

## Chapter 19

1. Each atom has a mass of  $m = M/N_A$ , where  $M$  is the molar mass and  $N_A$  is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or  $74.9 \times 10^{-3}$  kg/mol. Therefore,  $7.50 \times 10^{24}$  arsenic atoms have a total mass of

$$(7.50 \times 10^{24}) (74.9 \times 10^{-3} \text{ kg/mol}) / (6.02 \times 10^{23} \text{ mol}^{-1}) = 0.933 \text{ kg}.$$

2. (a) Equation 19-3 yields  $n = M_{\text{sam}}/M = 2.5/197 = 0.0127$  mol.

(b) The number of atoms is found from Eq. 19-2:

$$N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}.$$

3. **THINK** We treat the oxygen gas in this problem as ideal and apply the ideal-gas law.

**EXPRESS** In solving the ideal-gas law equation  $pV = nRT$  for  $n$ , we first convert the temperature to the Kelvin scale:  $T_i = (40.0 + 273.15) \text{ K} = 313.15 \text{ K}$ , and the volume to SI units:  $V_i = 1000 \text{ cm}^3 = 10^{-3} \text{ m}^3$ .

**ANALYZE** (a) The number of moles of oxygen present is

$$n = \frac{pV_i}{RT_i} = \frac{(1.01 \times 10^5 \text{ Pa})(1.000 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(313.15 \text{ K})} = 3.88 \times 10^{-2} \text{ mol}.$$

(b) Similarly, the ideal gas law  $pV = nRT$  leads to

$$T_f = \frac{pV_f}{nR} = \frac{(1.06 \times 10^5 \text{ Pa})(1.500 \times 10^{-3} \text{ m}^3)}{(3.88 \times 10^{-2} \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = 493 \text{ K}.$$

We note that the final temperature may be expressed in degrees Celsius as  $220^\circ\text{C}$ .

**LEARN** The final temperature can also be calculated by noting that  $\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$ , or

$$T_f = \left( \frac{p_f}{p_i} \right) \left( \frac{V_f}{V_i} \right) T_i = \left( \frac{1.06 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right) \left( \frac{1500 \text{ cm}^3}{1000 \text{ cm}^3} \right) (313.15 \text{ K}) = 493 \text{ K}.$$

4. (a) With  $T = 283 \text{ K}$ , we obtain

$$n = \frac{pV}{RT} = \frac{100 \times 10^3 \text{ Pa} \cdot 2.50 \text{ m}^3}{8.31 \text{ J/mol} \cdot \text{K} \cdot 283 \text{ K}} = 106 \text{ mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as:

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

(where  $n_i = n_f$  and thus cancels out), which yields a final volume of

$$V_f = V_i \left( \frac{p_i}{p_f} \right) \left( \frac{T_f}{T_i} \right) = (2.50 \text{ m}^3) \left( \frac{100 \text{ kPa}}{300 \text{ kPa}} \right) \left( \frac{303 \text{ K}}{283 \text{ K}} \right) = 0.892 \text{ m}^3.$$

5. With  $V = 1.0 \times 10^{-6} \text{ m}^3$ ,  $p = 1.01 \times 10^{-13} \text{ Pa}$ , and  $T = 293 \text{ K}$ , the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{(1.01 \times 10^{-13} \text{ Pa})(1.0 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 4.1 \times 10^{-23} \text{ mole}.$$

Consequently, Eq. 19-2 yields  $N = nN_A = 25$  molecules. We can express this as a ratio (with  $V$  now written as  $1 \text{ cm}^3$ )  $N/V = 25 \text{ molecules/cm}^3$ .

6. The initial and final temperatures are  $T_i = 5.00^\circ\text{C} = 278 \text{ K}$  and  $T_f = 75.0^\circ\text{C} = 348 \text{ K}$ , respectively. Using the ideal gas law with  $V_i = V_f$ , we find the final pressure to be

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \Rightarrow p_f = \frac{T_f}{T_i} p_i = \left( \frac{348 \text{ K}}{278 \text{ K}} \right) (1.00 \text{ atm}) = 1.25 \text{ atm}.$$

7. (a) Equation 19-45 (which gives 0) implies  $Q = W$ . Then Eq. 19-14, with  $T = (273 + 30.0) \text{ K}$  leads to gives  $Q = -3.14 \times 10^3 \text{ J}$ , or  $|Q| = 3.14 \times 10^3 \text{ J}$ .

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

8. (a) We solve the ideal gas law  $pV = nRT$  for  $n$ :

$$n = \frac{pV}{RT} = \frac{(100 \text{ Pa})(1.0 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(220 \text{ K})} = 5.47 \times 10^{-8} \text{ mol}.$$

(b) Using Eq. 19-2, the number of molecules  $N$  is

$$N = nN_A = (5.47 \times 10^{-6} \text{ mol}) (6.02 \times 10^{23} \text{ mol}^{-1}) = 3.29 \times 10^{16} \text{ molecules.}$$

9. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is  $p_i = 266 \text{ kPa}$ . Setting up the gas law in ratio form (where  $n_i = n_f$  and thus cancels out), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

which yields

$$p_f = p_i \left( \frac{V_i}{V_f} \right) \left( \frac{T_f}{T_i} \right) = (266 \text{ kPa}) \left( \frac{1.64 \times 10^{-2} \text{ m}^3}{1.67 \times 10^{-2} \text{ m}^3} \right) \left( \frac{300 \text{ K}}{273 \text{ K}} \right) = 287 \text{ kPa.}$$

Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

10. The pressure  $p_1$  due to the first gas is  $p_1 = n_1 RT/V$ , and the pressure  $p_2$  due to the second gas is  $p_2 = n_2 RT/V$ . So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} = (n_1 + n_2) \frac{RT}{V}.$$

The fraction of  $P$  due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2 RT/V}{(n_1 + n_2)(RT/V)} = \frac{n_2}{n_1 + n_2} = \frac{0.5}{2 + 0.5} = 0.2.$$

11. **THINK** The process consists of two steps: isothermal expansion, followed by isobaric (constant-pressure) compression. The total work done by the air is the sum of the works done for the two steps.

**EXPRESS** Suppose the gas expands from volume  $V_i$  to volume  $V_f$  during the isothermal portion of the process. The work it does is

$$W_1 = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law  $pV = nRT$  was used to replace  $p$  with  $nRT/V$ . Now  $V_i = nRT/p_i$  and  $V_f = nRT/p_f$ , so  $V_f/V_i = p_i/p_f$ . Also replace  $nRT$  with  $p_i V_i$  to obtain

$$W_1 = p_i V_i \ln \frac{p_i}{p_f}.$$

During the constant-pressure portion of the process the work done by the gas is  $W_2 = p_f(V_i - V_f)$ . The gas starts in a state with pressure  $p_f$ , so this is the pressure throughout this portion of the process. We also note that the volume decreases from  $V_f$  to  $V_i$ . Now  $V_f = p_i V_i / p_f$ , so

$$W_2 = p_f \left( V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i) V_i.$$

**ANALYZE** For the first portion, since the initial gauge pressure is  $1.03 \times 10^5$  Pa,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure:  $p_f = 1.013 \times 10^5$  Pa. Thus,

$$W_1 = (2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) \ln \left( \frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) = 2.00 \times 10^4 \text{ J}.$$

Similarly, for the second portion, we have

$$W_2 = (p_f - p_i) V_i = (1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44 \times 10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = W_1 + W_2 = 2.00 \times 10^4 \text{ J} + (-1.44 \times 10^4 \text{ J}) = 5.60 \times 10^3 \text{ J}.$$

**LEARN** The work done by the gas is positive when it expands, and negative when it contracts.

12. (a) At the surface, the air volume is

$$V_1 = Ah = \pi(1.00 \text{ m})^2(4.00 \text{ m}) = 12.57 \text{ m}^3 \approx 12.6 \text{ m}^3.$$

(b) The temperature and pressure of the air inside the submarine at the surface are  $T_1 = 20^\circ\text{C} = 293 \text{ K}$  and  $p_1 = p_0 = 1.00 \text{ atm}$ . On the other hand, at depth  $h = 80 \text{ m}$ , we have  $T_2 = -30^\circ\text{C} = 243 \text{ K}$  and

$$\begin{aligned} p_2 &= p_0 + \rho gh = 1.00 \text{ atm} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(80.0 \text{ m}) \frac{1.00 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \\ &= 1.00 \text{ atm} + 7.95 \text{ atm} = 8.95 \text{ atm}. \end{aligned}$$

Therefore, using the ideal gas law,  $pV = NkT$ , the air volume at this depth would be

$$\frac{p_1 V_1}{p_2 V_2} = \frac{T_1}{T_2} \Rightarrow V_2 = \left( \frac{p_1}{p_2} \right) \left( \frac{T_2}{T_1} \right) V_1 = \left( \frac{1.00 \text{ atm}}{8.95 \text{ atm}} \right) \left( \frac{243 \text{ K}}{293 \text{ K}} \right) (12.57 \text{ m}^3) = 1.16 \text{ m}^3.$$

(c) The decrease in volume is  $\Delta V = V_1 - V_2 = 11.44 \text{ m}^3$ . Using Eq. 19-5, the amount of air this volume corresponds to is

$$n = \frac{p \Delta V}{RT} = \frac{(8.95 \text{ atm}) (1.01 \times 10^5 \text{ Pa/atm}) (11.44 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K}) (243 \text{ K})} = 5.10 \times 10^3 \text{ mol}.$$

Thus, in order for the submarine to maintain the original air volume in the chamber,  $5.10 \times 10^3 \text{ mol}$  of air must be released.

13. (a) At point *a*, we know enough information to compute *n*:

$$n = \frac{pV}{RT} = \frac{(2500 \text{ Pa}) (1.0 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K}) (200 \text{ K})} = 1.5 \text{ mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law in terms of ratios (note:  $n_a = n_b$  and cancels out):

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a} \Rightarrow T_b = (200 \text{ K}) \left( \frac{7.5 \text{ kPa}}{2.5 \text{ kPa}} \right) \left( \frac{3.0 \text{ m}^3}{1.0 \text{ m}^3} \right)$$

which yields an absolute temperature at *b* of  $T_b = 1.8 \times 10^3 \text{ K}$ .

(c) As in the previous part, we choose to approach this using the gas law in ratio form:

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a} \Rightarrow T_c = (200 \text{ K}) \left( \frac{2.5 \text{ kPa}}{2.5 \text{ kPa}} \right) \left( \frac{3.0 \text{ m}^3}{1.0 \text{ m}^3} \right)$$

which yields an absolute temperature at *c* of  $T_c = 6.0 \times 10^2 \text{ K}$ .

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the *pV* diagram shown in Fig. 19-20). This, in turn, is related to  $\pm$  “area” inside that triangle (with  $\text{area} = \frac{1}{2}(\text{base})(\text{height})$ ), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2} (2.0 \text{ m}^3) (5.0 \times 10^3 \text{ Pa}) = 5.0 \times 10^3 \text{ J}.$$

14. Since the pressure is constant the work is given by  $W = p(V_2 - V_1)$ . The initial volume is  $V_1 = (AT_1 - BT_1^2)/p$ , where  $T_1 = 315 \text{ K}$  is the initial temperature,  $A = 24.9 \text{ J/K}$  and  $B = 0.00662 \text{ J/K}^2$ . The final volume is  $V_2 = (AT_2 - BT_2^2)/p$ , where  $T_2 = 325 \text{ K}$ . Thus

$$\begin{aligned} W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\ &= (24.9 \text{ J/K})(325 \text{ K} - 315 \text{ K}) - (0.00662 \text{ J/K}^2)[(325 \text{ K})^2 - (315 \text{ K})^2] = 207 \text{ J}. \end{aligned}$$

15. Using Eq. 19-14, we note that since it is an isothermal process (involving an ideal gas) then  $Q = W = nRT \ln(V_f/V_i)$  applies at any point on the graph. An easy one to read is  $Q = 1000 \text{ J}$  and  $V_f = 0.30 \text{ m}^3$ , and we can also infer from the graph that  $V_i = 0.20 \text{ m}^3$ . We are told that  $n = 0.825 \text{ mol}$ , so the above relation immediately yields  $T = 360 \text{ K}$ .

16. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If  $d$  is the depth of the lake and  $\rho$  is the density of water, then the pressure at the bottom of the lake is  $p_1 = p_0 + \rho g d$ , where  $p_0$  is atmospheric pressure. Since  $p_1 V_1 = nRT_1$ , the number of moles of gas in the bubble is

$$n = p_1 V_1 / RT_1 = (p_0 + \rho g d) V_1 / RT_1,$$

where  $V_1$  is the volume of the bubble at the bottom of the lake and  $T_1$  is the temperature there. At the surface of the lake the pressure is  $p_0$  and the volume of the bubble is  $V_2 = nRT_2/p_0$ . We substitute for  $n$  to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1} \frac{p_0 + \rho g d}{p_0} V_1 \\ &= \left( \frac{293 \text{ K}}{277 \text{ K}} \right) \left( \frac{1.013 \times 10^5 \text{ Pa} + (0.998 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(40 \text{ m})}{1.013 \times 10^5 \text{ Pa}} \right) (20 \text{ cm}^3) \\ &= 1.0 \times 10^2 \text{ cm}^3. \end{aligned}$$

17. When the valve is closed the number of moles of the gas in container  $A$  is  $n_A = p_A V_A / RT_A$  and that in container  $B$  is  $n_B = 4p_B V_A / RT_B$ . The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R} \left( \frac{p_A}{T_A} + \frac{4p_B}{T_B} \right) = \text{const.}$$

After the valve is opened, the pressure in container  $A$  is  $p'_A = Rn'_A T_A / V_A$  and that in container  $B$  is  $p'_B = Rn'_B T_B / 4V_A$ . Equating  $p'_A$  and  $p'_B$ , we obtain  $Rn'_A T_A / V_A = Rn'_B T_B / 4V_A$ , or  $n'_B = (4T_A/T_B)n'_A$ . Thus,

$$n = n'_A + n'_B = n'_A \left( 1 + \frac{4T_A}{T_B} \right) = n_A + n_B = \frac{V_A}{R} \left( \frac{p_A}{T_A} + \frac{4p_B}{T_B} \right).$$

We solve the above equation for  $n'_A$ :

$$n'_A = \frac{V}{R} \frac{p_A/T_A + 4p_B/T_B}{1 + 4T_A/T_B}.$$

Substituting this expression for  $n'_A$  into  $p'V_A = n'_AR T_A$ , we obtain the final pressure:

$$p' = \frac{n'_AR T_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B} = 2.0 \times 10^5 \text{ Pa}.$$

18. First we rewrite Eq. 19-22 using Eq. 19-4 and Eq. 19-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(kN_A)T}{(mN_A)}} = \sqrt{\frac{3kT}{m}}.$$

The mass of the electron is given in the problem, and  $k = 1.38 \times 10^{-23} \text{ J/K}$  is given in the textbook. With  $T = 2.00 \times 10^6 \text{ K}$ , the above expression gives  $v_{\text{rms}} = 9.53 \times 10^6 \text{ m/s}$ . The pressure value given in the problem is not used in the solution.

19. Table 19-1 gives  $M = 28.0 \text{ g/mol}$  for nitrogen. This value can be used in Eq. 19-22 with  $T$  in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using  $v$  for  $v_{\text{rms}}$ :

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With  $T_2 = (20.0 + 273.15) \text{ K} \approx 293 \text{ K}$ , we obtain  $v_2 = (517 \text{ m/s}) \sqrt{\frac{293 \text{ K}}{300 \text{ K}}} = 511 \text{ m/s}$ .

(b) In this case, we set  $v_3 = \frac{1}{2} v_2$  and solve  $v_3/v_2 = \sqrt{T_3/T_2}$  for  $T_3$ :

$$T_3 = T_2 \left( \frac{v_3}{v_2} \right)^2 = (293 \text{ K}) \left( \frac{1}{2} \right)^2 = 73.0 \text{ K}$$

which we write as  $73.0 - 273 = -200^\circ\text{C}$ .

(c) Now we have  $v_4 = 2v_2$  and obtain

$$T_4 = T_2 \left( \frac{v_4}{v_2} \right)^2 = (293 \text{ K})(4) = 1.17 \times 10^3 \text{ K}$$

which is equivalent to 899°C.

20. Appendix F gives  $M = 4.00 \times 10^{-3} \text{ kg/mol}$  (Table 19-1 gives this to fewer significant figures). Using Eq. 19-22, we obtain

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(1000 \text{ K})}{4.00 \times 10^{-3} \text{ kg/mol}}} = 2.50 \times 10^3 \text{ m/s}.$$

21. **THINK** According to kinetic theory, the rms speed is (see Eq. 19-34)  $v_{\text{rms}} = \sqrt{3RT/M}$ , where  $T$  is the temperature and  $M$  is the molar mass.

**EXPRESS** The rms speed is defined as  $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$ , where  $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v) dv$ , with the Maxwell's speed distribution function given by

$$P(v) = 4\pi \left( \frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}.$$

According to Table 19-1, the molar mass of molecular hydrogen is  $2.02 \text{ g/mol} = 2.02 \times 10^{-3} \text{ kg/mol}$ .

**ANALYZE** At  $T = 2.7 \text{ K}$ , we find the rms speed to be

$$v_{\text{rms}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(2.7 \text{ K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.8 \times 10^2 \text{ m/s}.$$

**LEARN** The corresponding average speed and most probable speed are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31 \text{ J/mol} \cdot \text{K})(2.7 \text{ K})}{\pi(2.02 \times 10^{-3} \text{ kg/mol})}} = 1.7 \times 10^2 \text{ m/s}$$

and

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.31 \text{ J/mol} \cdot \text{K})(2.7 \text{ K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.5 \times 10^2 \text{ m/s},$$

respectively.

22. The molar mass of argon is  $39.95 \text{ g/mol}$ . Eq. 19-22 gives



$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(313 \text{ K})}{39.95 \times 10^{-3} \text{ kg/mol}}} = 442 \text{ m/s}.$$

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is  $2mv \cos \theta$ , where  $m$  is the mass of the molecule,  $v$  is its speed, and  $\theta$  is the angle between its velocity and the normal to the wall. If  $N$  molecules collide with the wall, then the change in their total momentum is  $2Nmv \cos \theta$ , and if the total time taken for the collisions is  $\Delta t$ , then the average rate of change of the total momentum is  $2(N/\Delta t)mv \cos \theta$ . This is the average force exerted by the  $N$  molecules on the wall, and the pressure is the average force per unit area:

$$p = \frac{2}{A} \left( \frac{N}{\Delta t} \right) mv \cos \theta = \left( \frac{2}{2.0 \times 10^{-4} \text{ m}^2} \right) (1.0 \times 10^{23} \text{ s}^{-1}) (3.3 \times 10^{-27} \text{ kg}) (1.0 \times 10^3 \text{ m/s}) \cos 55^\circ$$

$$= 1.9 \times 10^3 \text{ Pa}.$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to  $\text{m}^2$ .

24. We can express the ideal gas law in terms of density using  $n = M_{\text{sam}}/M$ :

$$pV = \frac{M_{\text{sam}}RT}{M} \Rightarrow \rho = \frac{pM}{RT}.$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}}.$$

(a) We convert to SI units:  $\rho = 1.24 \times 10^{-2} \text{ kg/m}^3$  and  $p = 1.01 \times 10^3 \text{ Pa}$ . The rms speed is  $\sqrt{3(1010)/0.0124} = 494 \text{ m/s}$ .

(b) We find  $M$  from  $\rho = pM/RT$  with  $T = 273 \text{ K}$ .

$$M = \frac{\rho RT}{p} = \frac{(0.0124 \text{ kg/m}^3) (8.31 \text{ J/mol} \cdot \text{K}) (273 \text{ K})}{1.01 \times 10^3 \text{ Pa}} = 0.0279 \text{ kg/mol} = 27.9 \text{ g/mol}.$$

(c) From Table 19.1, we identify the gas to be  $\text{N}_2$ .

25. (a) Equation 19-24 gives  $K_{\text{avg}} = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K}) (273 \text{ K}) = 5.65 \times 10^{-21} \text{ J}.$

(b) For  $T = 373$  K, the average translational kinetic energy is  $K_{\text{avg}} = 7.72 \times 10^{-21}$  J .

(c) The unit mole may be thought of as a (large) collection:  $6.02 \times 10^{23}$  molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\text{mole}} = N_A K_{\text{avg}} = (6.02 \times 10^{23})(5.65 \times 10^{-21} \text{ J}) = 3.40 \times 10^3 \text{ J}.$$

(d) Similarly, the result from part (b) leads to

$$K_{\text{mole}} = (6.02 \times 10^{23})(7.72 \times 10^{-21} \text{ J}) = 4.65 \times 10^3 \text{ J}.$$

26. The average translational kinetic energy is given by  $K_{\text{avg}} = \frac{3}{2} kT$ , where  $k$  is the Boltzmann constant ( $1.38 \times 10^{-23}$  J/K) and  $T$  is the temperature on the Kelvin scale. Thus

$$K_{\text{avg}} = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(1600 \text{ K}) = 3.31 \times 10^{-20} \text{ J}.$$

27. (a) We use  $\varepsilon = L_V/N$ , where  $L_V$  is the heat of vaporization and  $N$  is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol, so the molar mass of  $\text{H}_2\text{O}$  is  $(1.0 + 1.0 + 16) = 18$  g/mol. There are  $N_A = 6.02 \times 10^{23}$  molecules in a mole, so the number of molecules in a gram of water is  $(6.02 \times 10^{23} \text{ mol}^{-1})/(18 \text{ g/mol}) = 3.34 \times 10^{22}$  molecules/g. Thus

$$\varepsilon = (539 \text{ cal/g})/(3.34 \times 10^{22}/\text{g}) = 1.61 \times 10^{-20} \text{ cal} = 6.76 \times 10^{-20} \text{ J}.$$

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2} kT = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})[(32.0 + 273.15) \text{ K}] = 6.32 \times 10^{-21} \text{ J}.$$

The ratio  $\varepsilon/K_{\text{avg}}$  is  $(6.76 \times 10^{-20} \text{ J})/(6.32 \times 10^{-21} \text{ J}) = 10.7$ .

28. Using  $v = f\lambda$  with  $v = 331$  m/s (see Table 17-1) with Eq. 19-2 and Eq. 19-25 leads to

$$\begin{aligned} f &= \frac{v}{\left( \frac{1}{\sqrt{2}\pi d^2 (N/V)} \right)} = (331 \text{ m/s}) \pi \sqrt{2} (3.0 \times 10^{-10} \text{ m})^2 \left( \frac{nN_A}{V} \right) \\ &= \left( 8.0 \times 10^7 \frac{\text{m}^3}{\text{s} \cdot \text{mol}} \right) \left( \frac{n}{V} \right) = \left( 8.0 \times 10^7 \frac{\text{m}^3}{\text{s} \cdot \text{mol}} \right) \left( \frac{1.01 \times 10^5 \text{ Pa}}{(8.31 \text{ J/mol} \cdot \text{K})(273.15 \text{ K})} \right) \\ &= 3.5 \times 10^9 \text{ Hz} \end{aligned}$$

where we have used the ideal gas law and substituted  $n/V = p/RT$ . If we instead use  $v = 343$  m/s (the “default value” for speed of sound in air, used repeatedly in Ch. 17), then the answer is  $3.7 \times 10^9$  Hz.

29. **THINK** Mean free path is the average distance traveled by a molecule between successive collisions.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where  $d$  is the diameter of a molecule and  $N$  is the number of molecules in volume  $V$ .

**ANALYZE** (a) Substituting  $d = 2.0 \times 10^{-10}$  m and  $N/V = 1 \times 10^6$  molecules/m<sup>3</sup>, we obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10} \text{ m})^2 (1 \times 10^6 \text{ m}^{-3})} = 6 \times 10^{12} \text{ m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

**LEARN** Mean free path is inversely proportional to the number density,  $N/V$ . The typical value of  $N/V$  at room temperature and atmospheric pressure for ideal gas is

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(298 \text{ K})} = 2.46 \times 10^{25} \text{ molecules/m}^3 = 2.46 \times 10^{19} \text{ molecules/cm}^3.$$

This is much higher than that in the outer space.

