Chapter 31

1. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U=\frac{Q^2}{2C}=\frac{\left(2.90\times10^{-6}\,\mathrm{C}\right)^2}{2\left(3.60\times10^{-6}\,\mathrm{F}\right)}=1.17\times10^{-6}\,\mathrm{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I=\sqrt{\frac{2U}{L}}=\sqrt{\frac{2\left(1.168\times10^{-6}\,\mathrm{J}\right)}{75\times10^{-3}\,\mathrm{H}}}=5.58\times10^{-3}\,\mathrm{A}.$$

2. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A=nT=\frac{n}{f}=\frac{n}{2.00\times10^3\,\mathrm{Hz}}=n(5.00\,\mu\mathrm{s}),$$

where n = 1, 2, 3, 4, $\ldots$. The earliest time is (n = 1) $t_A=5.00\,\mu$s.

(b) We note that it takes $t=\frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 31-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t=\frac{1}{2}T+(n-1)T=\frac{1}{2}(2n-1)T=\frac{(2n-1)}{2f}=\frac{(2n-1)}{2\left(2\times10^3\,\mathrm{Hz}\right)}=(2n-1)(2.50\,\mu\mathrm{s}),$$

where n = 1, 2, 3, 4, $\ldots$. The earliest time is (n = 1) $t=2.50\,\mu$s.

(c) At $t=\frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 31-1). Later this will repeat every half-period (compare steps c and g in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n = 1) $t = 1.25\,\mu$s.

3. (a) The period is $T = 4(1.50\ \mu\text{s}) = 6.00\ \mu\text{s}$.

(b) The frequency is the reciprocal of the period: $f = \dfrac{1}{T} = \dfrac{1}{6.00\,\mu\text{s}} = 1.67 \times 10^5\ \text{Hz}$.

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or 3.00 μs.

4. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{\left(1.60 \times 10^{-6}\ \text{C}\right)^2}{2\left(140 \times 10^{-6}\ \text{J}\right)} = 9.14 \times 10^{-9}\ \text{F}.$$

5. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\ \text{C}}{\sqrt{\left(1.10 \times 10^{-3}\ \text{H}\right)\left(4.00 \times 10^{-6}\ \text{F}\right)}} = 4.52 \times 10^{-2}\ \text{A}.$$

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\ \text{N}}{\left(2.0 \times 10^{-13}\ \text{m}\right)\left(0.50\ \text{kg}\right)}} = 89\ \text{rad/s}.$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{89\ \text{rad/s}} = 7.0 \times 10^{-2}\ \text{s}$.

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89\ \text{rad/s})^2(5.0\ \text{H})} = 2.5 \times 10^{-5}\ \text{F}.$$

7. **THINK** This problem explores the analogy between an oscillating LC system and an oscillating mass–spring system.

EXPRESS Table 31-1 provides a comparison of energies in the two systems. From the table, we see the following correspondences:

$$x \leftrightarrow q, \quad k \leftrightarrow \frac{1}{C}, \quad m \leftrightarrow L, \quad v = \frac{dx}{dt} \leftrightarrow \frac{dq}{dt} = i,$$
$$\frac{1}{2}kx^2 \leftrightarrow \frac{q^2}{2C}, \quad \frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}Li^2.$$

ANALYZE (a) The mass m corresponds to the inductance, so m = 1.25 kg.

(b) The spring constant k corresponds to the reciprocal of the capacitance, $1/C$. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance, we have

$$C = \frac{Q^2}{2U} = \frac{\left(175\times10^{-6}\text{ C}\right)^2}{2\left(5.70\times10^{-6}\text{ J}\right)} = 2.69\times10^{-3}\text{ F}$$

and

$$k = \frac{1}{2.69\times10^{-3}\text{ m/N}} = 372\ \text{N/m}.$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\max} = 1.75\times10^{-4}$ m.

(d) The maximum speed $v_{\max}$ corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175\times10^{-6}\text{ C}}{\sqrt{(1.25\text{ H})\left(2.69\times10^{-3}\text{ F}\right)}} = 3.02\times10^{-3}\text{ A}.$$

Consequently, $v_{\max} = 3.02 \times 10^{-3}$ m/s.

LEARN The correspondences suggest that an oscillating LC system is mathematically equivalent to an oscillating mass–spring system. The electrical mechanical analogy can also be seen by comparing their angular frequencies of oscillation:

$$\omega = \sqrt{\frac{k}{m}} \text{ (mass-spring system)}, \quad \omega = \frac{1}{\sqrt{LC}} \text{ (}LC\text{ circuit)}$$

8. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j}\right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) = L\frac{di}{dt} + \frac{q}{C} + iR$$

with

$$L = \sum_j L_j, \quad \frac{1}{C} = \sum_j \frac{1}{C_j}, \quad R = \sum_j R_j$$

and we require $\varepsilon_{\text{total}} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-27(b).

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}.$ Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0\times10^{-6}\,\text{F})}}{4} = 7.0\times10^{-4}\,\text{s}.$$

10. We find the inductance from $f = \omega/2\pi = \left(2\pi\sqrt{LC}\right)^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(10\times10^3\,\text{Hz}\right)^2\left(6.7\times10^{-6}\,\text{F}\right)} = 3.8\times10^{-5}\,\text{H}.$$

11. **THINK** The frequency of oscillation *f* in an *LC* circuit is related to the inductance *L* and capacitance *C* by $f = 1/2\pi\sqrt{LC}$.

EXPRESS Since $f \sim 1/\sqrt{C}$, the smaller value of *C* gives the larger value of *f*, while the larger value of *C* gives the smaller value of *f*. Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, and $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$.

ANALYZE (a) The ratio of the maximum frequency to the minimum frequency is

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0.$$

(b) An additional capacitance *C* is chosen so the desired ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If *C* is in picofarads (pF), then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96.$$

The solution for *C* is

$$C = \frac{(365\,\text{pF}) - (2.96)^2 (10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}.$$

(c) We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency, C = 365 pF + 36 pF = 401 pF and f = 0.54 MHz. Thus, the inductance is

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2 (401 \times 10^{-12}\,\text{F})(0.54 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\,\text{H}.$$

LEARN One could also use the maximum frequency condition to solve for the inductance of the coil in (d). The capacitance is C = 10 pF + 36 pF = 46 pF and f = 1.60 MHz, so

$$L = \frac{1}{(2\pi)^2 Cf^2} = \frac{1}{(2\pi)^2 (46 \times 10^{-12}\,\text{F})(1.60 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\,\text{H}.$$

12. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1 - 75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q/Q = \sqrt{0.250} = 0.500.$

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\%,$$

we find $i/I = \sqrt{0.750} = 0.866.$

13. (a) The charge (as a function of time) is given by $q = Q\sin\omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that q = 0 at time t = 0. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q\cos\omega t,$$

and at t = 0, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC},$

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00 \times 10^{-3}\,\text{H})(2.70 \times 10^{-6}\,\text{F})} = 1.80 \times 10^{-4}\,\text{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2\omega \sin \omega t \cos \omega t}{C}$$

We use the trigonometric identity $\cos \omega t \sin \omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when sin(2ωt) = 1 or 2ωt = π/2 rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 7.07\times10^{-5}\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and sin(2ωt) = 1 into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\text{ H})(2.70\times10^{-6}\text{ F})} = 5.655\times10^{-4}\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{\pi\left(1.80\times10^{-4}\text{ C}\right)^2}{(5.655\times10^{-4}\text{ s})(2.70\times10^{-6}\text{ F})} = 66.7\,\text{W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

14. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\text{ H})(2.0\times10^{-6}\text{ F}+5.0\times10^{-6}\text{ F})}}.$$
$$= 6.0\times10^2\text{ Hz}$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(5.0\times10^{-6}\,\text{F})}} = 7.1\times10^{2}\,\text{Hz}.$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})}} = 1.1\times10^{3}\,\text{Hz}.$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\,\text{F}+5.0\times10^{-6}\,\text{F}}{(1.0\times10^{-2}\,\text{H})(2.0\times10^{-6}\,\text{F})(5.0\times10^{-6}\,\text{F})}} = 1.3\times10^{3}\,\text{Hz}.$$

15. (a) The maximum charge is

$$Q = CV_{\text{max}} = (1.0\times10^{-9}\ \text{F})(3.0\ \text{V}) = 3.0\times10^{-9}\ \text{C}.$$

