m}$) is given by Eq. 23-1: $\vec{F}_E = qE_y \hat{j}$. Using $q = 5.0 \times 10^{-6} \text{ C}$, the net force on the particle is

$$\vec{F} = (0.00080\hat{j} - 0.0011\hat{k}) \text{ N}.$$

82. (a) We use Eq. 28-10: $v_d = E/B = (10 \times 10^{-6} \text{ V}/1.0 \times 10^{-2} \text{ m})/(1.5 \text{ T}) = 6.7 \times 10^{-4} \text{ m/s}$.

(b) We rewrite Eq. 28-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

where we use $A = \ell d$. In this experiment, $A = (0.010 \text{ m})(10 \times 10^{-6} \text{ m}) = 1.0 \times 10^{-7} \text{ m}^2$. By Eq. 28-10, v_d equals the ratio of the fields (as noted in part (a)), so we are led to

$$n = \frac{Bi}{E A e} = \frac{i}{v_d A e} = \frac{3.0 \text{ A}}{(6.7 \times 10^{-4} \text{ m/s})(1.0 \times 10^{-7} \text{ m}^2)(1.6 \times 10^{-19} \text{ C})} = 2.8 \times 10^{29} / \text{m}^3.$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north*, *south*, *east*, *west* and vertical *up* and *down* directions. We assume \vec{B} points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage that becomes established).

83. **THINK** The force on the charged particle is given by $\vec{F} = q\vec{v} \times \vec{B}$, where q is the charge, \vec{B} is the magnetic field, and \vec{v} is the velocity of the electron.

EXPRESS We write $\vec{B} = B\hat{i}$ and take the velocity of the particle to be $\vec{v} = v_x\hat{i} + v_y\hat{j}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_y\hat{j}) \times (B\hat{i}) = -qv_y B\hat{k}.$$

For the force to point along $+\hat{k}$, we must have $q < 0$.

ANALYZE The charge of the particle is

$$q = -\frac{F}{v_y B} = -\frac{0.48 \text{ N}}{(4.0 \times 10^3 \text{ m/s})(\sin 37^\circ)(0.0050 \text{ T})} = -4.0 \times 10^{-2} \text{ C}.$$

LEARN The component of the velocity, v_x , being parallel to the magnetic field, does not contribute to the magnetic force \vec{F} ; only v_y , the component of \vec{v} that is perpendicular to \vec{B} , contributes to \vec{F} .

84. The current is in the $+\hat{i}$ direction. Thus, the \hat{i} component of \vec{B} has no effect, and (with x in meters) we evaluate

$$\vec{F} = (3.00 \text{ A}) \int_0^1 (-0.600 \text{ T/m}^2) x^2 dx (\hat{i} \times \hat{j}) = \left(-1.80 \frac{1^3}{3} \text{ A} \cdot \text{T} \cdot \text{m} \right) \hat{k} = (-0.600 \text{ N}) \hat{k}.$$

85. (a) We use Eq. 28-2 and Eq. 3-30:

$$\begin{aligned}
\vec{F} &= q\vec{v} \times \vec{B} = (+e) \left((v_y B_z - v_z B_y) \hat{i} + (v_z B_x - v_x B_z) \hat{j} + (v_x B_y - v_y B_x) \hat{k} \right) \\
&= (1.60 \times 10^{-19}) \left(((4)(0.008) - (-6)(-0.004)) \hat{i} + \right. \\
&\quad \left((-6)(0.002) - (-2)(0.008) \right) \hat{j} + \left((-2)(-0.004) - (4)(0.002) \right) \hat{k} \right) \\
&= (1.28 \times 10^{-21}) \hat{i} + (6.41 \times 10^{-22}) \hat{j}
\end{aligned}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v} \perp \vec{F}$. This is easily verified by taking the dot (scalar) product of \vec{v} with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between \vec{B} and \vec{v} is presumably “close” to 180° . Here, we use Eq. 3-20:

$$\theta = \cos^{-1} \left(\frac{\vec{v} \cdot \vec{B}}{|\vec{v}| |\vec{B}|} \right) = \cos^{-1} \left(\frac{-68}{\sqrt{56} \sqrt{84}} \right) = 173^\circ.$$

86. (a) We are given $\vec{B} = B_x \hat{i} = (6 \times 10^{-5} \text{ T}) \hat{i}$, so that $\vec{v} \times \vec{B} = -v_y B_x \hat{k}$ where $v_y = 4 \times 10^4 \text{ m/s}$.