30. We solve Eq. 19-25 for  $d$ :

$$d = \sqrt{\frac{1}{\lambda \pi \sqrt{2} (N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5 \text{ cm}) \pi \sqrt{2} (2.7 \times 10^{19} / \text{cm}^3)}}$$

which yields  $d = 3.2 \times 10^{-8}$  cm, or 0.32 nm.

31. (a) We use the ideal gas law  $pV = nRT = NkT$ , where  $p$  is the pressure,  $V$  is the volume,  $T$  is the temperature,  $n$  is the number of moles, and  $N$  is the number of molecules. The substitutions  $N = nN_A$  and  $k = R/N_A$  were made. Since 1 cm of mercury = 1333 Pa, the pressure is  $p = (10^{-7})(1333 \text{ Pa}) = 1.333 \times 10^{-4}$  Pa. Thus,

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.333 \times 10^{-4} \text{ Pa}}{(1.38 \times 10^{-23} \text{ J/K})(295 \text{ K})} = 3.27 \times 10^{16} \text{ molecules/m}^3 = 3.27 \times 10^{10} \text{ molecules/cm}^3.$$

(b) The molecular diameter is  $d = 2.00 \times 10^{-10} \text{ m}$ , so, according to Eq. 19-25, the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi (2.00 \times 10^{-10} \text{ m})^2 (3.27 \times 10^{16} \text{ m}^{-3})} = 172 \text{ m}.$$

32. (a) We set up a ratio using Eq. 19-25:

$$\frac{\lambda_{\text{Ar}}}{\lambda_{\text{N}_2}} = \frac{1/(\pi\sqrt{2}d_{\text{Ar}}^2(N/V))}{1/(\pi\sqrt{2}d_{\text{N}_2}^2(N/V))} = \left(\frac{d_{\text{N}_2}}{d_{\text{Ar}}}\right)^2.$$

Therefore, we obtain

$$\frac{d_{\text{Ar}}}{d_{\text{N}_2}} = \sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}} = \sqrt{\frac{27.5 \times 10^{-6} \text{ cm}}{9.9 \times 10^{-6} \text{ cm}}} = 1.7.$$

(b) Using Eq. 19-2 and the ideal gas law, we substitute  $N/V = N_A n/V = N_A p/RT$  into Eq. 19-25 and find

$$\lambda = \frac{RT}{\pi\sqrt{2}d^2 p N_A}.$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right).$$

With  $\lambda_1 = 9.9 \times 10^{-6} \text{ cm}$ ,  $T_1 = 293 \text{ K}$  (the same as  $T_2$  in this part),  $p_1 = 750 \text{ torr}$ , and  $p_2 = 150 \text{ torr}$ , we find  $\lambda_2 = 5.0 \times 10^{-5} \text{ cm}$ .

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to  $\lambda_2 = 7.9 \times 10^{-6} \text{ cm}$  for  $T_2 = 233 \text{ K}$  and  $p_2 = 750 \text{ torr}$ .

33. **THINK** We're given the speeds of 10 molecules. The speed distribution is discrete.

**EXPRESS** The average speed is  $\bar{v} = \frac{\sum v}{N}$ , where the sum is over the speeds of the particles and  $N$  is the number of particles. Similarly, the rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}}.$$

**ANALYZE** (a) From the equation above, we find the average speed to be

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0) \text{ km/s}}{10} = 6.5 \text{ km/s}.$$

(b) With

$$\begin{aligned} \sum v^2 &= [(2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2 \\ &\quad + (7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2] \text{ km}^2/\text{s}^2 = 505 \text{ km}^2/\text{s}^2 \end{aligned}$$

the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{505 \text{ km}^2/\text{s}^2}{10}} = 7.1 \text{ km/s}.$$

**LEARN** Each speed is weighted equally in calculating the average and the rms values.

34. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{[2(1.0) + 4(2.0) + 6(3.0) + 8(4.0) + 2(5.0)] \text{ cm/s}}{2 + 4 + 6 + 8 + 2} = 3.2 \text{ cm/s}.$$

(b) From  $v_{\text{rms}} = \sqrt{\sum n_i v_i^2 / \sum n_i}$  we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2 + 4(2.0)^2 + 6(3.0)^2 + 8(4.0)^2 + 2(5.0)^2}{2 + 4 + 6 + 8 + 2}} \text{ cm/s} = 3.4 \text{ cm/s}.$$

(c) There are eight particles at  $v = 4.0 \text{ cm/s}$ , more than the number of particles at any other single speed. So  $4.0 \text{ cm/s}$  is the most probable speed.

35. (a) The average speed is

$$v_{\text{avg}} = \frac{1}{N} \sum_{i=1}^N v_i = \frac{1}{10} [4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s}.$$

(b) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^N v_i^2} = \sqrt{\frac{1}{10} [4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2]} = 458 \text{ m/s}$$

(c) Yes,  $v_{\text{rms}} > v_{\text{avg}}$ .

36. We divide Eq. 19-35 by Eq. 19-22:

$$\frac{v_P}{v_{\text{rms}}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which, for  $v_P = v_{\text{rms}}$ , leads to

$$\frac{T_2}{T_1} = \frac{3}{2} \left( \frac{v_P}{v_{\text{rms}}} \right)^2 = \frac{3}{2}.$$

**37. THINK** From the distribution function  $P(v)$ , we can calculate the average and rms speeds.

**EXPRESS** The distribution function gives the fraction of particles with speeds between  $v$  and  $v + dv$ , so its integral over all speeds is unity:  $\int P(v) dv = 1$ . The average speed is defined as  $v_{\text{avg}} = \int_0^\infty vP(v)dv$ . Similarly, the rms speed is given by  $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$ , where  $(v^2)_{\text{avg}} = \int_0^\infty v^2P(v)dv$ .

**ANALYZE** (a) Evaluate the integral by calculating the area under the curve in Fig. 19-23. The area of the triangular portion is half the product of the base and altitude, or  $\frac{1}{2}av_0$ . The area of the rectangular portion is the product of the sides, or  $av_0$ . Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0,$$

so  $\frac{3}{2}av_0 = 1$  and  $av_0 = 2/3 = 0.67$ .

(b) For the triangular portion of the distribution  $P(v) = av/v_0$ , and the contribution of this portion is

$$\frac{a}{v_0} \int_0^{v_0} v^2 dv = \frac{a}{3v_0} v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where  $2/3v_0$  was substituted for  $a$ .  $P(v) = a$  in the rectangular portion, and the contribution of this portion is

$$a \int_{v_0}^{2v_0} v dv = \frac{a}{2} (4v_0^2 - v_0^2) = \frac{3a}{2} v_0^2 = v_0.$$

Therefore, we have

$$v_{\text{avg}} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \Rightarrow \frac{v_{\text{avg}}}{v_0} = 1.22.$$

(c) In calculating  $v_{\text{avg}}^2 = \int v^2 P(v) dv$ , we note that the contribution of the triangular section is

$$\frac{a}{v_0} \int_0^{v_0} v^3 dv = \frac{a}{4v_0} v_0^4 = \frac{1}{6} v_0^2.$$

The contribution of the rectangular portion is

$$a \int_{v_0}^{2v_0} v^2 dv = \frac{a}{3} (8v_0^3 - v_0^3) = \frac{7a}{3} v_0^3 = \frac{14}{9} v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6} v_0^2 + \frac{14}{9} v_0^2} = 1.31v_0 \Rightarrow \frac{v_{\text{rms}}}{v_0} = 1.31.$$

(d) The number of particles with speeds between  $1.5v_0$  and  $2v_0$  is given by  $N \int_{1.5v_0}^{2v_0} P(v) dv$ .

The integral is easy to evaluate since  $P(v) = a$  throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N av_0 = N/3,$$

where  $2/3v_0$  was substituted for  $a$ . In other words, the fraction of particles in this range is  $1/3$  or  $0.33$ .

**LEARN** From the distribution function shown in Fig. 19-23, it is clear that there are more particles with a speed in the range  $v_0 < v < 2v_0$  than  $0 < v < v_0$ . In fact, straightforward calculation shows that the fraction of particles with speeds between  $1.0v_0$  and  $2v_0$  is

$$\int_{1.0v_0}^{2v_0} P(v) dv = a(2v_0 - 1.0v_0) = av_0 = \frac{2}{3}.$$

38. (a) From the graph we see that  $v_p = 400$  m/s. Using the fact that  $M = 28$  g/mol =  $0.028$  kg/mol for nitrogen ( $\text{N}_2$ ) gas, Eq. 19-35 can then be used to determine the absolute temperature. We obtain  $T = \frac{1}{2} M v_p^2 / R = 2.7 \times 10^2$  K.

(b) Comparing with Eq. 19-34, we conclude  $v_{\text{rms}} = \sqrt{3/2} v_p = 4.9 \times 10^2$  m/s.

39. The rms speed of molecules in a gas is given by  $v_{\text{rms}} = \sqrt{3RT/M}$ , where  $T$  is the temperature and  $M$  is the molar mass of the gas. See Eq. 19-34. The speed required for escape from Earth's gravitational pull is  $v = \sqrt{2gr_e}$ , where  $g$  is the acceleration due to gravity at Earth's surface and  $r_e (= 6.37 \times 10^6$  m) is the radius of Earth. To derive this

expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass  $m$  at Earth's surface is

$$U = -GMm/r_e^2 = -mgr_e,$$

where  $g = GM/r_e^2$  was used. If  $v$  is the speed of the particle, then its total energy is  $E = -mgr_e + \frac{1}{2}mv^2$ . If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means  $E = 0$  and  $v = \sqrt{2gr_e}$ . We equate the expressions for the speeds to obtain  $\sqrt{3RT/M} = \sqrt{2gr_e}$ . The solution for  $T$  is  $T = 2gr_eM/3R$ .

(a) The molar mass of hydrogen is  $2.02 \times 10^{-3}$  kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 1.0 \times 10^4 \text{ K}.$$

(b) The molar mass of oxygen is  $32.0 \times 10^{-3}$  kg/mol, so for that gas

$$T = \frac{2(9.8 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 1.6 \times 10^5 \text{ K}.$$

(c) Now,  $T = 2g_m r_m M / 3R$ , where  $r_m = 1.74 \times 10^6$  m is the radius of the Moon and  $g_m = 0.16g$  is the acceleration due to gravity at the Moon's surface. For hydrogen, the temperature is

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(2.02 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 4.4 \times 10^2 \text{ K}.$$

(d) For oxygen, the temperature is

$$T = \frac{2(0.16)(9.8 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})(32.0 \times 10^{-3} \text{ kg/mol})}{3(8.31 \text{ J/mol} \cdot \text{K})} = 7.0 \times 10^3 \text{ K}.$$

(e) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result, the atmosphere is depleted of hydrogen.

(f) On the other hand, very few oxygen atoms escape. So there should be much oxygen high in Earth's upper atmosphere.



40. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg}2}}{v_{\text{rms}1}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which, for  $v_{\text{avg}2} = 2v_{\text{rms}1}$ , leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8} \left( \frac{v_{\text{avg}2}}{v_{\text{rms}1}} \right)^2 = \frac{3\pi}{2} = 4.7.$$

41. (a) The root-mean-square speed is given by  $v_{\text{rms}} = \sqrt{3RT/M}$ . See Eq. 19-34. The molar mass of hydrogen is  $2.02 \times 10^{-3}$  kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(4000 \text{ K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 7.0 \times 10^3 \text{ m/s}.$$

(b) When the surfaces of the spheres that represent an  $\text{H}_2$  molecule and an Ar atom are touching, the distance between their centers is the sum of their radii:

$$d = r_1 + r_2 = 0.5 \times 10^{-8} \text{ cm} + 1.5 \times 10^{-8} \text{ cm} = 2.0 \times 10^{-8} \text{ cm}.$$

(c) The argon atoms are essentially at rest so in time  $t$  the hydrogen atom collides with all the argon atoms in a cylinder of radius  $d$ , and length  $vt$ , where  $v$  is its speed. That is, the number of collisions is  $\pi d^2 vt N/V$ , where  $N/V$  is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 v N}{V} = \pi (2.0 \times 10^{-10} \text{ m})^2 (7.0 \times 10^3 \text{ m/s}) (4.0 \times 10^{25} \text{ m}^{-3}) = 3.5 \times 10^{10} \text{ collisions/s}.$$

42. The internal energy is

$$E_{\text{int}} = \frac{3}{2} nRT = \frac{3}{2} (1.0 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (273 \text{ K}) = 3.4 \times 10^3 \text{ J}.$$

43. (a) From Table 19-3,  $C_V = \frac{5}{2} R$  and  $C_p = \frac{7}{2} R$ . Thus, Eq. 19-46 yields

$$Q = nC_p \Delta T = (3.00) \left( \frac{7}{2} (8.31) \right) (40.0) = 3.49 \times 10^3 \text{ J}.$$

(b) Equation 19-45 leads to

$$\Delta E_{\text{int}} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2.49 \times 10^3 \text{ J}.$$

(c) From either  $W = Q - \Delta E_{\text{int}}$  or  $W = p\Delta T = nR\Delta T$ , we find  $W = 997 \text{ J}$ .