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times10^{-9}\ \text{C}}{\sqrt{(3.0\times10^{-3}\ \text{H})(1.0\times10^{-9}\ \text{F})}} = 1.7\times10^{-3}\ \text{A}.$$

(c) When the current is at a maximum, the magnetic energy is at a maximum also:

$$U_{B,\text{max}} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}\ \text{H})(1.7\times10^{-3}\ \text{A})^2 = 4.5\times10^{-9}\ \text{J}.$$

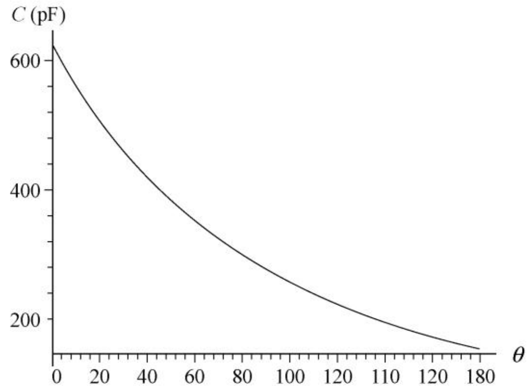
16. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1+\frac{\theta}{180°}\right) \Rightarrow \theta = 180°\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2\times10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2Lf_0^2(1+\theta/180°)^2} = \frac{81}{400000\pi^2(180°+\theta)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted next:



17. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(54.0\times10^{-3}\text{ H}\right)\left(6.20\times10^{-6}\text{ F}\right)}} = 275\text{ Hz.}$$

(b) When the switch is thrown, the capacitor is charged to V = 34.0 V and the current is zero. Thus, the maximum charge on the capacitor is

$$Q = VC = (34.0\text{ V})(6.20 \times 10^{-6}\text{ F}) = 2.11 \times 10^{-4}\text{ C.}$$

The current amplitude is

$$I = \omega Q = 2\pi fQ = 2\pi\left(275\text{ Hz}\right)\left(2.11\times10^{-4}\text{ C}\right) = 0.365\text{ A.}$$

18. (a) From $V = IX_C$ we find $\omega = I/CV$. The period is then $T = 2\pi/\omega = 2\pi CV/I$ = 46.1 μs.

(b) The maximum energy stored in the capacitor is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}(2.20\times10^{-7}\text{ F})(0.250\text{ V})^2 = 6.88\times10^{-9}\text{ J}.$$

(c) The maximum energy stored in the inductor is also $U_B = LI^2/2 = 6.88\text{ nJ}$.

(d) We apply Eq. 30-35 as $V = L(di/dt)_{max}$. We can substitute $L = CV^2/I^2$ (combining what we found in part (a) with Eq. 31-4) into Eq. 30-35 (as written above) and solve for $(di/dt)_{max}$. Our result is

$$\left(\frac{di}{dt}\right)_{max} = \frac{V}{L} = \frac{V}{CV^2/I^2} = \frac{I^2}{CV} = \frac{(7.50\times10^{-3}\text{ A})^2}{(2.20\times10^{-7}\text{ F})(0.250\text{ V})} = 1.02\times10^{3}\text{ A/s}.$$

(e) The derivative of $U_B = \frac{1}{2}Li^2$ leads to

$$\frac{dU_B}{dt} = LI^2\omega \sin \omega t \cos \omega t = \frac{1}{2} LI^2 \omega \sin 2\omega t \,.$$

Therefore, $\left(\frac{dU_B}{dt}\right)_{\max} = \frac{1}{2} LI^2\omega = \frac{1}{2} IV = \frac{1}{2}(7.50\times10^{-3}\text{ A})(0.250\text{ V}) = 0.938\text{ mW}.$

19. The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Equation 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\, d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

20. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$L = \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\max}}{I}\right)^2 = C\left(\frac{V_{\max}}{I}\right)^2 = \left(4.00\times10^{-6}\,\text{F}\right)\left(\frac{1.50\,\text{V}}{50.0\times10^{-3}\,\text{A}}\right)^2 = 3.60\times10^{-3}\,\text{H}.$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\,\text{H}\right)\left(4.00\times10^{-6}\,\text{F}\right)}} = 1.33\times10^{3}\,\text{Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^{3}\ \text{Hz}\right)} = 1.88\times10^{-4}\ \text{s}.$$

21. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0\times10^{-6}\ \text{F})} = 2.50\times10^{-3}\ \text{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times10^{-3}\ \text{H})(1.60\,\text{A})^2 = 3.20\times10^{-3}\ \text{J}.$$

22. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$
$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

23. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2L}{2} = \frac{(3.80\times10^{-6}\,\text{C})^2}{2(7.80\times10^{-6}\,\text{F})} + \frac{(9.20\times10^{-3}\,\text{A})^2(25.0\times10^{-3}\,\text{H})}{2} = 1.98\times10^{-6}\,\text{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80\times10^{-6}\ \text{F})(1.98\times10^{-6}\ \text{J})} = 5.56\times10^{-6}\ \text{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98\times10^{-6}\ \text{J})}{25.0\times10^{-3}\ \text{H}}} = 1.26\times10^{-2}\ \text{A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\ \text{C}}{5.56\times10^{-6}\ \text{C}}\right) = \pm46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$.

We obtain $-\omega Q\sin\phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9°$.

24. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$
\begin{aligned}
q &= Qe^{-Rt/2L}\cos(\omega' t + \phi) = Qe^{-RNT/2L}\cos\left[\omega'(2\pi N/\omega') + \phi\right] \\
&= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos(2\pi N + \phi) \\
&= Qe^{-N\pi R\sqrt{C/L}}\cos\phi.
\end{aligned}
$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\ \mu$C is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0\exp\left(-N\pi R\sqrt{C/L}\right)$.

(a) For $N = 5$, $q_5 = (6.2\mu\text{C})\exp\left(-5\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.85\mu\text{C}.$

(b) For $N = 10$, $q_{10} = (6.2\mu\text{C})\exp\left(-10\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 5.52\mu\text{C}.$

(c) For $N = 100$, $q_{100} = (6.2\mu\text{C})\exp\left(-100\pi(7.2\Omega)\sqrt{0.0000032\,\text{F}/12\,\text{H}}\right) = 1.93\mu\text{C}.$

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$
\begin{aligned}
t &= 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{(220\times10^{-3}\,\text{H})(12.0\times10^{-6}\,\text{F})}\right) \\
&= 0.5104\,\text{s}.
\end{aligned}
$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in Section 31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R=-\frac{2L}{t}\ \ln\left(\frac{q_{\max}}{Q}\right)=-\frac{2\left(220\times10^{-3}\ \text{H}\right)}{0.5104\,\text{s}}\ \ln\left(0.99\right)=8.66\times10^{-3}\ \Omega\,.$$

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{\max}^2/2C$, where $q_{\max}$ is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{\max}^2}{2C}=\frac{1}{2}\frac{Q^2}{2C}\ \Rightarrow\ \ q_{\max}=\frac{Q}{\sqrt{2}}.$$

Now $q_{\max}$ (referred to as the *exponentially decaying amplitude* in Section 31-5) is related to Q (and the other parameters of the circuit) by

$$q_{\max}=Qe^{-Rt/2L}\Rightarrow\ln\left(\frac{q_{\max}}{Q}\right)=-\frac{Rt}{2L}.$$

Setting $q_{\max}=Q/\sqrt{2}$, we solve for t:

$$t=-\frac{2L}{R}\ln\left(\frac{q_{\max}}{Q}\right)=-\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right)=\frac{L}{R}\ln 2\ .$$

The identities $\ln(1/\sqrt{2})=-\ln\sqrt{2}=-\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

27. **THINK** With the presence of a resistor in the *RLC* circuit, oscillation is damped, and the total electromagnetic energy of the system is no longer conserved, as some energy is transferred to thermal energy in the resistor.