We note that the magnetic force on the electron is $(-e)(-v_y B_x \hat{k})$ and therefore points in the $+\hat{k}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 28-16 becomes

$$r = \frac{m_e v_y}{e B_x} = 0.0038 \text{ m}.$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60 \mu\text{s}$, and during that time the “drift” of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_x T = 0.019 \text{ m}$ where $v_x = 32 \times 10^3 \text{ m/s}$.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call “upward”) but “pushed” in the $+z$ direction (to his right). Hence, he describes the electron’s spiral as clockwise.

87. (a) The magnetic force on the electrons is given by $\vec{F} = q\vec{v} \times \vec{B}$. Since the field \vec{B} points to the left, and an electron (with $q = -e$) is forced to rotate clockwise (out of the page at the top of the rotor), using the right-hand-rule, the direction of the magnetic force is up the figure.

(b) The magnitude of the magnetic force can be written as $F = evB = e\omega rB$, where ω is the angular velocity and r is the distance from the axis. Since $F \sim r$, the force is greater near the rim.

(c) The work per unit charge done by the force in moving the charge along the radial line from the center to the rim, or the voltage, is

$$\begin{aligned} V = \frac{W}{e} &= \frac{1}{e} \int_0^R e\omega Br dr = \frac{1}{2} \omega BR^2 = \frac{1}{2} (2\pi f) BR^2 = \pi f BR^2 \\ &= \pi (4000 \text{ /s}) (60 \times 10^{-3} \text{ T}) (0.250 \text{ m})^2 = 47.1 \text{ V}. \end{aligned}$$

(d) The emf of the device is simply equal to the voltage calculated in part (c): $\mathcal{E} = 47.1 \text{ V}$.

(e) The power produced is $P = iV = (50.0 \text{ A})(47.1 \text{ V}) = 2.36 \times 10^3 \text{ W}$.

88. The magnetic force exerted on the U-shaped wire is given by $F = iLB$. Using the impulse-momentum theorem, we have

$$\Delta p = m\Delta v = \int F dt = \int iLB dt = LB \int i dt = LBq,$$

where q is the charge in the pulse. Since the wire is initially at rest, the speed at which the wire jumps is $v = LBq/m$. On the other hand, energy conservation gives $\frac{1}{2}mv^2 = mgh$.

Combining the above expressions leads to

$$h = \frac{v^2}{2g} = \frac{1}{2g} \left(\frac{LBq}{m} \right)^2$$

Solving for q , we find

$$q = \frac{m\sqrt{2gh}}{LB} = \frac{(0.0100 \text{ kg})\sqrt{2(9.80 \text{ m/s}^2)(3.00 \text{ m})}}{(0.200 \text{ m})(0.100 \text{ T})} = 3.83 \text{ C}.$$

89. Just before striking the plate, the electric force on the electron is $F_E = eE = eV/d$, in the upward direction. Since the kinetic energy of the electron is $K = \frac{1}{2}mv^2 = eV$, $v = \sqrt{2eV/m}$. On the other hand, the magnetic force is

$$F_B = evB = eB\sqrt{\frac{2eV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B > F_E$, or

$$eB\sqrt{\frac{2eV}{m}} > \frac{eV}{d} \Rightarrow B > \frac{V}{d}\sqrt{\frac{m}{2eV}} = \sqrt{\frac{mV}{2ed^2}}$$

90. The average current in the loop is $i = \frac{q}{T} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$ and its magnetic dipole moment is

$$\mu = iA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{1}{2} qvr.$$

With $\vec{\tau} = \vec{\mu} \times \vec{B}$, we find the maximum torque exerted on the loop by a uniform magnetic field to be

$$\tau_{\max} = \mu B = \frac{1}{2} qvrB.$$

91. When the electric and magnetic forces are in balance, $eE = ev_d B$, where v_d is the drift speed of the electrons. In addition, since the current density is $J = nev_d$, we solve for n and find

$$n = \frac{J}{ev_d} = \frac{J}{e(E/B)} = \frac{JB}{eE}.$$

92. With $F_z = v_z = B_x = 0$, Eq. 28-2 (and Eq. 3-30) gives

$$F_x \hat{i} + F_y \hat{j} = q (v_y B_z \hat{i} - v_x B_z \hat{j} + v_x B_y \hat{k})$$

where $q = -e$ for the electron. The last term immediately implies $B_y = 0$, and either of the other two terms (along with the values stated in the problem, bearing in mind that “fN” means femto-newtons or 10^{-15} N) can be used to solve for B_z :

$$B_z = \frac{F_x}{-ev_y} = \frac{-4.2 \times 10^{-15} \text{ N}}{-(1.6 \times 10^{-19} \text{ C})(35,000 \text{ m/s})} = 0.75 \text{ T}.$$

We therefore find that the magnetic field is given by $\vec{B} = (0.75 \text{ T})\hat{k}$.