(d) Equation 19-24 is written in more convenient form (for this problem) in Eq. 19-38. Thus, the increase in kinetic energy is

$$\Delta K_{\text{trans}} = \Delta(NK_{\text{avg}}) = n\left(\frac{3}{2}R\right)\Delta T \approx 1.49 \times 10^3 \text{ J}.$$

Since  $\Delta E_{\text{int}} = \Delta K_{\text{trans}} + \Delta K_{\text{rot}}$ , the increase in rotational kinetic energy is

$$\Delta K_{\text{rot}} = \Delta E_{\text{int}} - \Delta K_{\text{trans}} = 2.49 \times 10^3 \text{ J} - 1.49 \times 10^3 \text{ J} = 1.00 \times 10^3 \text{ J}.$$

Note that had there been no rotation, all the energy would have gone into the translational kinetic energy.

44. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the  $pV$  diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases,  $W = p\Delta V$  for constant-pressure processes and  $W = 0$  for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in joules, pressure should be in pascals ( $\text{N/m}^2$ ) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is  $f = 5$ .

(a) The internal energy change is

$$\begin{aligned} E_{\text{int } c} - E_{\text{int } a} &= \frac{5}{2}(p_c V_c - p_a V_a) = \frac{5}{2}((2.0 \times 10^3 \text{ Pa})(4.0 \text{ m}^3) - (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)) \\ &= -5.0 \times 10^3 \text{ J}. \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left( \frac{p_a + p_c}{2} \right) (V_c - V_a) = (3.5 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)$$

which yields  $W_{\text{diag}} = 7.0 \times 10^3 \text{ J}$ . Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0 \times 10^3 + 7.0 \times 10^3) \text{ J} = 2.0 \times 10^3 \text{ J}.$$

(c) The fact that  $\Delta E_{\text{int}}$  only depends on the initial and final states, and not on the details of the “path” between them, means we can write  $\Delta E_{\text{int}} = E_{\text{int } c} - E_{\text{int } a} = -5.0 \times 10^3 \text{ J}$  for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3) + 0 = 1.0 \times 10^4 \text{ J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0 \times 10^3 + 1.0 \times 10^4) \text{ J} = 5.0 \times 10^3 \text{ J}.$$

45. Argon is a monatomic gas, so  $f = 3$  in Eq. 19-51, which provides

$$C_V = \frac{3}{2} R = \frac{3}{2} (8.31 \text{ J/mol} \cdot \text{K}) \left( \frac{1 \text{ cal}}{4.186 \text{ J}} \right) = 2.98 \frac{\text{cal}}{\text{mol} \cdot \text{C}^\circ}$$

where we have converted joules to calories, and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance)  $M$  is effectively a conversion factor between grams and moles, we see that  $c_V$  (see units specified in the problem statement) is related to  $C_V$  by  $C_V = c_V M$  where  $M = mN_A$ , and  $m$  is the mass of a single atom (see Eq. 19-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_A} = \frac{C_V / c_V}{N_A} = \frac{2.98 / 0.075}{6.02 \times 10^{23}} = 6.6 \times 10^{-23} \text{ g} = 6.6 \times 10^{-26} \text{ kg}.$$

(b) The molar mass is found to be

$$M = C_V / c_V = 2.98 / 0.075 = 39.7 \text{ g/mol}$$

which should be rounded to 40 g/mol since the given value of  $c_V$  is specified to only two significant figures.

46. (a) Since the process is a constant-pressure expansion,

$$W = p\Delta V = nR\Delta T = (2.02 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(15 \text{ K}) = 249 \text{ J}.$$

(b) Now,  $C_p = \frac{5}{2}R$  in this case, so  $Q = nC_p\Delta T = +623 \text{ J}$  by Eq. 19-46.

(c) The change in the internal energy is  $\Delta E_{\text{int}} = Q - W = +374 \text{ J}$ .

(d) The change in the average kinetic energy per atom is

$$\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22} \text{ J}.$$

47. (a) The work is zero in this process since volume is kept fixed.

(b) Since  $C_V = \frac{3}{2}R$  for an ideal monatomic gas, then Eq. 19-39 gives  $Q = +374 \text{ J}$ .

(c)  $\Delta E_{\text{int}} = Q - W = +374 \text{ J}$ .

(d) Two moles are equivalent to  $N = 12 \times 10^{23}$  particles. Dividing the result of part (c) by  $N$  gives the average translational kinetic energy change per atom:  $3.11 \times 10^{-22} \text{ J}$ .

48. (a) According to the first law of thermodynamics  $Q = \Delta E_{\text{int}} + W$ . When the pressure is a constant  $W = p\Delta V$ . So

$$\Delta E_{\text{int}} = Q - p\Delta V = 20.9 \text{ J} - (1.01 \times 10^5 \text{ Pa})(100 \text{ cm}^3 - 50 \text{ cm}^3) \left( \frac{1 \times 10^{-6} \text{ m}^3}{1 \text{ cm}^3} \right) = 15.9 \text{ J}.$$

(b) The molar specific heat at constant pressure is

$$C_p = \frac{Q}{n\Delta T} = \frac{Q}{n(p\Delta V/nR)} = \frac{R}{p} \frac{Q}{\Delta V} = \frac{(8.31 \text{ J/mol} \cdot \text{K})(20.9 \text{ J})}{(1.01 \times 10^5 \text{ Pa})(50 \times 10^{-6} \text{ m}^3)} = 34.4 \text{ J/mol} \cdot \text{K}.$$

(c) Using Eq. 19-49,  $C_V = C_p - R = 26.1 \text{ J/mol} \cdot \text{K}$ .

49. **THINK** The molar specific heat at constant volume for a gas is given by Eq. 19-41:  $C_V = \Delta E_{\text{int}}/n\Delta T$ . Our system consists of three non-interacting gases.

**EXPRESS** When the temperature changes by  $\Delta T$  the internal energy of the first gas changes by  $n_1 C_{V1} \Delta T$ , the internal energy of the second gas changes by  $n_2 C_{V2} \Delta T$ , and the internal energy of the third gas changes by  $n_3 C_{V3} \Delta T$ . The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1 C_{V1} + n_2 C_{V2} + n_3 C_{V3}) \Delta T.$$

This must be  $(n_1 + n_2 + n_3) C_V \Delta T$ , where  $C_V$  is the molar specific heat of the mixture. Thus,

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2} + n_3 C_{V3}}{n_1 + n_2 + n_3}.$$

**ANALYZE** With  $n_1=2.40$  mol,  $C_{V1}=12.0$  J/mol·K for gas 1,  $n_2=1.50$  mol,  $C_{V2}=12.8$  J/mol·K for gas 2, and  $n_3=3.20$  mol,  $C_{V3}=20.0$  J/mol·K for gas 3, we obtain

$$\begin{aligned} C_V &= \frac{(2.40 \text{ mol})(12.0 \text{ J/mol} \cdot \text{K}) + (1.50 \text{ mol})(12.8 \text{ J/mol} \cdot \text{K}) + (3.20 \text{ mol})(20.0 \text{ J/mol} \cdot \text{K})}{2.40 \text{ mol} + 1.50 \text{ mol} + 3.20 \text{ mol}} \\ &= 15.8 \text{ J/mol} \cdot \text{K} \end{aligned}$$

for the mixture.

**LEARN** The molar specific heat of the mixture  $C_V$  is the sum of each individual  $C_{Vi}$  weighted by the molar fraction.

50. Referring to Table 19-3, Eq. 19-45 and Eq. 19-46, we have

$$\Delta E_{\text{int}} = n C_V \Delta T = \frac{5}{2} n R \Delta T, \quad Q = n C_p \Delta T = \frac{7}{2} n R \Delta T.$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7}.$$

Thus, the given value  $Q = 70$  J leads to  $\Delta E_{\text{int}} = 50$  J.

51. The fact that they rotate but do not oscillate means that the value of  $f$  given in Table 19-3 is relevant. Thus, Eq. 19-46 leads to

$$Q = n C_p \Delta T = n \left( \frac{7}{2} R \right) (T_f - T_i) = n R T_i \left( \frac{7}{2} \right) \left( \frac{T_f}{T_i} - 1 \right)$$

where  $T_i = 273$  K and  $n = 1.0$  mol. The ratio of absolute temperatures is found from the gas law in ratio form. With  $p_f = p_i$  we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2.$$

Therefore, the energy added as heat is

$$Q = (1.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})\left(\frac{7}{2}\right)(2-1) \approx 8.0 \times 10^3 \text{ J}.$$

52. (a) Using  $M = 32.0 \text{ g/mol}$  from Table 19-1 and Eq. 19-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0 \text{ g}}{32.0 \text{ g/mol}} = 0.375 \text{ mol}.$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 19-46 and Table 19-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$Q = nC_p \Delta T = n\left(\frac{7}{2}R\right)\Delta T = (0.375 \text{ mol})\left(\frac{7}{2}\right)(8.31 \text{ J/mol} \cdot \text{K})(100 \text{ K}) = 1.09 \times 10^3 \text{ J}.$$

(c) We could compute a value of  $\Delta E_{\text{int}}$  from Eq. 19-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\Delta T}{n\left(\frac{7}{2}R\right)\Delta T} = \frac{5}{7} \approx 0.714.$$

53. **THINK** The molecules are diatomic, with translational and rotational degrees of freedom. The temperature change is under constant pressure.

**EXPRESS** Since the process is at constant pressure, energy transferred as heat to the gas is given by  $Q = nC_p \Delta T$ , where  $n$  is the number of moles in the gas,  $C_p$  is the molar specific heat at constant pressure, and  $\Delta T$  is the increase in temperature. Similarly, the change in the internal energy is given by  $\Delta E_{\text{int}} = nC_v \Delta T$ , where  $C_v$  is the specific heat at constant volume. For a diatomic ideal gas,  $C_p = \frac{7}{2}R$  and  $C_v = \frac{5}{2}R$  (see Table 19-3).

**ANALYZE** (a) The heat transferred is

$$Q = nC_p \Delta T = n\left(\frac{7R}{2}\right)\Delta T = \frac{7}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 6.98 \times 10^3 \text{ J}.$$

(b) From the above, we find the change in the internal energy to be

$$\Delta E_{\text{int}} = nC_v \Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(4.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(60.0 \text{ K}) = 4.99 \times 10^3 \text{ J}.$$

(c) According to the first law of thermodynamics,  $\Delta E_{\text{int}} = Q - W$ , so the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = 6.98 \times 10^3 \text{ J} - 4.99 \times 10^3 \text{ J} = 1.99 \times 10^3 \text{ J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2} n R \Delta T = \frac{3}{2} (4.00 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (60.0 \text{ K}) = 2.99 \times 10^3 \text{ J}.$$

**LEARN** The diatomic gas has three translational and two rotational degrees of freedom (making  $f = 3 + 2 = 5$ ). By equipartition theorem, each degree of freedom accounts for an energy of  $RT/2$  per mole. Thus,  $C_V = (f/2)R = 5R/2$  and  $C_p = C_V + R = 7R/2$ .