EXPRESS Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t)=\frac{q_{\max 1}^2}{2C}=\frac{Q^2}{2C}e^{-Rt/L}$$

where

$$q_{\max 1}=Qe^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in Section 31-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2}=Qe^{-R(t+T)2/L}$$

where $T=\dfrac{2\pi}{\omega'}$, and the energy is

$$U(t+T)=\frac{q_{\max 2}^2}{2C}=\frac{Q^2}{2C}e^{-R(t+T)/L}.$$

ANALYZE The fractional loss in energy is

$$\frac{|\Delta U|}{U}=\frac{U(t)-U(t+T)}{U(t)}=\frac{e^{-Rt/L}-e^{-R(t+T)/L}}{e^{-Rt/L}}=1-e^{-RT/L}.$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L}\approx 1-\frac{RT}{L}+\frac{R^2T^2}{2L^2}+\cdots.$$

If we approximate $\omega\approx\omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U}\approx 1-\left(1-\frac{RT}{L}+\cdots\right)\approx\frac{RT}{L}=\frac{2\pi R}{\omega L}.$$

LEARN The ratio $|\Delta U|/U$ can be rewritten as

$$\frac{|\Delta U|}{U}=\frac{2\pi}{Q}$$

where $Q=\omega L/R$ (not to confuse Q with charge) is called the "quality factor" of the oscillating circuit. A high-Q circuit has low resistance and hence, low fractional energy loss.

28. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I=\omega_d C\varepsilon_m=2\pi f_d C\varepsilon_m=2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\text{ V})=0.283\text{ A}.$$

(b) $I = 2\pi(8.00\times10^3\text{ Hz})(1.50\times10^{-6}\text{ F})(30.0\text{ V}) = 2.26\text{ A}$.

29. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I=\frac{V_L}{X_L}=\frac{\varepsilon_m}{2\pi f_d L}=\frac{30.0\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\text{H})}=0.0955\text{A}=95.5\text{ mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955\text{ A})/8 = 0.0119\text{ A} = 11.9\text{ mA}.$$

30. (a) The current through the resistor is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\text{ V}}{50.0\ \Omega} = 0.600\text{ A}.$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600\text{ A}$.

31. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\text{H})(10\times10^{-6}\text{F})}} = 6.5\times10^2\text{ Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650\text{ Hz})(6.0\times10^{-3}\text{ H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

32. (a) The circuit consists of one generator across one inductor; therefore, $\varepsilon_m = V_L$. The current amplitude is

$$I = \frac{\varepsilon_m}{X_L} = \frac{\varepsilon_m}{\omega_d L} = \frac{25.0\text{ V}}{(377\text{ rad/s})(12.7\text{ H})} = 5.22\times10^{-3}\text{ A}.$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\varepsilon_L = 0$ at that instant. Stated another way, since $\varepsilon(t)$ and $i(t)$ have a 90° phase difference, then $\varepsilon(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 31-28 with $\varepsilon = -\varepsilon_m/2$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 31-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22\times10^{-3}\text{A})\left(\frac{\sqrt{3}}{2}\right) = 4.51\times10^{-3}\text{ A} .$$

33. **THINK** Our circuit consists of an ac generator that produces an alternating current, as well as a load that could be purely resistive, capacitive, or inductive. The nature of the load can be determined by the phase angle between the current and the emf.

EXPRESS The generator emf and the current are given by

$$\varepsilon = \varepsilon_m \sin(\omega_d - \pi/4), \quad i(t) = I\sin(\omega_d - 3\pi/4).$$

The expressions show that the emf is maximum when $\sin(\omega_d t - \pi/4) = 1$ or

$$\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

Similarly, the current is maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or

$$\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi \quad [n = \text{integer}].$$

ANALYZE (a) The first time the emf reaches its maximum after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\text{ rad/s})} = 6.73\times10^{-3}\text{ s} .$$

(b) The first time the current reaches its maximum after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$, as in part (a) with $n = 0$. Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\text{ rad/s})} = 1.12\times10^{-2}\text{ s}.$$

(c) The current lags the emf by $+\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \varepsilon_m$. Thus, $\varepsilon_m = I\omega_d L$ and

$$L = \frac{\varepsilon_m}{I\omega_d} = \frac{30.0\text{ V}}{(620\times10^{-3}\text{A})(350\text{ rad/s})} = 0.138\text{ H}.$$

LEARN The current in the circuit can be rewritten as

$$i(t) = I\sin\left(\omega_d - \frac{3\pi}{4}\right) = I\sin\left(\omega_d - \frac{\pi}{4} - \phi\right)$$

where $\phi = +\pi/2$. In a purely inductive circuit, the current lags the voltage by $90°$.

34. (a) The circuit consists of one generator across one capacitor; therefore, $\varepsilon_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\varepsilon_m}{X_C} = \omega C\varepsilon_m = (377\ \text{rad/s})(4.15\times10^{-6}\,\text{F})(25.0\ \text{V}) = 3.91\times10^{-2}\,\text{A}\ .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\text{max}}$), but rather as it passes through the (momentary) states of being uncharged (q = 0). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\varepsilon(t)$ and current $i(t)$ have a ϕ = –90° phase relation, implying $\varepsilon(t)$ = 0 when $i(t) = I$. The fact that ϕ = –90° = $-\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\varepsilon = -\frac{1}{2}\varepsilon_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I\sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = (3.91\times10^{-3}\,\text{A})\left(-\frac{\sqrt{3}}{2}\right) = -3.38\times10^{-2}\,\text{A},$$

or $|i| = 3.38\times10^{-2}$ A.

35. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi\ ,$$

which we solve for R:

$$R = \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) = \frac{1}{\tan 75°}\left[(2\pi)(930\,\text{Hz}(8.8\times10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times10^{-6}\,\text{F}}\right]$$
$$= 89\,\Omega.$$

36. (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: $R = 500\ \Omega$.

(b) We describe three methods here (each using information from different points on the graph):

method 1: At $\omega_d = 50$ rad/s, we have $Z \approx 700\ \Omega$, which gives $C = (\omega_d\sqrt{Z^2 - R^2}\ )^{-1} = 41\ \mu$F.

method 2: At $\omega_d = 50$ rad/s, we have $X_C \approx 500\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

method 3: At $\omega_d = 250$ rad/s, we have $X_C \approx 100\ \Omega$, which gives $C = (\omega_d X_C)^{-1} = 40\ \mu$F.

37. The rms current in the motor is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\text{A}.$$

38. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance Z becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find R: 8.0/4.0 = 2.0 Ω.

39. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

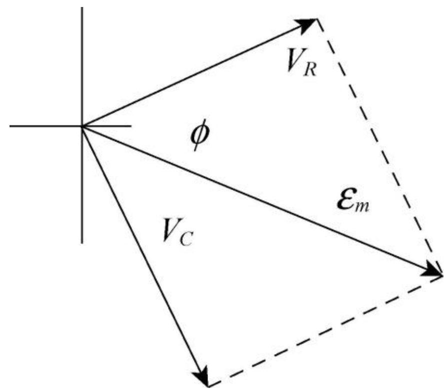
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{267\,\Omega} = 0.135\text{ A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\,\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\,\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so I leads ε_m. The phasor diagram is drawn to scale next.



40. A phasor diagram very much like Fig. 31-14(d) leads to the condition:

$$V_L - V_C = (6.00\text{ V})\sin(30^\circ) = 3.00\text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V. Since the capacitor and inductor voltage phasors are 180° out of phase, the potential difference across the inductor is -8.00 V.

41. **THINK** We have a series *RLC* circuit. Since *R*, *L*, and *C* are in series, the same current is driven in all three of them.

EXPRESS The capacitive and the inductive reactances can be written as

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C}, \quad X_L = \omega_d L = 2\pi f_d L.$$

The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the current amplitude is given by $I = \varepsilon_m / Z$.

ANALYZE (a) Substituting the values given, we find the capacitive reactance to be

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{ F})} = 37.9\ \Omega.$$

Similarly, the inductive reactance is

$$X_L = 2\pi f_d L = 2\pi(60.0\text{ Hz})(230\times10^{-3}\text{ H}) = 86.7\ \Omega.$$

Thus, the impedance is

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{(200\ \Omega)^2+(37.9\ \Omega-86.7\ \Omega)^2}=206\ \Omega.$$

(b) The phase angle is

$$\phi=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)=\tan^{-1}\left(\frac{86.7\ \Omega-37.9\ \Omega}{200\ \Omega}\right)=13.7°.$$

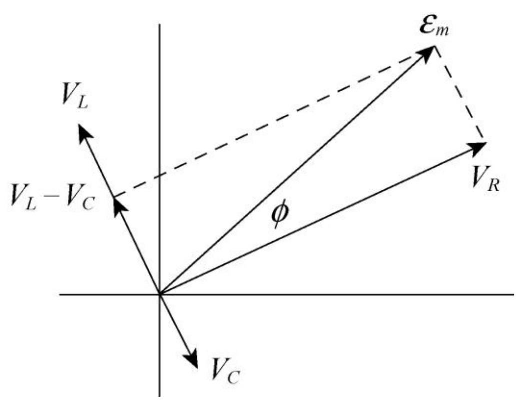
(c) The current amplitude is

$$I=\frac{\varepsilon_m}{Z}=\frac{36.0\text{ V}}{206\Omega}=0.175\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.175\text{ A})(200\ \Omega) = 35.0\text{ V}$$
$$V_L = IX_L = (0.175\text{ A})(86.7\ \Omega) = 15.2\text{ V}$$
$$V_C = IX_C = (0.175\text{ A})(37.9\ \Omega) = 6.62\text{ V}$$

Note that $X_L > X_C$, so that ε_m leads I. The phasor diagram is drawn to scale below.