54. The fact that they rotate but do not oscillate means that the value of  $f$  given in Table 19-3 is relevant. In Section 19-11, it is noted that  $\gamma = C_p/C_V$  so that we find  $\gamma = 7/5$  in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{1.01 \times 10^5 \text{ N/m}^2} = 0.049 \text{ m}^3.$$

Consequently,

$$pV^\gamma = (1.01 \times 10^5 \text{ N/m}^2)(0.049 \text{ m}^3)^{1.4} = 1.5 \times 10^3 \text{ N} \cdot \text{m}^{2.2}.$$

55. (a) Let  $p_i$ ,  $V_i$ , and  $T_i$  represent the pressure, volume, and temperature of the initial state of the gas. Let  $p_f$ ,  $V_f$ , and  $T_f$  represent the pressure, volume, and temperature of the final state. Since the process is adiabatic  $p_i V_i^\gamma = p_f V_f^\gamma$ , so

$$p_f = \left( \frac{V_i}{V_f} \right)^\gamma p_i = \left( \frac{4.3 \text{ L}}{0.76 \text{ L}} \right)^{1.4} (1.2 \text{ atm}) = 13.6 \text{ atm} \approx 14 \text{ atm}.$$

We note that since  $V_i$  and  $V_f$  have the same units, their units cancel and  $p_f$  has the same units as  $p_i$ .

(b) The gas obeys the ideal gas law  $pV = nRT$ , so  $p_i V_i / p_f V_f = T_i / T_f$  and

$$T_f = \frac{p_f V_f}{p_i V_i} T_i = \left[ \frac{(13.6 \text{ atm})(0.76 \text{ L})}{(1.2 \text{ atm})(4.3 \text{ L})} \right] (310 \text{ K}) = 6.2 \times 10^2 \text{ K}.$$

56. (a) We use Eq. 19-54 with  $V_f/V_i = \frac{1}{2}$  for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{p_f}{p_i} = \left( \frac{V_i}{V_f} \right)^\gamma = (2.00)^{1.3}$$

which yields  $p_f = (2.46)(1.0 \text{ atm}) = 2.46 \text{ atm}$ .

(b) Similarly, Eq. 19-56 leads to

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma-1} = (273 \text{ K})(1.23) = 336 \text{ K}.$$

(c) We use the gas law in ratio form and note that when  $p_1 = p_2$  then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273 \text{ K}}{336 \text{ K}} = 0.813.$$

The volume  $V_1$  is half the original volume of one liter, so

$$V_2 = 0.813(0.500 \text{ L}) = 0.406 \text{ L}.$$

57. (a) Equation 19-54,  $p_i V_i^\gamma = p_f V_f^\gamma$ , leads to

$$p_f = p_i \left( \frac{V_i}{V_f} \right)^\gamma \Rightarrow 4.00 \text{ atm} = (1.00 \text{ atm}) \left( \frac{200 \text{ L}}{74.3 \text{ L}} \right)^\gamma$$

which can be solved to yield

$$\gamma = \frac{\ln(p_f/p_i)}{\ln(V_i/V_f)} = \frac{\ln(4.00 \text{ atm}/1.00 \text{ atm})}{\ln(200 \text{ L}/74.3 \text{ L})} = 1.4 = \frac{7}{5}.$$

This implies that the gas is diatomic (see Table 19-3).

(b) One can now use either Eq. 19-56 or use the ideal gas law itself. Here we illustrate the latter approach:

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i} \Rightarrow T_f = 446 \text{ K}.$$



(c) Again using the ideal gas law:  $n = P_i V_i / RT_i = 8.10$  moles. The same result would, of course, follow from  $n = P_f V_f / RT_f$ .

58. Let  $p_i$ ,  $V_i$ , and  $T_i$  represent the pressure, volume, and temperature of the initial state of the gas, and let  $p_f$ ,  $V_f$ , and  $T_f$  be the pressure, volume, and temperature of the final state. Since the process is adiabatic  $p_i V_i^\gamma = p_f V_f^\gamma$ . Combining with the ideal gas law,  $pV = NkT$ , we obtain

$$p_i V_i^\gamma = p_i (T_i / p_i)^\gamma = p_i^{1-\gamma} T_i^\gamma = \text{constant} \Rightarrow p_i^{1-\gamma} T_i^\gamma = p_f^{1-\gamma} T_f^\gamma$$

With  $\gamma = 4/3$ , which gives  $(1-\gamma)/\gamma = -1/4$ , the temperature at the end of the adiabatic expansion is

$$T_f = \left( \frac{p_i}{p_f} \right)^{\frac{1-\gamma}{\gamma}} T_i = \left( \frac{5.00 \text{ atm}}{1.00 \text{ atm}} \right)^{-1/4} (278 \text{ K}) = 186 \text{ K} = -87^\circ\text{C}.$$

59. Since  $\Delta E_{\text{int}}$  does not depend on the type of process,

$$(\Delta E_{\text{int}})_{\text{path 2}} = (\Delta E_{\text{int}})_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so  $\Delta E_{\text{int}} = 0$  for isotherms), then

$$(\Delta E_{\text{int}})_{\text{path 1}} = \sum (\Delta E_{\text{int}})_{\text{adiabat}}.$$

Finally, since  $Q = 0$  for adiabatic processes, then (for path 1)

$$\begin{aligned} (\Delta E_{\text{int}})_{\text{adiabatic expansion}} &= -W = -40 \text{ J} \\ (\Delta E_{\text{int}})_{\text{adiabatic compression}} &= -W = -(-25) \text{ J} = 25 \text{ J}. \end{aligned}$$

Therefore,  $(\Delta E_{\text{int}})_{\text{path 2}} = -40 \text{ J} + 25 \text{ J} = -15 \text{ J}$ .

60. Let  $p_1$ ,  $V_1$ , and  $T_1$  represent the pressure, volume, and temperature of the air at  $y_1 = 4267$  m. Similarly, let  $p$ ,  $V$ , and  $T$  be the pressure, volume, and temperature of the air at  $y = 1567$  m. Since the process is adiabatic,  $p_1 V_1^\gamma = p V^\gamma$ . Combining with the ideal gas law,  $pV = NkT$ , we obtain

$$pV^\gamma = p(T / p)^\gamma = p^{1-\gamma} T^\gamma = \text{constant} \Rightarrow p^{1-\gamma} T^\gamma = p_1^{1-\gamma} T_1^\gamma.$$

With  $p = p_0 e^{-\alpha y}$  and  $\gamma = 4/3$  (which gives  $(1-\gamma)/\gamma = -1/4$ ), the temperature at the end of the descent is

$$T = \left(\frac{p_1}{p}\right)^{\frac{1-\gamma}{\gamma}} T_1 = \left(\frac{p_0 e^{-ay_1}}{p_0 e^{-ay}}\right)^{\frac{1-\gamma}{\gamma}} T_1 = e^{-a(y-y_1)/4} T_1 = e^{-(1.16 \times 10^{-4}/\text{m})(1567 \text{ m} - 4267 \text{ m})/4} (268 \text{ K})$$

$$= (1.08)(268 \text{ K}) = 290 \text{ K} = 17^\circ\text{C}.$$

61. The aim of this problem is to emphasize what it means for the internal energy to be a state function. Since path 1 and path 2 start and stop at the same places, then the internal energy change along path 1 is equal to that along path 2. Now, during isothermal processes (involving an ideal gas) the internal energy change is zero, so the only step in path 1 that we need to examine is step 2. Equation 19-28 then immediately yields  $-20 \text{ J}$  as the answer for the internal energy change.

62. Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^\gamma \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma}.$$

Using Eq. 19-54 we can write this as

$$W = p_i V_i \frac{1 - (p_f / p_i)^{1-\gamma}}{1-\gamma}$$

In this problem,  $\gamma = 7/5$  (see Table 19-3) and  $p_f/p_i = 2$ . Converting the initial pressure to pascals we find  $p_i V_i = 24240 \text{ J}$ . Plugging in, then, we obtain  $W = -1.33 \times 10^4 \text{ J}$ .

63. In the following,  $C_v = \frac{3}{2}R$  is the molar specific heat at constant volume,  $C_p = \frac{5}{2}R$  is the molar specific heat at constant pressure,  $\Delta T$  is the temperature change, and  $n$  is the number of moles.

The process  $1 \rightarrow 2$  takes place at constant volume.

(a) The heat added is

$$Q = nC_v \Delta T = \frac{3}{2}nR \Delta T = \frac{3}{2}(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 3.74 \times 10^3 \text{ J}.$$

(b) Since the process takes place at constant volume, the work  $W$  done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74 \times 10^3 \text{ J}.$$

(c) The work  $W$  done by the gas is zero.

The process  $2 \rightarrow 3$  is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (455 \text{ K} - 600 \text{ K}) = -1.81 \times 10^3 \text{ J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81 \times 10^3 \text{ J}.$$

The process  $3 \rightarrow 1$  takes place at constant pressure.

(g) The heat added is

$$Q = nC_p \Delta T = \frac{5}{2} nR \Delta T = \frac{5}{2} (1.00 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (300 \text{ K} - 455 \text{ K}) = -3.22 \times 10^3 \text{ J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (300 \text{ K} - 455 \text{ K}) = -1.93 \times 10^3 \text{ J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3 \text{ J} + 1.93 \times 10^3 \text{ J} = -1.29 \times 10^3 \text{ J}.$$

(j) For the entire process the heat added is

$$Q = 3.74 \times 10^3 \text{ J} + 0 - 3.22 \times 10^3 \text{ J} = 520 \text{ J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74 \times 10^3 \text{ J} - 1.81 \times 10^3 \text{ J} - 1.93 \times 10^3 \text{ J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81 \times 10^3 \text{ J} - 1.29 \times 10^3 \text{ J} = 520 \text{ J}.$$

(m) We first find the initial volume. Use the ideal gas law  $p_1 V_1 = nRT_1$  to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{(1.013 \times 10^5 \text{ Pa})} = 2.46 \times 10^{-2} \text{ m}^3.$$

(n) Since  $1 \rightarrow 2$  is a constant volume process,  $V_2 = V_1 = 2.46 \times 10^{-2} \text{ m}^3$ . The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K})}{2.46 \times 10^{-2} \text{ m}^3} = 2.02 \times 10^5 \text{ Pa}.$$

This is approximately equal to 2.00 atm.

(o)  $3 \rightarrow 1$  is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(455 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 3.73 \times 10^{-2} \text{ m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1:  $p_3 = p_1 = 1.013 \times 10^5 \text{ Pa}$  (1.00 atm)

64. We write  $T = 273 \text{ K}$  and use Eq. 19-14:

$$W = (1.00 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (273 \text{ K}) \ln \left( \frac{16.8}{22.4} \right)$$

which yields  $W = -653 \text{ J}$ . Recalling the sign conventions for work stated in Chapter 18, this means an external agent does 653 J of work *on* the ideal gas during this process.

65. (a) We use  $p_i V_i^\gamma = p_f V_f^\gamma$  to compute  $\gamma$ :

$$\gamma = \frac{\ln(p_i/p_f)}{\ln(V_f/V_i)} = \frac{\ln(1.0 \text{ atm}/1.0 \times 10^5 \text{ atm})}{\ln(1.0 \times 10^3 \text{ L}/1.0 \times 10^6 \text{ L})} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form, the final temperature is

$$T_f = T_i \frac{p_f V_f}{p_i V_i} = (273 \text{ K}) \frac{(1.0 \times 10^5 \text{ atm})(1.0 \times 10^3 \text{ L})}{(1.0 \text{ atm})(1.0 \times 10^6 \text{ L})} = 2.7 \times 10^4 \text{ K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{(1.01 \times 10^5 \text{ Pa})(1.0 \times 10^3 \text{ cm}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})} = 4.5 \times 10^4 \text{ mol.}$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2} RT_i = \frac{3}{2} (8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K}) = 3.4 \times 10^3 \text{ J.}$$

(e) After the compression,

$$K_f = \frac{3}{2} RT_f = \frac{3}{2} (8.31 \text{ J/mol} \cdot \text{K})(2.7 \times 10^4 \text{ K}) = 3.4 \times 10^5 \text{ J.}$$

(f) Since  $v_{\text{rms}}^2 \propto T$ , we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273 \text{ K}}{2.7 \times 10^4 \text{ K}} = 0.010.$$

66. Equation 19-25 gives the mean free path:

$$\lambda = \frac{1}{\sqrt{2} d^2 \pi \epsilon_0 (N/V)} = \frac{n R T}{\sqrt{2} d^2 \pi \epsilon_0 P N}$$

where we have used the ideal gas law in that last step. Thus, the change in the mean free path is

$$\Delta \lambda = \frac{n R \Delta T}{\sqrt{2} d^2 \pi \epsilon_0 P N} = \frac{R Q}{\sqrt{2} d^2 \pi \epsilon_0 P N C_p}$$

where we have used Eq. 19-46. The constant pressure molar heat capacity is  $(7/2)R$  in this situation, so (with  $N = 9 \times 10^{23}$  and  $d = 250 \times 10^{-12} \text{ m}$ ) we find

$$\Delta \lambda = 1.52 \times 10^{-9} \text{ m} = 1.52 \text{ nm.}$$

67. (a) The volume has increased by a factor of 3, so the pressure must decrease accordingly (since the temperature does not change in this process). Thus, the final pressure is one-third of the original 6.00 atm. The answer is 2.00 atm.