LEARN The circuit in this problem is more inductive since $X_L > X_C$. The phase angle is positive, so the current lags behind the applied emf.

42. (a) Since $Z = \sqrt{R^2 + X_L^2}$ and $X_L = \omega_d L$, then as $\omega_d \to 0$ we find $Z \to R$ = 40 Ω.

(b) $L = X_L/\omega_d$ = *slope* = 60 mH.

43. (a) Now X_C = 0, while R = 200 Ω and

$$X_L = \omega L = 2\pi f_d L = 86.7\ \Omega$$

both remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200\ \Omega)^2 + (86.7\ \Omega)^2} = 218\ \Omega\ .$$

(b) The phase angle is, from Eq. 31-65,

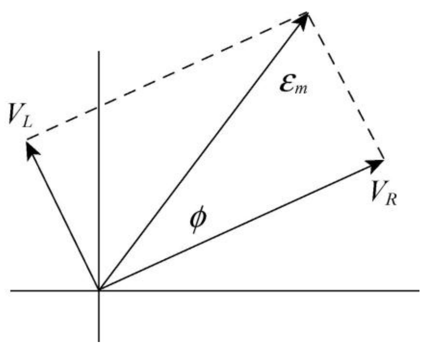
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{200\ \Omega}\right) = 23.4°\ .$$

(c) The current amplitude is now found to be $I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{218\ \Omega} = 0.165\ \text{A}$.

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.165\ \text{A})(200\Omega) \approx 33\text{V}$$
$$V_L = IX_L = (0.165\text{A})(86.7\Omega) \approx 14.3\text{V}.$$

This is an inductive circuit, so ε_m leads I. The phasor diagram is drawn to scale next.



44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\ \text{Hz})(24.0\times10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2}$$
$$= \sqrt{(220\,\Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 16.6\ \Omega]^2} = 422\ \Omega\ .$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\ \text{A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\ \text{Hz})(150\times10^{-3}\ \text{H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\,.$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\ \text{H}}{\sqrt{(1.0\ \text{H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega\,.$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

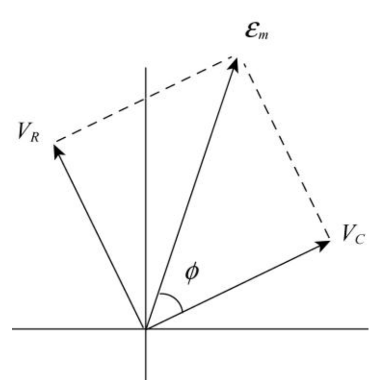
$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\ \text{V}}{10\ \Omega} = 1.0\ \text{A}\,.$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\ \Omega) = 1.0\times10^3\ \text{V}$$

which is much larger than the amplitude of the generator emf.

46. (a) A sketch of the phasor diagram is shown to the right.



(b) We have $I\,R = I\,X_C$, or

$$I\,R = I\,X_C \;\rightarrow\; R = \frac{1}{\omega_d C}$$

which yields

$$f = \frac{\omega_d}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi(50.0\,\Omega)(2.00\times10^{-5}\ \text{F})} = 159\ \text{Hz}\,.$$

(c) $\phi = \tan^{-1}(-V_C/V_R) = -45°$.

(d) $\omega_d = 1/RC = 1.00 \times 10^3$ rad/s.

(e) $I = (12\text{ V})/\sqrt{R^2 + X_C^2} = 6/(25\sqrt{2}) \approx 170$ mA.

47. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency.

EXPRESS For a given amplitude ε_m of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}.$$

To explicitly show that I is maximum when $\omega_d = \omega = 1/\sqrt{LC}$, we differentiate I with respect to ω_d and set the derivative to zero:

$$\frac{dI}{d\omega_d} = -(E)_m[R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2}\left(\omega_d L - \frac{1}{\omega_d C}\right)\left(L + \frac{1}{\omega_d^2 C}\right).$$

The only factor that can equal zero is when $\omega_d L - (1/\omega_d C)$, or $\omega_d = 1/\sqrt{LC} = \omega$.

ANALYZE (a) For this circuit, the driving angular frequency is

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\text{ H})(20.0\times 10^{-6}\text{ F})}} = 224\text{ rad/s}.$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0\text{ V}}{5.00\ \Omega} = 6.00\text{ A}.$$

(c) We want to find the (positive) values of ω_d for which $I = \varepsilon_m/2R$:

$$\frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\varepsilon_m}{2R}.$$

This may be rearranged to give

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2.$$

Taking the square root of both sides (acknowledging the two ± roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2(LC) \pm \omega_d\left(\sqrt{3}CR\right) - 1 = 0.$$

Using the quadratic formula, we find the smallest positive solution

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{-\sqrt{3}(20.0\times10^{-6}\text{ F})(5.00\ \Omega)}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$+\frac{\sqrt{3(20.0\times10^{-6}\text{ F})^2(5.00\ \Omega)^2 + 4(1.00\text{ H})(20.0\times10^{-6}\text{ F})}}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$= 219\text{ rad/s}.$$

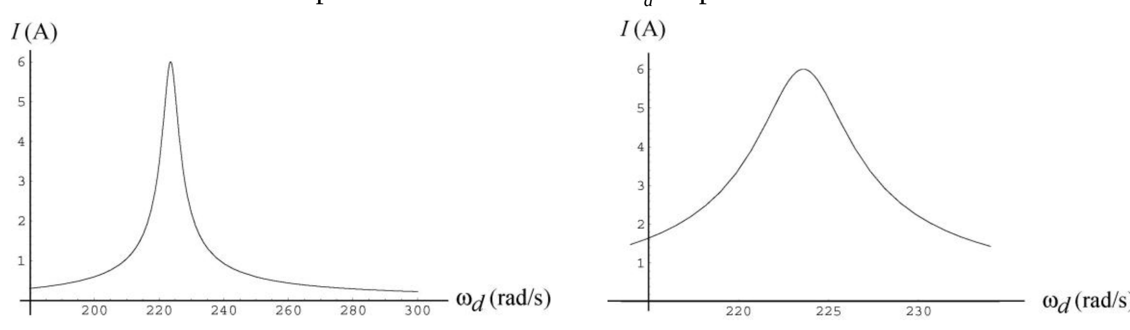
(d) The largest positive solution

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{+\sqrt{3}(20.0\times10^{-6}\text{ F})(5.00\,\Omega)}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$+\frac{\sqrt{3(20.0\times10^{-6}\text{ F})^2(5.00\ \Omega)^2 + 4(1.00\text{ H})(20.0\times10^{-6}\text{F})}}{2(1.00\text{ H})(20.0\times10^{-6}\text{ F})}$$
$$= 228\text{ rad/s}.$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega} = \frac{228\text{ rad/s} - 219\text{ rad/s}}{224\text{ rad/s}} = 0.040.$$

LEARN The current amplitude as a function of ω_d is plotted below.



We see that I is a maximum at $\omega_d = \omega = 224$ rad/s, and is at half maximum (3 A) at 219 rad/s and 228 rad/s.

48. (a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to

$$X_{\text{net}} = (12\text{ V})/(0.447\text{ A}) = 26.85\ \Omega.$$

With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = \frac{X_{\text{net}}}{\tan\phi} = \frac{26.85\,\Omega}{\tan 15°} = 100\,\Omega\,.$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find

$$X_{\text{net first}} = R\tan\phi' = (100\ \Omega)\tan(-30.9°) = -59.96\ \Omega.$$

We observe that the effect of switch 1 implies

$$X_C = X_{\text{net}} - X_{\text{net first}} = 26.85\ \Omega - (-59.96\ \Omega) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{\text{net}} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH .

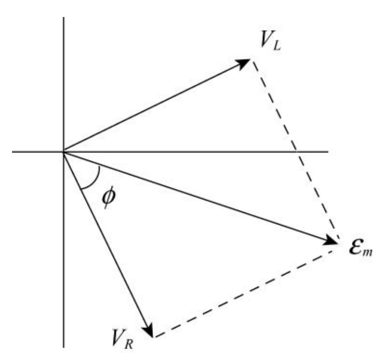
49. (a) Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\begin{aligned}\omega &= \frac{1}{2\pi\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}} \\ &= \frac{1}{2\pi\sqrt{\left(1.70\times10^{-3}\text{ H}+2.30\times10^{-3}\text{ H}\right)\left(4.00\times10^{-6}\text{ F}+2.50\times10^{-6}\text{ F}+3.50\times10^{-6}\text{ F}\right)}} \\ &= 796\,\text{Hz}.\end{aligned}$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{\text{eq}}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.



50. (a) A sketch of the phasor diagram is shown to the right.

(b) We have $V_R = V_L$, which implies

$$I\,R = I\,X_L \;\rightarrow\; R = \omega_d\,L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times10^3$ rad/s.

(e) $I = (6\text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

51. **THINK** In a driven *RLC* circuit, the current amplitude is maximum at resonance, where the driven angular frequency is equal to the natural angular frequency. It then falls off rapidly away from resonance.

EXPRESS We use the expressions found in Problem 31-47:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}, \quad \omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}.$$

The resonance angular frequency is $\omega = 1/\sqrt{LC}$.

ANALYZE Thus, the fractional half width is

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

LEARN Note that the value of $\Delta\omega_d/\omega$ increases linearly with R; that is, the larger the resistance, the broader the peak. As an example, the data of Problem 31-47 gives

$$\frac{\Delta\omega_d}{\omega} = (5.00\ \Omega)\sqrt{\frac{3\left(20.0\times10^{-6}\text{ F}\right)}{1.00\text{ H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 31-47. The method used there, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

52. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

53. **THINK** Energy is supplied by the 120 V rms ac line to keep the air conditioner running.

EXPRESS The impedance of the circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the average rate of energy delivery is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_{\text{rms}}}{Z}\right)^2 R = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2}.$$

ANALYZE (a) Substituting the values given, the impedance is

$$Z = \sqrt{(12.0\ \Omega)^2 + (1.30\ \Omega - 0)^2} = 12.1\ \Omega.$$

(b) The average rate at which energy has been supplied is

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\ \text{V})^2 (12.0\ \Omega)}{(12.07\ \Omega)^2} = 1.186\times 10^3\ \text{W} \approx 1.19\times 10^3\ \text{W}.$$

LEARN In a steady-state operation, the total energy stored in the capacitor and the inductor stays constant. Thus, the net energy transfer is from the generator to the resistor, where electromagnetic energy is dissipated in the form of thermal energy.

54. The amplitude (peak) value is

$$V_{\text{max}} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(100\ \text{V}) = 141\ \text{V}.$$

55. The average power dissipated in resistance R when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R,$ where I_{rms} is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R.$ Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\ \text{A}}{\sqrt{2}} = 1.84\ \text{A}.$$

56. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\text{max}}/P_{\text{min}} = (I/I_{\text{min}})^2 = 5$, or

$$\left(\frac{I}{I_{\text{min}}}\right)^2 = \left(\frac{\varepsilon_m / Z_{\text{min}}}{\varepsilon_m / Z_{\text{max}}}\right)^2 = \left(\frac{Z_{\text{max}}}{Z_{\text{min}}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\text{max}})^2}}{R}\right)^2 = 5.$$

We solve for L_{max}:

$$L_{\text{max}} = \frac{2R}{\omega} = \frac{2(120\ \text{V})^2 / 1000\ \text{W}}{2\pi(60.0\ \text{Hz})} = 7.64\times 10^{-2}\ \text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{\max} + R_{\text{bulb}}}{R_{\text{bulb}}}\right)^2 = 5,$$

or

$$R_{\max} = \left(\sqrt{5} - 1\right) R_{\text{bulb}} = \left(\sqrt{5} - 1\right) \frac{(120\,\text{V})^2}{1000\,\text{W}} = 17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

57. We shall use

$$P_{\text{avg}} = \frac{\varepsilon_m^2 R}{2Z^2} = \frac{\varepsilon_m^2 R}{2\left[R^2 + \left(\omega_d L - 1/\omega_d C\right)^2\right]}.$$

where $Z = \sqrt{R^2 + \left(\omega_d L - 1/\omega_d C\right)^2}$ is the impedance.

(a) Considered as a function of C, P_{avg} has its largest value when the factor $R^2 + \left(\omega_d L - 1/\omega_d C\right)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 \left(60.0 \times 10^{-3}\ \text{H}\right)} = 1.17 \times 10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is cos ϕ = cos 0° = 1.

(f) The minimum average power is P_{avg} = 0 (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that ϕ = –90°.

(h) At minimum power, the power factor is cos ϕ = cos(–90°) = 0.

58. This circuit contains no reactances, so $\varepsilon_{rms} = I_{rms}R_{total}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{\rm rms}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_m^2(r-R)}{(r+R)^3} = 0 \quad \Rightarrow \quad R = r$$

59. (a) The rms current is

$$\begin{aligned} I_{\rm rms} &= \frac{\varepsilon_{\rm rms}}{Z} = \frac{\varepsilon_{\rm rms}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}} \\ &= \frac{75.0\,\text{V}}{\sqrt{(15.0\,\Omega)^2 + \left\{2\pi(550\,\text{Hz})(25.0\,\text{mH}) - 1/\left[2\pi(550\,\text{Hz})(4.70\,\mu\text{F})\right]\right\}^2}} \\ &= 2.59\,\text{A}. \end{aligned}$$

(b) The rms voltage across R is $V_{ab} = I_{\rm rms}R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\,\text{V}$.

(c) The rms voltage across C is

$$V_{bc} = I_{\rm rms}X_C = \frac{I_{\rm rms}}{2\pi fC} = \frac{2.59\,\text{A}}{2\pi(550\,\text{Hz})(4.70\,\mu\text{F})} = 159\,\text{V}.$$

(d) The rms voltage across L is

$$V_{cd} = I_{\rm rms} X_L = 2\pi I_{\rm rms} fL = 2\pi(2.59\,{\rm A})(550\,{\rm Hz})(25.0\,{\rm mH}) = 224\,{\rm V}\,.$$

(e) The rms voltage across *C* and *L* together is

$$V_{bd} = |V_{bc} - V_{cd}| = |159.5\,{\rm V} - 223.7\,{\rm V}| = 64.2\,{\rm V}\,.$$

(f) The rms voltage across *R*, *C*, and *L* together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,{\rm V})^2 + (64.2\,{\rm V})^2} = 75.0\,{\rm V}\,.$$

(g) For the resistor *R*, the power dissipated is $P_R = \dfrac{V_{ab}^2}{R} = \dfrac{(38.8\,{\rm V})^2}{15.0\,\Omega} = 100\,{\rm W}.$

(h) No energy dissipation in *C*.

(i) No energy dissipation in *L*.