(b) We note that Eq. 19-14 can be written as  $P_i V_i \ln(V_f/V_i)$ . Converting “atm” to “Pa” (a pascal is equivalent to a  $\text{N/m}^2$ ) we obtain  $W = 333 \text{ J}$ .

(c) The gas is monatomic so  $\gamma = 5/3$ . Equation 19-54 then yields  $P_f = 0.961 \text{ atm}$ .

(d) Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^\gamma \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma}$$

where in the last step Eq. 19-54 has been used. Converting “atm” to “Pa,” we obtain  $W = 236 \text{ J}$ .

68. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50.0)}{1.00 \times 10^{-8}} = 4.16 \times 10^{10} \text{ m}^3.$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = \frac{(1)(6.02 \times 10^{23})}{4.16 \times 10^{10}} = 1.45 \times 10^{13} \frac{\text{molecules}}{\text{m}^3}.$$

Using  $d = 20.0 \times 10^{-9} \text{ m}$ , Eq. 19-25 yields

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 \left(\frac{N}{V}\right)} = 38.8 \text{ m}.$$

69. **THINK** The net upward force is the difference between the buoyant force and the weight of the balloon with air inside.

**EXPRESS** Let  $\rho_c$  be the density of the cool air surrounding the balloon and  $\rho_h$  be the density of the hot air inside the balloon. The magnitude of the buoyant force on the balloon is  $F_b = \rho_c g V$ , where  $V$  is the volume of the envelope. The force of gravity is  $F_g = W + \rho_h g V$ , where  $W$  is the combined weight of the basket and the envelope. Thus, the net upward force is

$$F_{\text{net}} = F_b - F_g = \rho_c g V - W - \rho_h g V.$$

**ANALYZE** With  $F_{\text{net}} = 2.67 \times 10^3 \text{ N}$ ,  $W = 2.45 \times 10^3 \text{ N}$ ,  $V = 2.18 \times 10^3 \text{ m}^3$ , and  $\rho_c g = 11.9 \text{ N/m}^3$ , we obtain

$$\rho_h g = \frac{\rho_c g V - W - F_{\text{net}}}{V} = \frac{(11.9 \text{ N/m}^3)(2.18 \times 10^3 \text{ m}^3) - 2.45 \times 10^3 \text{ N} - 2.67 \times 10^3 \text{ N}}{2.18 \times 10^3 \text{ m}^3} = 9.55 \text{ N/m}^3$$

The ideal gas law gives  $p/RT = n/V$ . Multiplying both sides by the “molar weight”  $Mg$  then leads to

$$\frac{pMg}{RT} = \frac{nMg}{V} = \rho_h g.$$

With  $p = 1.01 \times 10^5$  Pa and  $M = 0.028$  kg/mol, we find the temperature to be

$$T = \frac{pMg}{R\rho_h g} = \frac{(1.01 \times 10^5 \text{ Pa})(0.028 \text{ kg/mol})(9.8 \text{ m/s}^2)}{(8.31 \text{ J/mol} \cdot \text{K})(9.55 \text{ N/m}^3)} = 349 \text{ K}.$$

**LEARN** As can be seen from the results above, increasing the temperature of the gas inside the balloon increases the value of  $F_{\text{net}}$ , i.e., the lifting capacity.

70. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with  $V_2 = 4 \text{ m}^3$ ; then continues on to state 3 isothermally, with  $V_3 = 10 \text{ m}^3$ ; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process  $1 \rightarrow 2$   $p_1 V_1^\gamma = p_2 V_2^\gamma$ , for the isothermal process  $2 \rightarrow 3$   $p_2 V_2 = p_3 V_3$ , and finally for the adiabatic process  $3 \rightarrow 4$   $p_3 V_3^\gamma = p_4 V_4^\gamma$ . These equations yield

$$p_4 = p_3 \left( \frac{V_3}{V_4} \right)^\gamma = p_2 \left( \frac{V_2}{V_3} \right) \left( \frac{V_3}{V_4} \right)^\gamma = p_1 \left( \frac{V_1}{V_2} \right)^\gamma \left( \frac{V_2}{V_3} \right) \left( \frac{V_3}{V_4} \right)^\gamma.$$

We substitute this expression for  $p_4$  into the equation  $p_1 V_1 = p_4 V_4$  (since  $T_1 = T_4$ ) to obtain  $V_1 V_3 = V_2 V_4$ . Solving for  $V_4$  we obtain

$$V_4 = \frac{V_1 V_3}{V_2} = \frac{(2.0 \text{ m}^3)(10 \text{ m}^3)}{4.0 \text{ m}^3} = 5.0 \text{ m}^3.$$

71. **THINK** An adiabatic process is a process in which the energy transferred as heat is zero.

**EXPRESS** The change in the internal energy is given by  $\Delta E_{\text{int}} = nC_V \Delta T$ , where  $C_V$  is the specific heat at constant volume,  $n$  is the number of moles in the gas, and  $\Delta T$  is the change in temperature. According to the first law of thermodynamics, the work done by the gas is  $W = Q - \Delta E_{\text{int}}$ . For an adiabatic process,  $Q = 0$ , and  $W = -\Delta E_{\text{int}}$ .

**ANALYZE** (a) The work done by the gas is

$$W = -\Delta E_{\text{int}} = -nC_V \Delta T = -\frac{3}{2} nR \Delta T = -\frac{3}{2} (2.0 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(15.0 \text{ K}) = -374 \text{ J}.$$

(b)  $Q = 0$  since the process is adiabatic.

(c) The change in internal energy is  $\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T = 374 \text{ J}$ .

(d) The number of atoms in the gas is  $N = nN_A$ , where  $N_A$  is the Avogadro's number. Thus, the change in the average kinetic energy per atom is

$$\Delta K_1 = \frac{\Delta E_{\text{int}}}{N} = \frac{\Delta E_{\text{int}}}{nN_A} = \frac{374 \text{ J}}{(2.00)(6.02 \times 10^{23} / \text{mol})} = 3.11 \times 10^{-22} \text{ J}.$$

**LEARN** The work done *on* the system is the negative of the work done *by* the system:  $W_{\text{on}} = -W = \Delta E_{\text{int}} = +374 \text{ J}$ . By work-kinetic energy theorem:  $\Delta K = \Delta W_{\text{on}} = \Delta E_{\text{int}}$ .

72. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293 \text{ K})}{M_{\text{hydrogen}}}}$$

for  $T$ . With the molar masses found in Table 19-1, we obtain

$$T = (293 \text{ K}) \left( \frac{4.0}{2.02} \right) = 580 \text{ K}$$

which is equivalent to  $307^\circ\text{C}$ .

73. **THINK** The collision frequency is related to the mean free path and average speed of the molecules.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where  $d$  is the diameter of a molecule and  $N$  is the number of molecules in volume  $V$ . Using ideal gas law, the number density can be written as  $N/V = p/kT$ , where  $p$  is the pressure,  $T$  is the temperature on the Kelvin scale and  $k$  is the Boltzmann constant. The average time between collisions is  $\tau = \lambda / v_{\text{avg}}$ , where  $v_{\text{avg}} = \sqrt{8RT/\pi M}$ , where  $R$  is the universal gas constant and  $M$  is the molar mass. The collision frequency is simply given by  $f = 1/\tau$ .

**ANALYZE** With  $p = 2.02 \times 10^3 \text{ Pa}$  and  $d = 290 \times 10^{-12} \text{ m}$ , we find the mean free path to be



$$\lambda = \frac{1}{\sqrt{2}\pi d^2(p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} = \frac{(1.38 \times 10^{-23} \text{ J/K})(400 \text{ K})}{\sqrt{2}\pi(290 \times 10^{-12} \text{ m})^2(1.01 \times 10^5 \text{ Pa})} = 7.31 \times 10^{-8} \text{ m}.$$

Similarly, with  $M = 0.032 \text{ kg/mol}$ , we find the average speed to be

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31 \text{ J/mol} \cdot \text{K})(400 \text{ K})}{\pi(32 \times 10^{-3} \text{ kg/mol})}} = 514 \text{ m/s}.$$

Thus, the collision frequency is  $f = \frac{v_{\text{avg}}}{\lambda} = \frac{514 \text{ m/s}}{7.31 \times 10^{-8} \text{ m}} = 7.04 \times 10^9 \text{ collisions/s}.$

**LEARN** This is very similar to the Sample Problem 19.04 – “Mean free path, average speed and collision frequency.” A general expression for  $f$  is

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{pd^2}{k} \sqrt{\frac{16\pi R}{MT}}.$$

74. (a) Since  $n/V = p/RT$ , the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_A}{V} = N_A \left( \frac{p}{RT} \right) (6.02 \times 10^{23}) \frac{1.01 \times 10^5 \text{ Pa}}{(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}})(293 \text{ K})} = 2.5 \times 10^{25} \frac{\text{molecules}}{\text{m}^3}.$$

(b) Three-fourths of the  $2.5 \times 10^{25}$  value found in part (a) are nitrogen molecules with  $M = 28.0 \text{ g/mol}$  (using Table 19-1), and one-fourth of that value are oxygen molecules with  $M = 32.0 \text{ g/mol}$ . Consequently, we generalize the  $M_{\text{sam}} = NM/N_A$  expression for these two species of molecules and write

$$\frac{3}{4}(2.5 \times 10^{25}) \frac{28.0}{6.02 \times 10^{23}} + \frac{1}{4}(2.5 \times 10^{25}) \frac{32.0}{6.02 \times 10^{23}} = 1.2 \times 10^3 \text{ g} = 1.2 \text{ kg}.$$

75. We note that  $\Delta K = n(\frac{3}{2}R)\Delta T$  according to the discussion in Sections 19-5 and 19-9. Also,  $\Delta E_{\text{int}} = nC_V\Delta T$  can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 19-49 leads to  $C_p = C_V + R \approx 8.0 \text{ cal/mol} \cdot \text{K}$  after we convert joules to calories in the ideal gas constant value (Eq. 19-6):  $R \approx 2.0 \text{ cal/mol} \cdot \text{K}$ . The first law of thermodynamics  $Q = \Delta E_{\text{int}} + W$  applies to each process.

• Constant volume process with  $\Delta T = 50 \text{ K}$  and  $n = 3.0 \text{ mol}$ .

(a) Since the change in the internal energy is  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900 \text{ cal}$ , and the work done by the gas is  $W = 0$  for constant volume processes, the first law gives  $Q = 900 + 0 = 900 \text{ cal}$ .

(b) As shown in part (a),  $W = 0$ .

(c) The change in the internal energy is, from part (a),  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900 \text{ cal}$ .

(d) The change in the total translational kinetic energy is

$$\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450 \text{ cal}.$$

• Constant pressure process with  $\Delta T = 50 \text{ K}$  and  $n = 3.0 \text{ mol}$ .

(e)  $W = p\Delta V$  for constant pressure processes, so (using the ideal gas law)

$$W = nR\Delta T = (3.0)(2.0)(50) = 300 \text{ cal}.$$

The first law gives  $Q = (900 + 300) \text{ cal} = 1200 \text{ cal}$ .

(f) From (e), we have  $W = 300 \text{ cal}$ .

(g) The change in the internal energy is  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900 \text{ cal}$ .

(h) The change in the translational kinetic energy is  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450 \text{ cal}$ .

• Adiabatic process with  $\Delta T = 50 \text{ K}$  and  $n = 3.0 \text{ mol}$ .

(i)  $Q = 0$  by definition of “adiabatic.”

(j) The first law leads to  $W = Q - E_{\text{int}} = 0 - 900 \text{ cal} = -900 \text{ cal}$ .

(k) The change in the internal energy is  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900 \text{ cal}$ .