60. The current in the circuit satisfies $i(t) = I\sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\,{\rm V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\,{\rm rad/s})(9.20\,{\rm mH}) - 1/[(3000\,{\rm rad/s})(31.2\,\mu{\rm F})]\}^2}} \\ &= 1.93\,{\rm A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\,{\rm rad/s})(9.20\,{\rm mH})}{16.0\,\Omega} - \frac{1}{(3000\,{\rm rad/s})(16.0\,\Omega)(31.2\,\mu{\rm F})}\right] \\ &= 46.5^\circ. \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\varepsilon(t) = I\sin(\omega_d t - \phi)\,\varepsilon_m \sin\omega_d t \\ &= (1.93\,{\rm A})(45.0\,{\rm V})\sin[(3000\,{\rm rad/s})(0.442\,{\rm ms})]\sin[(3000\,{\rm rad/s})(0.442\,{\rm ms}) - 46.5^\circ] \\ &= 41.4\,{\rm W}. \end{aligned}$$

(b) With

$$v_c(t) = V_c \sin(\omega_d t - \phi - \pi/2) = -V_c \cos(\omega_d t - \phi)$$

where $V_c = I/\omega_d C$, the rate at which the energy in the capacitor changes is

$$\begin{aligned}P_c &= \frac{d}{dt}\left(\frac{q^2}{2C}\right) = i\frac{q}{C} = iv_c \\ &= -I\sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin\left[2(\omega_d t - \phi)\right] \\ &= -\frac{(1.93\,\text{A})^2}{2(3000\,\text{rad/s})(31.2\times 10^{-6}\,\text{F})}\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\ &= -17.0\text{ W}.\end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned}P_L &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}\left[I\sin(\omega_d t - \phi)\right] = \frac{1}{2}\omega_d LI^2\sin\left[2(\omega_d t - \phi)\right] \\ &= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right] \\ &= 44.1\text{ W}.\end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned}P_R &= i^2R = I^2R\sin^2(\omega_d t - \phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right] \\ &= 14.4\text{ W}.\end{aligned}$$

(e) Equal. $P_L + P_R + P_c = 44.1\text{W} - 17.0\text{ W} + 14.4\text{ W} = 41.5\text{ W} = P_g$.

61. **THINK** We have an ac generator connected to a “black box,” whose load is of the form of an *RLC* circuit. Given the functional forms of the emf and the current in the circuit, we can deduce the nature of the load.

EXPRESS In general, the driving emf and the current can be written as

$$\varepsilon(t) = \varepsilon_m \sin\omega_d t, \quad i(t) = I\sin(\omega_d t - \phi).$$

Thus, we have $\varepsilon_m = 75$ V, I = 1.20 A, and $\phi = -42°$ for this circuit. The power factor of the circuit is simply given by $\cos\phi$.

ANALYZE (a) With $\phi = -42.0°$, we obtain $\cos\phi = \cos(-42.0°) = 0.743$.

(b) Since the phase constant is negative, $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan\phi = (X_L - X_C)/R$. We have

$$\tan\phi = \tan(-42.0°) = -0.900,$$

a negative number. Therefore, $X_L - X_C$ is negative, which implies that $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance, X_L would be the same as X_C, then $\tan\phi$ would be zero, and ϕ would be zero as well. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance is zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}.$$

(i) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L, and C, then the value of the frequency would also be needed to compute the power factor.

LEARN The phase constant ϕ allows us to calculate the power factor and deduce the nature of the load in the circuit. In (f) we stated that the inductance may be set to zero. If there is an inductor, then its reactance must be smaller than the capacitive reactance, $X_L < X_C$.

62. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00\times10^3\,\text{V}.$$

63. **THINK** The transformer in this problem is a step-down transformer.

EXPRESS If N_p is the number of primary turns, and N_s is the number of secondary turns, then the step-down voltage in the secondary circuit is

$$V_s = V_p\left(\frac{N_s}{N_p}\right).$$

By Ohm's law, the current in the secondary circuit is given by $I_s = V_s / R_s$.

ANALYZE (a) The step-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) The current in the secondary is $I_s = \dfrac{V_s}{R_s} = \dfrac{2.4\,\text{V}}{15\,\Omega} = 0.16\,\text{A}.$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times 10^{-3}\,\text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}.$

LEARN In a transformer, the voltages and currents in the secondary circuit are related to that in the primary circuit by

$$V_s = V_p\left(\frac{N_s}{N_p}\right), \qquad I_s = I_p\left(\frac{N_p}{N_s}\right).$$

64. For step-up transformer:

(a) The smallest value of the ratio V_s / V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: V_{13}/V_{23} = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: V_{23}/V_{13} = 800/200 = 4.00.

(c) The largest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: V_{13}/V_{12} = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

65. (a) The rms current in the cable is $I_{\text{rms}} = P / V_t = 250 \times 10^3 \text{W} / \left(80 \times 10^3 \text{ V}\right) = 3.125 \text{A}$. Therefore, the rms voltage drop is $\Delta V = I_{\text{rms}} R = (3.125 \text{A})(2)(0.30 \,\Omega) = 1.9 \text{ V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2 R = (3.125 \text{A})(2)(0.60 \,\Omega) = 5.9 \text{W}$.

(c) Now $I_{\text{rms}} = 250 \times 10^3 \text{ W} / \left(8.0 \times 10^3 \text{ V}\right) = 31.25 \text{A}$, so $\Delta V = (31.25 \text{A})(0.60 \Omega) = 19 \text{V}$.

(d) $P_d = (3.125 \text{A})^2 (0.60 \,\Omega) = 5.9 \times 10^2 \text{ W}$.

(e) $I_{\text{rms}} = 250 \times 10^3 \text{ W} / \left(0.80 \times 10^3 \text{ V}\right) = 312.5 \text{ A}$, so $\Delta V = (312.5 \text{A})(0.60 \,\Omega) = 1.9 \times 10^2 \text{ V}$.

(f) $P_d = (312.5 \text{A})^2 (0.60 \ \Omega) = 5.9 \times 10^4 \text{W}$.

66. (a) The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings.

(b) If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = \left(N_p / N_s\right) I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R.$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\varepsilon_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\varepsilon_{\text{rms}}}{r + \left(N_p / N_s\right)^2 R}$$

where Eq. 31-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\varepsilon^2 (N_p / N_s)^2 R}{\left[r + (N_p / N_s)^2 R\right]^2}.$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\varepsilon^2 R x}{(r + xR)^2},$$

and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\varepsilon^2 R(r - xR)}{(r + xR)^3}.$$

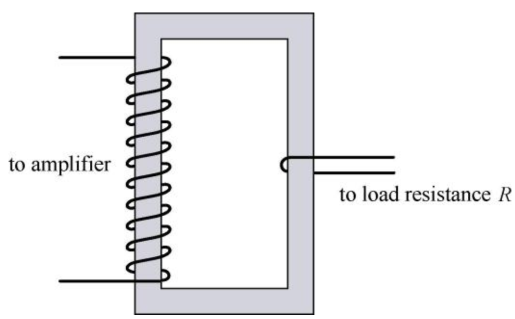
This is zero for

$$x = r/R = (1000\,\Omega)/(10\,\Omega) = 100.$$

We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for

$$N_p / N_s = \sqrt{x} = 10.$$

The diagram that follows is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



67. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/\left[4(350\,\text{rad/s})\right] = 6.73\times10^{-3}$ s.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/\left[4(350\,\text{rad/s})\right] = 2.24\times10^{-3}$ s.

(c) Since i leads ε in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \varepsilon_m / I$:

$$C = \frac{I}{\varepsilon_m \omega} = \frac{6.20\times10^{-3}\text{ A}}{(30.0\text{ V})(350\,\text{rad/s})} = 5.90\times10^{-5}\text{ F}.$$

68. (a) We observe that ω_d = 12566 rad/s. Consequently, X_L = 754 Ω and X_C = 199 Ω. Hence, Eq. 31-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22 \text{ rad} .$$

(b) We find the current amplitude from Eq. 31-60:

$$I = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288\,\text{A} .$$

69. (a) Using $\omega = 2\pi f$, $X_L = \omega L$, $X_C = 1/\omega C$ and $\tan(\phi) = (X_L - X_C)/R$, we find

$$\phi = \tan^{-1}[(16.022 - 33.157)/40.0] = -0.40473 \approx -0.405 \text{ rad}.$$

(b) Equation 31-63 gives $I = 120/\sqrt{40^2 + (16\text{-}33)^2} = 2.7576 \approx 2.76$ A.

(c) $X_C > X_L \Rightarrow$ capacitive.

70. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^3\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^3\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\left(4.60\times10^3\,\text{Hz}\right)\left(1.30\times10^3\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus,

$$X_L = 2(1.30\times10^3\ \Omega) = 2.60\times10^3\ \Omega .$$

(d) $X_C = 1.30\times10^3\ \Omega/2 = 6.50\times10^2\ \Omega$.

71. (a) The impedance is Z = (80.0 V)/(1.25 A) = 64.0 Ω.

(b) We can write $\cos\phi = R/Z$. Therefore,

$$R = (64.0\ \Omega)\cos(0.650 \text{ rad}) = 50.9\ \Omega.$$

(c) Since the current leads the emf, the circuit is capacitive.

72. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L / 1.50)}{(V_L / 2.00)}\right)$$

which becomes $\tan^{-1}(2/3) = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60, we have

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(9.98\text{ V})^2 + (20.0\text{ V} - 13.3\text{ V})^2} = 12.0\text{ V}.$$

73. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu$H.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{max} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{max} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{max}} = 82.2$ nC.

74. (a) Equation 31-4 directly gives $1/\sqrt{LC} \approx 5.77\times10^3$ rad/s.

(b) Equation 16-5 then yields $T = 2\pi/\omega = 1.09$ ms.

(c) Although we do not show the graph here, we describe it: it is a cosine curve with amplitude 200 μC and period given in part (b).

75. (a) The impedance is $Z = \dfrac{\varepsilon_m}{I} = \dfrac{125\text{ V}}{3.20\text{ A}} = 39.1\ \Omega.$

(b) From $V_R = IR = \varepsilon_m \cos\phi$, we get

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(125\text{ V})\cos(0.982\text{ rad})}{3.20\text{ A}} = 21.7\ \Omega.$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\text{ rad})$, we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

76. (a) Equation 31-39 gives $f = \omega/2\pi = (2\pi C X_C)^{-1} = 8.84$ kHz.

(b) Because of its inverse relationship with frequency, the reactance will go down by a factor of 2 when f increases by a factor of 2. The answer is $X_C = 6.00\ \Omega$.

77. **THINK** The three-phase generator has three ac voltages that are 120° out of phase with each other.

EXPRESS To calculate the potential difference between any two wires, we use the following trigonometric identity:

$$\sin\alpha - \sin\beta = 2\sin\left[(\alpha-\beta)/2\right]\cos\left[(\alpha+\beta)/2\right],$$

where α and β are any two angles.

ANALYZE (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}A\cos(\omega_d t - 60°)$$

where $\sin 60° = \sqrt{3}/2$. Similarly,

$$\begin{aligned}\Delta V_{13} &= A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 120°)\end{aligned}$$

and

$$\begin{aligned}\Delta V_{23} &= A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 180°).\end{aligned}$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

LEARN A three-phase generator provides a smoother flow of power than a single-phase generator.

78. (a) The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\ \text{hp})(746\ \text{W/hp})}{(0.650\ \text{A})^2} = 177\ \Omega.$$

(b) This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\text{rms}}^2 R$ would not give $P_{\text{mechanical}}$ but rather the rate of energy loss due to thermal dissipation.

79. **THINK** The total energy in the LC circuit is the sum of electrical energy stored in the capacitor, and the magnetic energy stored in the inductor. Energy is conserved.

EXPRESS Let U_E be the electrical energy in the capacitor and U_B be the magnetic energy in the inductor. The total energy is $U = U_E + U_B$. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor.

ANALYZE (a) Thus,

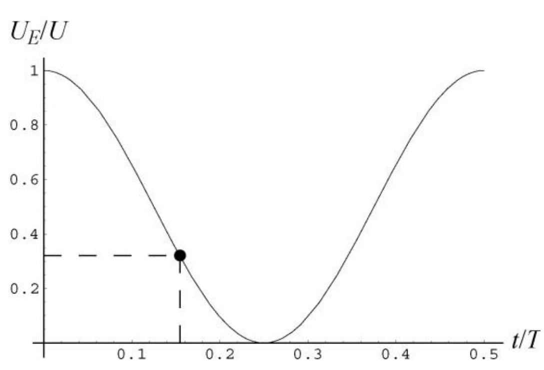
$$\frac{Q^2}{2C} = \frac{3.00q^2}{2C} \quad \Rightarrow \quad q = \frac{Q}{\sqrt{3.00}} = 0.577Q.$$

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t = 0.557$, or $\omega t = 0.955$ rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$, or $t/T = 0.152$.

LEARN The fraction of total energy that is of electrical nature at a given time t is given by

$$\frac{U_E}{U} = \frac{(Q^2/2C)\cos^2\omega t}{Q^2/2C} = \cos^2\omega t = \cos^2\left(\frac{2\pi t}{T}\right).$$

A plot of U_E/U as a function of t/T is given below.



From the plot, we see that $U_E/U = 1/3$ at $t/T = 0.152$.

80. (a) The reactances are as follows:

$$X_L = 2\pi f_d L = 2\pi(400\text{ Hz})(0.0242\text{ H}) = 60.82\ \Omega$$
$$X_C = (2\pi f_d C)^{-1} = [2\pi(400\text{ Hz})(1.21\times10^{-5}\text{ F})]^{-1} = 32.88\ \Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0\ \Omega)^2 + (60.82\ \Omega - 32.88\ \Omega)^2} = 34.36\ \Omega\,.$$

With $\varepsilon = 90.0\text{ V},$ we have

$$I = \frac{\varepsilon}{Z} = \frac{90.0\text{ V}}{34.36\ \Omega} = 2.62\text{ A} \ \Rightarrow\ I_{\text{rms}} = \frac{I}{\sqrt{2}} = \frac{2.62\text{ A}}{\sqrt{2}} = 1.85\text{ A}\,.$$

Therefore, the rms potential difference across the resistor is $V_{R\text{ rms}} = I_{\text{rms}} R = 37.0$ V.

(b) Across the capacitor, the rms potential difference is $V_{C\text{ rms}} = I_{\text{rms}} X_C = 60.9$ V.

(c) Similarly, across the inductor, the rms potential difference is $V_{L\text{ rms}} = I_{\text{rms}} X_L = 113$ V.

(d) The average rate of energy dissipation is $P_{\text{avg}} = (I_{\text{rms}})^2 R = 68.6$ W.

81. **THINK** Since the current lags the generator emf, the phase angle is positive and the circuit is more inductive than capacitive.

EXPRESS Let V_L be the maximum potential difference across the inductor, V_C be the maximum potential difference across the capacitor, and V_R be the maximum potential difference across the resistor. The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right).$$

The maximum emf is related to the current amplitude by $\varepsilon_m = IZ$, where Z is the impedance.

ANALYZE (a) With $V_C = V_L/2.00$ and $V_R = V_L/2.00,$ we find the phase constant to be

$$\phi = \tan^{-1}\left(\frac{V_L - V_L/2.00}{V_L/2.00}\right) = \tan^{-1}\left(1.00\right) = 45.0°.$$

(b) The resistance is related to the impedance by $R = Z\cos\phi.$ Thus,

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(30.0\text{ V})(\cos 45°)}{300\times10^{-3}\text{ A}} = 70.7\ \Omega.$$

LEARN With R and I known, the inductive and capacitive reactances are, respectively, $X_L = 2.00R = 141\ \Omega,$ and $X_C = R = 70.7\ \Omega$. Similarly, the impedance of the circuit is

$$Z = \frac{\varepsilon_m}{I} = (30.0 \text{ V})/(300 \times 10^{-3} \text{ A}) = 100\ \Omega .$$

82. From $U_{\max} = \frac{1}{2}LI^2$ we get I = 0.115 A.

83. From Eq. 31-4 we get $f = 1/2\pi\sqrt{LC}$ = 1.84 kHz.

84. (a) With a phase constant of 45º the (net) reactance must equal the resistance in the circuit, which means the circuit impedance becomes

$$Z = R\sqrt{2} \Rightarrow R = Z/\sqrt{2} = 707\ \Omega.$$

(b) Since f = 8000 Hz, then $\omega_d = 2\pi(8000)$ rad/s. The net reactance (which, as observed, must equal the resistance) is therefore

$$X_L - X_C = \omega_d L - (\omega_d C)^{-1} = 707\ \Omega.$$

We are also told that the resonance frequency is 6000 Hz, which (by Eq. 31-4) means

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 (6000 \text{ Hz})^2 L}.$$

Substituting this for C in our previous expression (for the net reactance) we obtain an equation that can be solved for the self-inductance. Our result is L = 32.2 mH.

(c) $C = ((2\pi(6000))^2 L)^{-1}$ = 21.9 nF.

85. **THINK** The current and the charge undergo sinusoidal oscillations in the LC circuit. Energy is conserved.

EXPRESS The angular frequency oscillation is related to the capacitance C and inductance L by $\omega = 1/\sqrt{LC}$. The electrical energy and magnetic energy in the circuit as a function of time are given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

The maximum value of U_E is $Q^2/2C$, which is the total energy in the circuit, U. Similarly, the maximum value of U_B is also $Q^2/2C$, which can also be written as $LI^2/2$ using $I = \omega Q$.