(l) As in part (d) and (h),  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450 \text{ cal}$ .

76. (a) With work being given by

$$W = p\Delta V = (250)(-0.60) \text{ J} = -150 \text{ J},$$

and the heat transfer given as  $-210 \text{ J}$ , then the change in internal energy is found from the first law of thermodynamics to be  $[-210 - (-150)] \text{ J} = -60 \text{ J}$ .

(b) Since the pressures (and also the number of moles) don't change in this process, then the volume is simply proportional to the (absolute) temperature. Thus, the final temperature is  $\frac{1}{4}$  of the initial temperature. The answer is  $90 \text{ K}$ .

77. **THINK** From the distribution function  $P(v)$ , we can calculate the average and rms speeds of the gas.

**EXPRESS** The distribution function gives the fraction of particles with speeds between  $v$  and  $v + dv$ , so its integral over all speeds is unity:  $\int P(v) dv = 1$ . The average speed is defined as  $v_{\text{avg}} = \int_0^\infty vP(v)dv$ . Similarly, the rms speed is given by  $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$ , where  $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$ .

**ANALYZE** (a) By normalizing the distribution function:

$$1 = \int_0^{v_0} P(v) dv = \int_0^{v_0} C v^2 dv = \frac{C}{3} v_0^3$$

we find the constant  $C$  to be  $C = 3/v_0^3$ .

(b) The average speed is

$$v_{\text{avg}} = \int_0^{v_0} vP(v) dv = \int_0^{v_0} v \left( \frac{3v^2}{v_0^3} \right) dv = \frac{3}{v_0^3} \int_0^{v_0} v^3 dv = \frac{3}{4} v_0.$$

(c) Similarly, the rms speed is the square root of

$$\int_0^{v_0} v^2 P(v) dv = \int_0^{v_0} v^2 \left( \frac{3v^2}{v_0^3} \right) dv = \frac{3}{v_0^3} \int_0^{v_0} v^4 dv = \frac{3}{5} v_0^2.$$

Therefore,  $v_{\text{rms}} = \sqrt{3/5} v_0 \approx 0.775 v_0$ .

**LEARN** The maximum speed of the gas is  $v_{\text{max}} = v_0$ , as indicated by the distribution function. Using Eq. 19-29, we find the fraction of molecules with speed between  $v_1$  and  $v_2$  to be

$$\text{frac} = \int_{v_1}^{v_2} P(v) dv = \int_{v_1}^{v_2} \left( \frac{3v^2}{v_0^3} \right) dv = \frac{3}{v_0^3} \int_{v_1}^{v_2} v^2 dv = \frac{v_2^3 - v_1^3}{v_0^3}.$$

78. (a) In the free expansion from state 0 to state 1 we have  $Q = W = 0$ , so  $\Delta E_{\text{int}} = 0$ , which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3.00 V_0} = \frac{1}{3.00} p_0 \Rightarrow \frac{p_1}{p_0} = \frac{1}{3.00} = 0.333.$$

(b) For the adiabatic process from state 1 to 2 we have  $p_1 V_1^\gamma = p_2 V_2^\gamma$ , that is,

$$\frac{1}{3.00} p_0 (3.00V_0)^\gamma = (3.00)^{\frac{1}{3}} p_0 V_0^\gamma$$

which gives  $\gamma = 4/3$ . The gas is therefore polyatomic.

(c) From  $T = pV/nR$  we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^{1/3} = 1.44.$$

79. **THINK** The compression is isothermal so  $\Delta T = 0$ . In addition, since the gas is ideal, we can use the ideal gas law:  $pV = nRT$ .

**EXPRESS** The work done by the gas during the isothermal compression process from volume  $V_i$  to volume  $V_f$  is given by

$$W = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \left( \frac{V_f}{V_i} \right),$$

where we use the ideal gas law to replace  $p$  with  $nRT/V$ .

**ANALYZE** (a) The temperature is  $T = 10.0^\circ\text{C} = 283 \text{ K}$ . Then, with  $n = 3.50 \text{ mol}$ , we obtain

$$W = nRT \ln \left( \frac{V_f}{V_0} \right) = (3.50 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(283 \text{ K}) \ln \left( \frac{3.00 \text{ m}^3}{4.00 \text{ m}^3} \right) = -2.37 \times 10^3 \text{ J}.$$

(b) The internal energy change  $\Delta E_{\text{int}}$  vanishes (for an ideal gas) when  $\Delta T = 0$  so that the First Law of Thermodynamics leads to  $Q = W = -2.37 \text{ kJ}$ .

**LEARN** The work done by the gas is negative since  $V_f < V_i$ . Also, the negative value in  $Q$  implies that the heat transfer is from the sample to its environment.

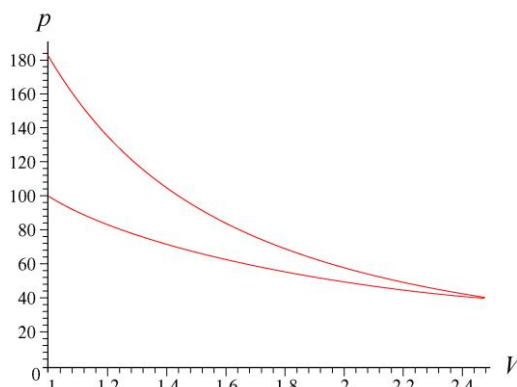
80. The ratio is

$$\frac{mgh}{mv_{\text{rms}}^2/2} = \frac{2gh}{v_{\text{rms}}^2} = \frac{2Mgh}{3RT}$$

where we have used Eq. 19-22 in that last step. With  $T = 273 \text{ K}$ ,  $h = 0.10 \text{ m}$  and  $M = 32 \text{ g/mol} = 0.032 \text{ kg/mol}$ , we find the ratio equals  $9.2 \times 10^{-6}$ .

81. (a) The  $p$ - $V$  diagram is shown next. Note that to obtain the graph, we have chosen  $n = 0.37 \text{ moles}$  for concreteness, in which case the horizontal axis (which we note starts not at zero but at 1) is to be interpreted in units of cubic centimeters, and the vertical axis (the absolute pressure) is in kilopascals. However, the constant volume temperature-increase

process described in the third step (see the problem statement) is difficult to see in this graph since it coincides with the pressure axis.



(b) We note that the change in internal energy is zero for an ideal gas isothermal process, so (since the net change in the internal energy must be zero for the entire cycle) the increase in internal energy in step 3 must equal (in magnitude) its decrease in step 1. By Eq. 19-28, we see this number must be 125 J.

(c) As implied by Eq. 19-29, this is equivalent to heat being added *to the gas*.

82. (a) The ideal gas law leads to

$$V = \frac{nRT}{p} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(273 \text{ K})}{1.01 \times 10^5 \text{ Pa}}$$

which yields  $V = 0.0225 \text{ m}^3 = 22.5 \text{ L}$ . If we use the standard pressure value given in Appendix D,  $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ , then our answer rounds more properly to 22.4 L.

(b) From Eq. 19-2, we have  $N = 6.02 \times 10^{23}$  molecules in the volume found in part (a) (which may be expressed as  $V = 2.24 \times 10^4 \text{ cm}^3$ ), so that

$$\frac{N}{V} = \frac{6.02 \times 10^{23}}{2.24 \times 10^4 \text{ cm}^3} = 2.69 \times 10^{19} \text{ molecules/cm}^3.$$

83. **THINK** For an isothermal expansion,  $\Delta T = 0$ . However, if the expansion is adiabatic, then  $\Delta Q = 0$ .

**EXPRESS** Using ideal gas law:  $pV = nRT$ , we have  $\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$ . For isothermal

process,  $T_f = T_i$ , which gives  $p_f = \frac{p_i V_i}{V_f}$ . The work done by the gas is

$$W = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \left( \frac{V_f}{V_i} \right).$$

Now, for an adiabatic process,  $p_i V_i^\gamma = p_f V_f^\gamma$ . The final pressures and temperatures are

$$p_f = p_i \left( \frac{V_i}{V_f} \right)^\gamma, \quad T_f = \frac{p_f V_f T_i}{p_i V_i}$$

The work done is  $W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}}$ .

**ANALYZE** (a) For the isothermal process, the final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32 \text{ atm})(1.0 \text{ L})}{4.0 \text{ L}} = 8.0 \text{ atm}.$$

(b) The final temperature of the gas is the same as the initial temperature:  $T_f = T_i = 300 \text{ K}$ .

(c) The work done is

$$\begin{aligned} W &= nRT_i \ln \left( \frac{V_f}{V_i} \right) = p_i V_i \ln \left( \frac{V_f}{V_i} \right) = (32 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(1.0 \times 10^{-3} \text{ m}^3) \ln \left( \frac{4.0 \text{ L}}{1.0 \text{ L}} \right) \\ &= 4.4 \times 10^3 \text{ J}. \end{aligned}$$

(d) For the adiabatic process, the final pressure is ( $\gamma = 5/3$  for monatomic gas)

$$p_f = p_i \left( \frac{V_i}{V_f} \right)^\gamma = (32 \text{ atm}) \left( \frac{1.0 \text{ L}}{4.0 \text{ L}} \right)^{5/3} = 3.2 \text{ atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 120 \text{ K}.$$

(f) The work done is

$$\begin{aligned} W &= -\Delta E_{\text{int}} = -\frac{3}{2} nR\Delta T = -\frac{3}{2} (p_f V_f - p_i V_i) \\ &= -\frac{3}{2} [(3.2 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})] (1.01 \times 10^5 \text{ Pa/atm})(10^{-3} \text{ m}^3/\text{L}) \\ &= 2.9 \times 10^3 \text{ J}. \end{aligned}$$

(g) If the gas is diatomic, then  $\gamma = 1.4$ , and the final pressure is

$$p_f = p_i \left( \frac{V_i}{V_f} \right)^\gamma = (32 \text{ atm}) \left( \frac{1.0 \text{ L}}{4.0 \text{ L}} \right)^{1.4} = 4.6 \text{ atm}.$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6 \text{ atm})(4.0 \text{ L})(300 \text{ K})}{(32 \text{ atm})(1.0 \text{ L})} = 170 \text{ K}.$$

(i) The work done is

$$\begin{aligned} W = Q - \Delta E_{\text{int}} &= -\frac{5}{2} n R \Delta T = -\frac{5}{2} (p_f V_f - p_i V_i) \\ &= -\frac{5}{2} [(4.6 \text{ atm})(4.0 \text{ L}) - (32 \text{ atm})(1.0 \text{ L})] (1.01 \times 10^5 \text{ Pa/atm}) (10^{-3} \text{ m}^3/\text{L}) \\ &= 3.4 \times 10^3 \text{ J}. \end{aligned}$$

**LEARN** Comparing (c) with (f), we see that more work is done by the gas if the expansion is isothermal rather than adiabatic.

84. (a) With  $P_1 = (20.0)(1.01 \times 10^5 \text{ Pa})$  and  $V_1 = 0.0015 \text{ m}^3$ , the ideal gas law gives

$$P_1 V_1 = n R T_1 \quad \Rightarrow \quad T_1 = 121.54 \text{ K} \approx 122 \text{ K}.$$

(b) From the information in the problem, we deduce that  $T_2 = 3T_1 = 365 \text{ K}$ .

(c) We also deduce that  $T_3 = T_1$ , which means  $\Delta T = 0$  for this process. Since this involves an ideal gas, this implies the change in internal energy is zero here.