ANALYZE (a) Using the fact that $\omega = 2\pi f$, the inductance is

$$L = \frac{1}{\omega^2 C} = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10.4\times10^3\text{ Hz}\right)^2 \left(340\times10^{-6}\text{ F}\right)} = 6.89\times10^{-7}\text{ H.}$$

(b) The total energy may be calculated from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(6.89\times10^{-7}\text{ H}\right)\left(7.20\times10^{-3}\text{ A}\right)^2 = 1.79\times10^{-11}\text{ J.}$$

(c) We solve for Q from $U = \frac{1}{2}Q^2/C$:

$$Q = \sqrt{2CU} = \sqrt{2\left(340\times10^{-6}\text{ F}\right)\left(1.79\times10^{-11}\text{ J}\right)} = 1.10\times10^{-7}\text{ C.}$$

LEARN Figure 31-4 of the textbook illustrates the oscillations of electrical and magnetic energies. The total energy $U = U_E + U_B = Q^2/2C$ remains constant. When U_E is maximum, U_B is zero, and vice versa.

86. From Eq. 31-60, we have $(220\text{ V}/3.00\text{ A})^2 = R^2 + X_L^2 \Rightarrow X_L = 69.3\,\Omega$.

87. When the switch is open, we have a series *LRC* circuit involving just the one capacitor near the upper right corner. Equation 31-65 leads to

$$\frac{\omega_d L - \dfrac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) = -\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \dfrac{1}{2\omega_d C}}{R} = \tan\phi_1 = \tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an *LC* circuit with current amplitude

$$I_2 = \frac{\varepsilon_m}{Z_{LC}} = \frac{\varepsilon_m}{\sqrt{\left(\omega_d L - \dfrac{1}{\omega_d C}\right)^2}} = \frac{\varepsilon_m}{\dfrac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $(\omega_d C)^{-1} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for L, R and C from the three equations above, and the results are as follows:

(a) $R = \dfrac{-\varepsilon_m}{I_2 \tan\phi_o} = \dfrac{-120\text{V}}{(2.00\,\text{A})\tan(-20.0°)} = 165\,\Omega$,

(b) $L = \dfrac{\varepsilon_m}{\omega_d I_2}\left(1-2\dfrac{\tan\phi_1}{\tan\phi_o}\right) = \dfrac{120\,\text{V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1-2\dfrac{\tan 10.0°}{\tan(-20.0°)}\right) = 0.313\text{ H}$,

(c) and

$$C = \frac{I_2}{2\omega_d\varepsilon_m\left(1-\tan\phi_1/\tan\phi_0\right)} = \frac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1-\tan 10.0°/\tan(-20.0°)\right)}$$
$$= 1.49\times10^{-5}\text{ F.}$$

88. (a) Eqs. 31-4 and 31-14 lead to

$$Q = \frac{1}{\omega} = I\sqrt{LC} = 1.27\times10^{-6}\text{C}.$$

(b) We choose the phase constant in Eq. 31-12 to be $\phi = -\pi/2$, so that $i_0 = I$ in Eq. 31-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2 .$$

Differentiating and using the fact that 2 sin θ cos θ = sin 2θ, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\omega\sin 2\omega t .$$

We find the maximum value occurs whenever sin $2\omega t = 1$, which leads (with n = *odd* integer) to

$$t = \frac{1}{2\omega}\frac{n\pi}{2} = \frac{n\pi}{4\omega} = \frac{n\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\text{s},\ 2.49\times10^{-4}\text{s},\ \ldots .$$

The earliest time is $t = 8.31\times10^{-5}$s.

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\frac{I}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\,\text{J/s}\,.$$

89. **THINK** In this problem, we demonstrate that in a driven *RLC* circuit, the energies stored in the capacitor and the inductor stay constant; however, energy is transferred from the driving emf device to the resistor.

EXPRESS The energy stored in the capacitor is given by $U_E = q^2/2C$. Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}Li^2$. The rate of energy supply by the driving emf device is $P_\varepsilon = i\varepsilon$, where $i = I\sin(\omega_d - \phi)$ and $\varepsilon = \varepsilon_m \sin\omega_d t$. The rate with which energy dissipates in the resistor is $P_R = i^2 R$.

ANALYZE (a) Since the charge q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Since the current i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the emf device over one cycle is

$$\begin{aligned}U_\varepsilon &= \int_0^T P_\varepsilon dt = I\varepsilon_m\int_0^T \sin(\omega_d t-\phi)\sin(\omega_d t)dt = I\varepsilon_m\int_0^T [\sin\omega_d t\cos\phi - \cos\omega_d t\sin\phi]\sin(\omega_d t)dt \\ &= \frac{T}{2}I\varepsilon_m\cos\phi,\end{aligned}$$

where we have used

$$\int_0^T \sin^2(\omega_d t)dt = \frac{T}{2}, \qquad \int_0^T \sin(\omega_d t)\cos(\omega_d t)dt = 0.$$

(d) Over one cycle, the energy dissipated in the resistor is

$$U_R = \int_0^T P_R dt = I^2R\int_0^T \sin^2(\omega_d t-\phi)dt = \frac{T}{2}I^2R.$$

(e) Since $\varepsilon_m I\cos\phi = \varepsilon_m I\left(V_R/\varepsilon_m\right) = \varepsilon_m I\left(IR/\varepsilon_m\right) = I^2R$, the two quantities are indeed the same.

LEARN In solving for (c) and (d), we could have used Eqs. 31-74 and 31-71: By doing so, we find the energy supplied by the generator to be

$$P_{\text{avg}}T = \left(I_{\text{rms}}\varepsilon_{\text{rms}}\cos\phi\right)T = \left(\frac{1}{2}T\right)\varepsilon_m I\cos\phi$$

where we substitute $I_{\text{rms}} = I/\sqrt{2}$ and $\varepsilon_{\text{rms}} = \varepsilon_m/\sqrt{2}$. Similarly, the energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\, T = \left(I_{\text{rms}} V_R\right) T = I_{\text{rms}}\left(I_{\text{rms}} R\right) T = \left(\frac{1}{2}T\right) I^2 R.$$

The same results are obtained without any integration.

90. From Eq. 31-4, we have $C = (\omega^2 L)^{-1} = ((2\pi f)^2 L)^{-1} = 1.59\ \mu\text{F}$.

91. Resonance occurs when the inductive reactance equals the capacitive reactance. Reactances of a certain type add (in series) just like resistances. Thus, since the resonance ω values are the same for both circuits, we have for each circuit:

$$\omega L_1 = \frac{1}{\omega C_1}, \qquad \omega L_2 = \frac{1}{\omega C_2}$$

and adding these equations we find

$$\omega\left(L_1 + L_2\right) = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

Since $L_{\text{eq}} = L_1 + L_2$ and $C_{\text{eq}}^{-1} = (C_1^{-1} + C_2^{-1})$,

$$\omega L_{\text{eq}} = \frac{1}{\omega C_{\text{eq}}} \quad \Rightarrow \quad \text{resonance in the combined circuit.}$$

92. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C \tau_L}$.

93. (a) We note that we obtain the maximum value in Eq. 31-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417\ \text{s}$$

or 4.17 ms. The result is $\varepsilon_m \sin(\pi/2) = \varepsilon_m \sin(90°) = 36.0\ \text{V}$.

(b) At $t = 4.17$ ms, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90° - (-24.3°)) = (0.164\,\text{A})\cos(24.3°)$$
$$= 0.1495\,\text{A} \approx 0.150\ \text{A}.$$

Ohm's law directly gives

$$v_R = iR = (0.1495\,\text{A})(200\,\Omega) = 29.9\,\text{V}.$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at t = 4.17 ms, we obtain

$$v_C = I\sin(90° - (-24.3°) - 90°)X_C = IX_C\sin(24.3°) = (0.164\,\text{A})(177\,\Omega)\sin(24.3°)$$
$$= 11.9\,\text{V}.$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at t = 4.17 ms, we find

$$v_L = I\sin(90° - (-24.3°) + 90°)X_L = -IX_L\sin(24.3°) = -(0.164\,\text{A})(86.7\,\Omega)\sin(24.3°)$$
$$= -5.85\,\text{V}.$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).