85. (a) We use  $pV = nRT$ . The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left( \frac{300 \text{ g}}{17 \text{ g/mol}} \right) (8.31 \text{ J/mol} \cdot \text{K}) (350 \text{ K})}{1.35 \times 10^6 \text{ Pa}} = 3.8 \times 10^{-2} \text{ m}^3 = 38 \text{ L}.$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7 \times 10^5 \text{ Pa})(3.8 \times 10^{-2} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 13.5 \text{ mol}.$$

The mass of the gas that leaked out is then

$$\Delta m = 300 \text{ g} - (13.5 \text{ mol})(17 \text{ g/mol}) = 71 \text{ g}.$$

86. To model the “uniform rates” described in the problem statement, we have expressed the volume and the temperature functions as follows:

$$V = V_i + \left( \frac{V_f - V_i}{\tau_f} \right) t, \quad T = T_i + \left( \frac{T_f - T_i}{\tau_f} \right) t$$

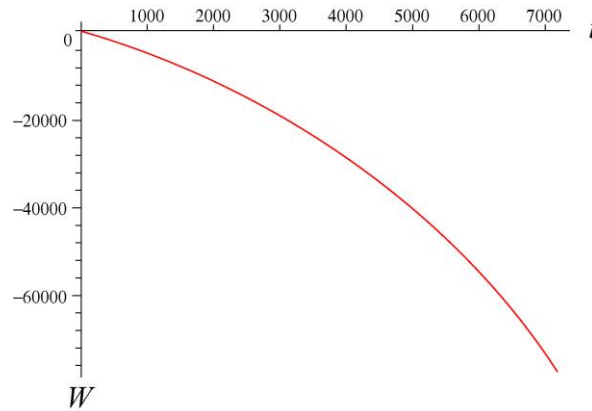
where  $V_i = 0.616 \text{ m}^3$ ,  $V_f = 0.308 \text{ m}^3$ ,  $\tau_f = 7200 \text{ s}$ ,  $T_i = 300 \text{ K}$ , and  $T_f = 723 \text{ K}$ .

(a) We can take the derivative of  $V$  with respect to  $t$  and use that to evaluate the cumulative work done (from  $t = 0$  until  $t = \tau$ ):

$$W = \int p dV = \int \left( \frac{nRT}{V} \right) \left( \frac{dV}{dt} \right) dt = 12.2 \tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With  $\tau = \tau_f$  our result is  $W = -77169 \text{ J} \approx -77.2 \text{ kJ}$ , or  $|W| \approx 77.2 \text{ kJ}$ .

The graph of cumulative work is shown below. The graph for work done is purely negative because the gas is being compressed (work is being done *on* the gas).



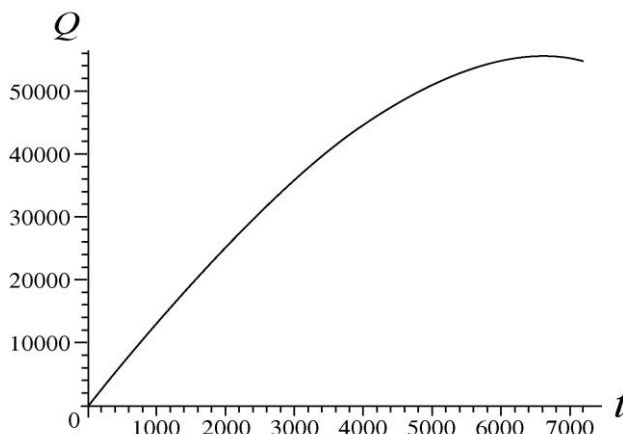
(b) With  $C_V = \frac{3}{2}R$  (since it's a monatomic ideal gas) then the (infinitesimal) change in internal energy is  $nC_V dT = \frac{3}{2}nR \left( \frac{dT}{dt} \right) dt$ , which involves taking the derivative of the temperature expression listed above. Integrating this and adding this to the work done gives the cumulative heat absorbed (from  $t = 0$  until  $t = \tau$ ):

$$Q = \int \left( \frac{nRT}{V} \right) \left( \frac{dV}{dt} \right) + \frac{3}{2}nR \left( \frac{dT}{dt} \right) dt = 30.5 \tau + 238113 \ln(14400 - \tau) - 2.28 \times 10^6$$

with SI units understood. With  $\tau = \tau_f$  our result is  $Q_{\text{total}} = 54649 \text{ J} \approx 5.46 \times 10^4 \text{ J}$ .



The graph cumulative heat is shown below. We see that  $Q > 0$ , since the gas is absorbing heat.



(c) Defining  $C = \frac{Q_{\text{total}}}{n(T_f - T_i)}$ , we obtain  $C = 5.17 \text{ J/mol}\cdot\text{K}$ . We note that this is considerably smaller than the constant-volume molar heat  $C_V$ .

We are now asked to consider this to be a two-step process (time dependence is no longer an issue) where the first step is isothermal and the second step occurs at constant volume (the ending values of pressure, volume, and temperature being the same as before).

(d) Equation 19-14 readily yields  $W = -43222 \text{ J} \approx -4.32 \times 10^4 \text{ J}$  (or  $|W| \approx 4.32 \times 10^4 \text{ J}$ ), where it is important to keep in mind that no work is done in a process where the volume is held constant.

(e) In step 1 the heat is equal to the work (since the internal energy does not change during an isothermal ideal gas process), and in step 2 the heat is given by Eq. 19-39. The total heat is therefore  $88595 \approx 8.86 \times 10^4 \text{ J}$ .

(f) Defining a molar heat capacity in the same manner as we did in part (c), we now arrive at  $C = 8.38 \text{ J/mol}\cdot\text{K}$ .

87. For convenience, the “int” subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that  $\sum_{\text{cycle}} E = 0$ , which gives

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow C} + \Delta E_{C \rightarrow D} + \Delta E_{D \rightarrow E} + \Delta E_{E \rightarrow A} = 0.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A \rightarrow B} = \Delta E_{D \rightarrow E} = 0.$$

Now, with  $\Delta E_{E \rightarrow A} = 8.0 \text{ J}$  given in the problem statement, we have

$$\Delta E_{B \rightarrow C} + \Delta E_{C \rightarrow D} + 8.0 \text{ J} = 0.$$

In an adiabatic process,  $\Delta E = -W$ , which leads to

$$-5.0 \text{ J} + \Delta E_{C \rightarrow D} + 8.0 \text{ J} = 0,$$

and we obtain  $\Delta E_{C \rightarrow D} = -3.0 \text{ J}$ .

88. (a) The work done in a constant-pressure process is  $W = p\Delta V$ . Therefore,

$$W = (25 \text{ N/m}^2) (1.8 \text{ m}^3 - 3.0 \text{ m}^3) = -30 \text{ J}.$$

The sign conventions discussed in the textbook for  $Q$  indicate that we should write  $-75 \text{ J}$  for the energy that leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\text{int}} = Q - W = (-75 \text{ J}) - (-30 \text{ J}) = -45 \text{ J}.$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form leads to

$$T_2 = T_1 \left( \frac{V_2}{V_1} \right) = (300 \text{ K}) \left( \frac{1.8 \text{ m}^3}{3.0 \text{ m}^3} \right) = 1.8 \times 10^2 \text{ K}.$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes  $C_V = \frac{3}{2}R$  and uses Eq. 19-45, one arrives at this same value for the final temperature).

89. Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that  $p + \rho_w g(L/2) = p_0 + \rho_w gh$ , where  $p_0$  is the atmospheric pressure, and  $p$  is the pressure of the air inside the pipe, which satisfies  $p(L/2) = p_0 L$ , or  $p = 2p_0$ . We solve for  $h$ :

$$h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01 \times 10^5 \text{ Pa}}{1.00 \times 10^3 \text{ kg/m}^3 \cdot 9.8 \text{ m/s}^2} + \frac{25.0 \text{ m}}{2} = 22.8 \text{ m}.$$

90. (a) For diatomic gas,  $\gamma = 7/5$ . Using  $pV^\gamma = \text{constant}$ , we find the final gas pressure to be

$$p_f = \left( \frac{V_i}{V_f} \right)^\gamma p_i = \left( \frac{50 \text{ cm}^3}{250 \text{ cm}^3} \right)^{7/5} (15 \text{ atm}) = 1.58 \text{ atm}.$$

The work done by the gas during the adiabatic expansion process is

$$\begin{aligned}
W &= p_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^\gamma \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma} \\
&= \frac{(1.58 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(250 \times 10^{-6} \text{ m}^3) - (15 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(50 \times 10^{-6} \text{ m}^3)}{1 - (7/5)} \\
&= 89.64 \text{ J}
\end{aligned}$$

The period for each cycle is  $\tau = (60 \text{ s})/(4000) = 0.015 \text{ s}$ . Since the time involved in the expansion is one-half of the total cycle:  $\Delta t = \tau/2 = 7.5 \times 10^{-3} \text{ s}$ , the average power for the expansion is

$$P = \frac{W}{\Delta t} = \frac{89.64 \text{ J}}{7.5 \times 10^{-3} \text{ s}} = 1.2 \times 10^4 \text{ W}.$$

(b) Using the conversion factor  $1 \text{ hp} = 746 \text{ W}$ , the power can also be expressed as 16 hp.

91. (a) For adiabatic process,  $pV^\gamma = \text{constant}$ , or  $p = CV^{-\gamma}$ . Thus,

$$B = -V \frac{dp}{dV} = -V \frac{d}{dV} (CV^{-\gamma}) = \gamma CV^{-\gamma} = \gamma p.$$

(b) Using  $p = nRT/V = (m/M)RT/V$  with  $\rho = m/V$ , we find the speed of sound in an ideal gas to be

$$v_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma(m/M)RT/V}{m/V}} = \sqrt{\frac{\gamma RT}{M}}.$$

92. With  $p = 1.01 \times 10^5 \text{ Pa}$  and  $\rho = 1.29 \text{ kg/m}^3$ , we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{(1.29 \text{ kg/m}^3)(331 \text{ m/s})^2}{1.01 \times 10^5 \text{ Pa}} = 1.40.$$

93. Using  $v_s = \sqrt{\gamma RT/M}$ , the result obtained in part (b) of problem 91, we find the ratio to be

$$\frac{v_1}{v_2} = \frac{\sqrt{\gamma RT/M_1}}{\sqrt{\gamma RT/M_2}} = \sqrt{\frac{M_2}{M_1}}.$$

94. The speed of sound in the gas is  $v_s = \sqrt{\gamma RT/M}$ , and the rms speed of the gas is  $v_{\text{rms}} = \sqrt{3RT/M}$ . Thus, the ratio is

$$\frac{v_s}{v_{\text{rms}}} = \frac{\sqrt{\gamma RT/M}}{\sqrt{3RT/M}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{C_p}{3C_v}} = \sqrt{\frac{C_v + R}{3C_v}} = \sqrt{\frac{5.0R + R}{3(5.0R)}} = \sqrt{\frac{2}{5}} = 0.63.$$

95. The speed of sound in an ideal gas is  $v_s = \sqrt{\gamma RT/M}$ , which gives

$$\gamma = \frac{Mv_s^2}{RT}.$$

Since the nodes of the standing waves are separated by half a wavelength, we have  $\lambda = 2(9.57 \text{ cm}) = 19.14 \text{ cm} = 0.1914 \text{ m}$ , and the corresponding speed of sound is

$$v_s = \lambda f = (0.1914 \text{ m})(1000 \text{ Hz}) = 191.4 \text{ m/s}.$$

Thus,

$$\gamma = \frac{Mv_s^2}{RT} = \frac{(0.127 \text{ kg/mol})(191.4 \text{ m/s})^2}{(8.314 \text{ J/mol} \cdot \text{K})(400 \text{ K})} = 1.40.$$

96. The speed of sound in an ideal gas is  $v_s = \sqrt{\gamma RT/M}$ . Differentiating  $v_s$  with respect to  $T$ , we obtain

$$\frac{dv_s}{dT} = \frac{1}{2} \sqrt{\frac{\gamma R}{M}} T^{-1/2} = \frac{1}{2T} \sqrt{\frac{\gamma RT}{M}} = \frac{v_s}{2T}$$

Near  $T = 0^\circ\text{C} = 273 \text{ K}$ , the speed of sound is  $331 \text{ m/s}$ . Thus, with  $\Delta T = 1^\circ\text{C} = 1 \text{ K}$ , the change in speed is

$$\Delta v_s = \frac{\Delta T}{2T} v_s = \frac{1 \text{ K}}{2(273 \text{ K})} (331 \text{ m/s}) = 0.606 \text{ m/s} \approx 0.61 \text{ m/s}.$$

97. The average speed and rms speed of an ideal gas are given by  $v_{\text{avg}} = \sqrt{8RT/\pi M}$  and  $v_{\text{rms}} = \sqrt{3RT/M}$ , respectively. Thus,

$$\frac{v_{\text{avg } 2}}{v_{\text{rms } 1}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}.$$

If  $v_{\text{avg } 2} = 2v_{\text{rms } 1}$ , then

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8} \left( \frac{v_{\text{avg } 2}}{v_{\text{rms } 1}} \right)^2 = \frac{3\pi}{2} = 4.71.$$